Vol. XVII
No. 3

PSYCHOLOGICAL REVIEW PUBLICATIONS

June, 1914
Whole No. 69

THE
Psychological Monographs

EDITED BY

UNIVERSITY OF IOWA STUDIES IN PSYCHOLOGY

No. VI

Edited
BY
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PSYCHOLOGICAL REVIEW COMPANY
PRINCETON, N. J.
AND LANCASTER, PA.

Agents: G. E. STECHERT & CO., London (2 Star Yard, Carey St., W. C.);
Leipzig (Hospital St., 10); Paris (76 rue de Rennes)

PRINCETON UNIVERSITY PRESS

THE TONOSCOPE

BY

CARL E. SEASHORE

In experimental psychology instruments have as a rule been designed to meet immediate needs, and have usually been described incidentally in reporting the results of psychological investigations. Much research has been wasted because done with untried apparatus. In fact most of our instruments are in a crude condition; and many fields of investigation lie untouched for want of measuring instruments. It is a sign of a higher stage in the science that the most essential psychological instruments are now being subjected to investigation apart from the specific pending psychological use. Only in this way can we properly develop instruments and standardize the technique of manipulation. I therefore take pleasure in presenting the description of an instrument and its use, as an object worth while in itself.

The tonoscope, Fig. 1, works on the principle of stroboscopic vision, the principle of moving pictures. Auditory vibrations of air, caused by voice or musical instrument, are converted directly and instantaneously into visual configurations on a screen, and the vibration frequency which denotes the pitch of the tone may be seen in plain figures on a scale. This enables us to measure the pitch of any tone by direct inspection while singing, speaking, or playing under normal conditions. The ability to do this opens up countless problems in the psychology of tonal expression.

There is a contrivance by which the vibration of the voice mechanically raises and lowers a flame for each sound wave. The oscillation of the flame results in corresponding exposures on the screen which it illuminates. The vibration being rapid, the retinal lag produces the effect of continuous vision, although the duration of the illumination for each vibration is very short in comparison with the corresponding period of non-illumination. In moving pictures it is well known that, if we have successive pictures which are alike thrown on the screen in the same place and in rapid succession they form one continuous picture which stands out clear and still. This is the principle here employed. The revolving screen,

rotating at the rate of one revolution per second, carries rows of dots, regularly spaced but varying in number for each row. When a tone is sounded, the row which has the dot-frequency that corresponds to the vibration-frequency of the tone will stand still and be clear while all other dots move and tend to blur. Each row runs under a number on the scale. The row which stands still, therefore, points to a number which designates the pitch of the tone. The screen contains a sufficient number of rows of dots, varying in number, to correspond directly, or by multiple, to all tones within the range of the voice. To see the pitch of the tone one has therefore only to see the number of the line that stands still.

Earlier models of the tonoscope have been described in a previous volume of these Studies (1); also in the Musician (2). Such radical changes have, however, been made since then that we are now dealing with an instrument very much modified and extended in its usefulness.[1] The present instrument is not the result of the work of one man but of many of whom, aside from those who have developed the principle of stroboscopic vision in physics, I desire to mention particularly Dr. E. W. Scripture who designed the first laboratory exercise using this principle in psychology (3); Dr. C. F. Lorenz (4), to whose ingenuity and most generous coöperation we owe the synchronous motor and the plan of using the selenium cell with the siren; Mr. E. W. Bechly, and Professor E. A. Jenner, who made the first tests with the tonoscope in determining its value for use in the musical conservatory (5); and Dr. Walter R. Miles (6), who has standardized procedure for various problems in the measurement of singers with this instrument.

Instead of giving merely a description of the commercial form of the instrument, I shall attempt to suggest, in a semi-technical way, its various possible forms on the basis of actual experiment from the laboratory point of view. The essential features which must be discussed in turn in the description are the speed regulation, the screen, the dot grouping, the sensitive light and sound transmitter, and the siren.

The synchronous motor.—The validity of stroboscopic frequency measurements depends upon the accuracy with which the movement

[1] Fig. 2, showing the 1902 model, is here reproduced because it shows the bare elements of construction better than they can be seen in the present encased model, as shown in Fig. 1.

FIG. 1. THE TONOSCOPE

(The instrument seen at the right is the acousticon.)

Fig. 2. Early Model of Tonoscope. (1902)

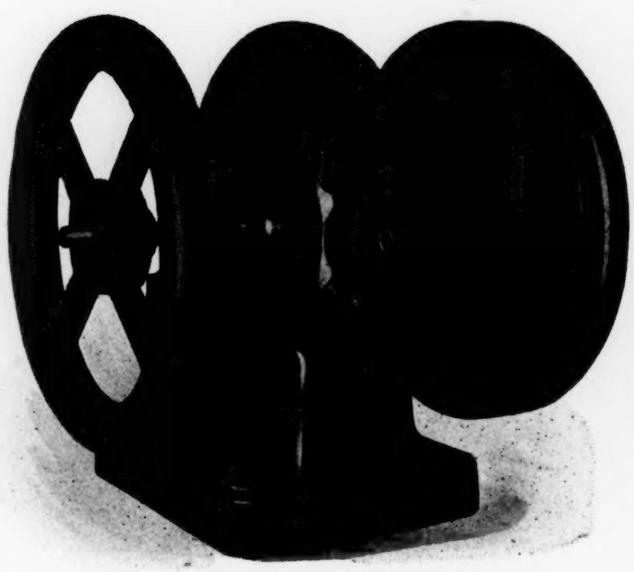

Fig. 3. The Synchronous Motor

of the exposed object is controlled. The method employed in the early models proved entirely too laborious and inconvenient.[2] The final solution was found in the use of a synchronous motor which drives the drum (screen) at a regulated speed. This motor (Fig. 3) works on the principle that, if a regularly interrupted current be sent through a multipolar field, and the needed initial momentum be given to the revolving multipolar armature, each closing of the circuit will synchronize with poles always in a corresponding position of approach, and the momentary pull will be sufficient to continue the rotation until the next pull occurs, at the next approach of poles. This is the principle of La Cour's "phonic wheel" (7), a principle also employed by Lord Rayleigh (8).

A motor of this type is mounted on the main shaft of the tonoscope drum.[3] The drum, serving as a balance wheel and being connected to the motor by a coil spring, furnishes the right degree of inertia and flexibility in the transfer of the pull.[4]

A large 10 v.d. tuning fork is used as an interrupter. It is energized by primary cells, and is encased in a box which is kept out of the way in a closet so that no noise shall come from it. A 16 c.p. lamp used for resistance in the motor circuit, is mounted

[2] In the early models the drum was driven by an ordinary direct current motor. An assistant at the back side of the drum observed and recorded the actual speed at the time of every reading. This was done by means of the stroboscopic effect of the intermittent light in a vacuum tube in circuit with a standard tuning fork. Every reading had to be adjusted with reference to an elaborate table of corrections calculated for different steps of variation from the true speed.

[3] In the early stages the motor was placed outside of the drum and the power was transferred by belt. This was neither as convenient nor as reliable as the present method, but it left the motor available for other laboratory purposes.

There is a great demand for such a synchronous motor in laboratories because, with constant speed and power, many of the laboratory problems are readily solved and we gain a higher degree of accuracy than can be obtained by any other form of electric motor or by kymograph clockworks. As the speed may be transformed up or down, this motor will take the place of many cruder devices in the physics and psychology laboratories for securing constant motion, marking intervals, measuring time, making exposures, etc.

[4] In the type of motor here needed this principle has not before been successfully developed for practical use. The hitch lies in the jerky nature of the pull. By attaching a flywheel to the axle by a flexible connector we get a flexible moment of inertia which solves the problem.

between the prongs of the fork and proves a convenient means of keeping it at a sufficiently constant temperature, the temperature being that to which the fork is raised by the heat of the lamp within the box.[5]

A 110 volt direct current is completed through the motor and a mercury contact interrupted by the fork. The current is reduced by the lamp resistance. The make-and-break is short-circuited with a condenser to avoid forming of an arc. A large amplitude of the fork, fully 10 mm., also helps in preventing the tendency to arc.

A rheostat inside the tonoscope case, with a switch on the surface, serves for the adjustment of current, as there may be fluctuations in the supply main. A small detachable crank for starting fits the end of the main shaft which comes out flush with the edge of the case on the side. To start the tonoscope one has only to start the fork, give the drum a turn up to approximately one revolution per second and close the switch. Once started, the instrument will run indefinitely and there is no care or distraction in the running of it.

The screen.—The stroboscopic screen is formed by mounting a sheet of aluminum in the shape of a drum over a heavy balance wheel. A section of this drum is seen through the opening on the front of the case (Fig. 1). This screen is 50 cm. wide and has a circumference of 242 cm. The balance wheel is heavily mounted on ball-bearings resting on a heavy iron frame. The whole instrument is enclosed in an oak case with doors on every facet.

The size of the drum is determined by the minimum area for the legible distribution of 18,500 markings, or stroboscopic dots. In the present screen the dots are bored holes, three and one-half mm. in diameter. The inside of the drum being dark, the holes show up clearly as black spots on the light aluminum surface. These holes are spaced with the highest mechanical accuracy and are arranged in 110 parallel rows, each completing the circumference of the drum in uniform spacings for each row (Cf. arrangement of dots in screen in Fig. 2). One row has 110 dots and the dot frequency in the remaining rows increases by one dot for each row up to and including 219. Thus we get frequencies to correspond

[5] Like the motor, this fork becomes a sort of "universal" apparatus in the laboratory. Being standardized and always connected up, it becomes the most convenient means for timing purposes. It is notably serviceable in connection with a multiple recorder which makes impressions on ticker tape.

to each intergral vibration-frequency in an octave of tones, the octave of 110 v.d. to 220 v.d. This is approximately the octave from *A* up to the *a* below middle ć

This octave was chosen after much experimenting as being the most serviceable, all factors taken into consideration. Within this octave the tones are read directly, and above and below it they are read by multiples. The number of holes in each row is shown in plain large figures on the scale. When the drum revolves the row formation stands out clearly and each row points to a number.

The "framing-effect".—As may be seen in Fig. 1, there is an upper and lower scale, one on each edge of the shield. It is necessary that the holes should be large enough to be easily legible under the prevailing conditions of fusion, and also that they shall be widely enough spaced in both directions to be easily read. It is also essential that a single little sensitive flame shall light up the whole exposed surface of the screen. This forces upon us the difficult problem of securing compactness. In the early models no other solution was seen than to restrict the instrument to a part of an octave, but a unique solution was finally found. This consists in alternating rows of widely different frequency as may be seen by observing the actual numbers of the adjacent lines on the screen in reading alternately on the upper and lower scale. The numbers on the upper scale are consecutive from 110 to 164, and the numbers on the lower scale from 165 to 219, the rows reading on one scale alternating with those reading on the other. To illustrate, if rows 150 and 151 were adjacent they would need to be separated by a wide space in order to be differentiated, because their movements are so nearly alike, but if they are separated by another row, for example the 205, the differentiation between the two original rows becomes clearer and the spaces between them may be materially reduced, for, when rows 150 and 151 stand approximately still and their individual dots stand out clear and distinct, row 205 moves so swiftly that it forms one continuous line or gray streak which has a most serviceable separating or framing effect for the adjacent rows. When the rows are arranged as shown on the scale, this differentiating, or framing effect will operate for each and every row that may be standing out for reading. This contrivance makes it possible to reduce the screen to about one-third of the size otherwise required, and still makes the reading more legible than it would

have been on a screen three times as large without some contrivance like this.

The sensitive light and sound transmitter.—A fundamental requirement in this principle of measurement is that the light shall be made intermittent through the action of sound waves. This may be accomplished in various ways. In the simplest arrangement an ordinary manometric capsule is used and the singer holds a funnel before his mouth in such a way as to effectively collect the vibrations. Acetylene gas supplied by a motorcycle tank is used for this sensitive flame. We have not yet determined the most effective form of capsule or the maximum upper and lower limit of its vibration response, but have found that this varies with numerous conditions, such as the vibration frequency, the volume, the smoothness, etc. of the tone. This capsule may be used in recording from such musical instruments as send a fairly concentrated volume of waves in one direction, such as tuning forks, wind instruments, reed instruments, and the siren. With all these it is, however, advantageous to use a Helmholtz or a Koenig resonator as a selector although it is not necessary in all cases. As a rule the shorter the speaking tube or horn, the less danger there is of interference in the sound waves.

While this mechanical transmission through a manometric capsule is for most purposes the simplest means, and is entirely satisfactory, especially in singing, we have electrical devices that have distinct advantages. The receiver of a microphone may be converted into a manometric capsule by building a gas chamber on the ear side and supplying it with a gas inlet and a jet nipple. The vibration of the receiver membrane controls the gas flame in the same way as in an ordinary capsule. The microphone transmitter is used with this as in ordinary speaking. The best type of commercial instrument that may be readily adapted for this purpose is the phonette or the acousticon made by the General Acoustic Company, New York. The acousticon known as type D seems to be the most serviceable.

While the electrical apparatus may be a little more delicate to handle it has the advantage that it is more sensitive and can be used for the recording of a tone which would not be strong enough to register in any other way. It also makes it possible to set this apparatus in front of the singer so that he may sing for a record

without being aware that a recording instrument is present in the room. The singer may be isolated in a quiet room or in familiar surroundings, in order not to be disturbed by the presence of another person and the main instrument. The measurement may even be made at any long distance covered by telephone connection, as all that is necessary is to put the microphone transmitter in front of the singer at one end of the telephone line, and connect it with the microphone receiver on the tonoscope at the other end.

When sound vibrations are strong enough completely to make and break the circuit, the ordinary telephone receiver may be used as a capsule in the manner just described. On certain musical instruments a mechanical interrupter resting on the resonating chamber of the instrument, for example, a violin may be used.

Under the same circumstances a vacuum tube may be used in place of the gas-flame capsule. The intermittent light is then caused by the interruption of the current in the primary circuit of an induction coil which has the vacuum tube in the secondary circuit.

If two simultaneous records are desired, one record may be taken on each side of the tonoscope. Indeed, four records may be taken simultaneously by using both the upper and the lower facets on the back and the face of the drum, there being doors on the case for this purpose.

The stroboscopic reading requires fairly complete darkness. To avoid darkening the room a hood (not shown in the figure) has been built to fit over the reading surface of the tonoscope. This hood forms a dark chamber and the inner surface, being bright, serves as an excellent reflector for the light. For intensive reading at a given point on the scale, a small sliding hood is made on the same principle. It has the advantage of centering the light upon the point of reading in the scale. A reflecting mirror (not shown in the figure) is used to distribute the light over the visible portion of the screen for ordinary use.

The siren.—For certain purposes it may be desirable to get a key-note or a standard pitch from the tonoscope itself. The dots on the screen were therefore made as holes. At the base of the front of the case is a siren blow-pipe supported on a horizontal carrier so that it may glide freely over the surface of the drum, while a pointer indicates on the scale just what hole-frequency is blown. This makes it possible to produce as siren tones all the tones from 110 to 219 by one-vibration steps, excepting those rows of holes

which happen to be closed by the contact with the balance wheel. The siren blow-pipe is connected with a compressed air tank or it may be blown directly by a mouth tube. A speaking tube is used to carry the sound to the observer's ear and the opening and closing of this tube by means of a clamp starts and stops the sound.

The siren tone is not a tone of good quality. But a beautiful tone may be produced by projecting a beam of light through the holes in the screen upon a selenium cell in circuit with a telephone receiver. It so happens that the fluctuation in the resistance of the selenium cell takes approximately the form of a sine curve, and that produces a tone of most remarkably clear and smooth timbre in the receiver. One may, however, use any sort of instrument for giving the standard tone, as the pitch of the instrument can be read off on the tonoscope at any moment.

The reading.—Although the reading is simple and direct, it is necessary to mention some of the underlying principles. The first task is to see which row stands still, or the nearest still. This row indicates the desired record and will be seen irresistibly the moment the tone is produced, because all other rows are blurred or in rapid motion. Having identified the line which stands still we must next know within what octave the tone lies. If, for example, row 128 stands still this may represent a tone of 64, 128, 256, 512 vibrations, or even higher. Now from 110 v.d. to 219 v.d. the correspondence is direct and the dots appear as actually spaced on the drum within an octave above this, the dots double in number and therefore stand only half as far apart; and, within the second octave above, they quadruple in number. It is therefore easy to see instantly from the spacing of the dots within which octave one is reading. If the spaces in row 128, *e.g.*, are one-half of the original the tone is 256 v.d.; if they are only one-fourth of the original steps, the tone is 512 v.d., etc.

But in fine reading we deal with fractions of vibrations. If instead of one row standing still, two rows move slowly in the opposite direction, the tone lies between these, and the fraction is determined by the relative rate of movement of the two rows. In a very accurate recording of instruments this may be done to a high degree of certainly by timing the movement with a stop watch over a considerable period of time. To do this we observe how many dot spaces are moving up or down and apply this general rule: if the ascend-

ing row has been counted, *add* the fraction of dot space per second; or, if the descending row has been counted, *subtract* it from the integral number of the row observed. It is best to count the faster moving of the two rows.

Sources of error.—There can be no time-error in the transformation of the sound wave into an illumination wave; they must synchronize, since one is the direct cause of the other. There can be no progressive change in the speed of the motor because if the motor does not run in step with the fork it must stop. The only possible error on the physical side lies in a tendency to pendular oscillation of the drum which may show a tendency to "hunt" when starting or when the current is too weak. By allowing a minute for the "finding" immediately after starting, and by securing a right adjustment of the strength of the current, this hunting movement may be reduced to an inappreciable or negligible quantity. The presence or absence of this source of error may, of course, be ascertained at any moment by registering the tone from a standard fork as a control. The limit of accuracy in the registering of the apparatus is therefore set by the limit of constancy in the driving fork. This fork being carefully balanced, firmly mounted, and kept in fairly constant temperature, shows a very high degree of constancy and compares favorably with a standard 100 v.d. fork.

But the main source of error lies in the reading, particularly of high tones. This need, however, be a source of error only in rapid reading. By making the tone long enough to observe the rate of fractional movement, one may secure any desired degree of accuracy in reading, as, *e.g.*, in registering a tuning fork, by timing the fractional movements of the dots for a sufficiently long time. In brief, without giving numerical records, we may say that the limit of accuracy in the use of the instrument is really set by the limit of accuracy of a tuning fork, the driving fork. For actual tests of accuracy in reading see Miles (6).

The use of the tonoscope.—The tonoscope furnishes us the first ready and, at the same time, reliable means of measuring directly the pitch of a tone as sung, spoken, or played with a musical instrument. Heretofore, graphic recording has been the only reliable method. This has the merit of accuracy but is entirely too indirect and laborious to be of general use in practical work. As we have seen, it registers the tone as sung or played under natural

conditions, and the record is simultaneous with the tone. The scope of its usefulness is therefore very great. It furnishes us an approach to countless problems both in pure and applied psychology. The psychology of tonal expression is a field practically unworked as compared with the psychology of the appreciation of tone, largely because we have not before had any convenient means of measurement.

A few concrete illustrations from the laboratory may be cited. In standardizing the pitch discrimination test (9) it was found necessary to compare the relative reliability of available instruments. Tuning forks, string instruments, reed instruments, wind instruments, and sirens were all tested by direct registration upon the tonoscope. Temperature coefficients, air pressure coefficients, and resonance coefficients were worked out by the same mode of registration. While most of these measurements could have been made in other ways, the tonoscope proved at least a good, labor-saving device.

The settling of disputed questicns of pitch has been interesting. For example, there was a pitch discrepancy in the playing of the oboe and the French horn in a symphony orchestra. Each player was given an opportunity to register a specific tone in the tonoscope, and it was found that the oboe was playing consistently 1.5 v.d. flat. A vocal soloist had a tendency to flat relatively high notes. She observed the error and learned to make the right correction. A singer was practicing to eliminate an undesirable fluctuation of the pitch of the voice and was much helped in practicing before the tonoscope as before a mirror. In a recent article (10) I have described some measurements which at the present time could be made successfully only with the tonoscope.

There is a conspicuous place for the tonoscope in the musical conservatory. The ear of the singer or player is too generous because it seldom has any objective correction. The pupil persists in constant errors because there is no objective check on the ear. But the tonoscope does for the ear what the microscope does for the eye. It magnifies and objectifies to the eye, bringing out even small details of the pitch of the tone.

An actual experiment in training of the voice by the use of the instrument revealed among other facts the following (5): A group of six singers practicing daily for twelve days, part of the time

with the instrument and part of the time without it, showed that the average result of training with the instrument was superior to the average result of training without it, by forty-two per cent. in the ability to strike a tone, by fifty-five per cent. in the ability to sing musical intervals, and by twenty-six per cent. in the voluntary control of the voice in sharping or flatting; and the ability gained by virtue of the aid of the instrument was transferred in large part to ordinary singing.

The entire article by Dr. Miles in the present volume (6) should be regarded as supplementary to this description and particularly as furnishing illustrations of the use of the instrument. The general trend of usefulness of the tonoscope in the psychological laboratory, the musical conservatory, and other situations in which the registration of the pitch of tones is desirable may be indicated in a partial outline of measurements which may be made with it, as follows:

I. Striking a tone.
   1. Voluntary control of the pitch of a vocal or instrumental tone—the first and simplest test of ability to sound a given tone in true pitch.
   2. The effect on the pitch of a tone of conditions varied under control; *e.g.*, the character of the standard tone, the absolute pitch, the mode of tone production, distraction, practice, seeing the registration when producing the tone, information about previous errors, deliberate correction, etc.
   3. The pitch and reliability of the pitch of an instrument, as in tuning, testing, standardizing, and comparing instruments.

II. Sustaining a tone.
   1. Degree of accuracy or ability in holding a tone.
   2. Constant tendencies—sharp or flat.
   3. Artistic effects in singing a "constant" tone, as in pitch tremulo.
   4. Inflection of speech.

III. Minimal producible change.

   A fundamental measure of discriminative action in the voluntary control of the pitch of a tone—a sort of psychophysic foot-rule which serves the same purpose on the motor side as the minimal producible change serves on the sensory side; *e.g.*, as a unit of measurement in the study of individual difference, or the effect of any other controlled variable.

IV. Tonal transition.

1. The mode and the accuracy of attack and release of tones in singing, in playing, or in inflection of speech (a) in measuring precision, (b) in registering some artistic effect, and (c) in musical or oratorical training.
2. Testing of instrument; *e.g.*, in proving that the piston pitch pipe is unfit for the sounding of a key-note because the note is necessarily attacked by a gradual sliding up to the key.

V. Tonal intervals.

Accuracy in producing musical intervals; *e.g.*, single steps, the natural scale, the chromatic scale, melody as in singing or playing of an air; (a) for the purpose of psychological or aesthetic study, or, (b) as a means of training in the musical conservatory.

VI. Transcribing of speech or musical records from the phonograph or any other recording instrument.

There are thus two quite distinct fields in which the tonoscope in its present form may be employed, namely, in the research laboratory and in the practical work of the musical conservatory.

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ACCURACY OF THE VOICE IN SIMPLE PITCH SINGING

BY

WALTER R. MILES

Contents

The experiments here reported deal with two phases of simple pitch singing: (1) the ability of the voice to reproduce the pitch of a tone, and (2) the ability to make faint shadings in pitch, sharp or flat. The aim has been to formulate, if possible, a standard test for the measurement of each, to establish norms, and to investigate some of the underlying psychological factors.[1]

[1] The extensive measurements made would have been impossible were it not for the previous labor of Professor Seashore in perfecting a recording apparatus, the Tonoscope. Dr. Seashore has furthermore proved himself an unfailing source of inspiration and suggestion throughout the experimentation. The author is also under heavy obligations to Assistant Professor Mabel C. Williams, Dr. Thomas F. Vance, Messrs. Bruene and Malmberg, and the many observers for their kind and prolonged assistance.

HISTORICAL

The first investigator to employ the experimental method in attacking the problem of the accuracy of the voice in singing pitch was *Klünder* (11) 1872.[2] He used a manometric flame with two connected speaking tubes, an organ tone sounding in one while the observer sang simultaneously in the other. The difference in vibration number between the standard and sung tones was determined by counting waves. The average ± errors found on three tones, 128, 192, 256, v.d. are 0.761, 0.434, and 0.257 per cent. (of standards) respectively. The difference between 0.761 and 0.257 was thought to be due to the vocal cords and not to hearing.

Klünder was not satisfied with his method or his results and continued working on the problem, publishing a second time in 1879 (12). Again he used organ tones as standards and had his observers sing simultaneously with them, either in unison or in specified interval. The recording was done on smoked paper by means of two phonautographs. The two records were compared directly, that for the organ tone being used as a standard, and deviation in the pitch of the voice from that of the standard was computed in terms of .25 v.d. That Klünder was primarily interested in the physiological side of the problem is indicated by the questions which he set himself:

(1) Does our ear control the voice or is it controlled by the feeling of tension in the larynx? (2) How firmly does the voice attack tones? (3) Are the fluctuations of the voice such that give proof of control by the ear? (4) How many stress degrees of muscular tetanus are we justified in accepting through the performance of the muscles of the larynx?

Klünder found that for the pitches 96, 128, 192, 256, v.d., respectively, he himself as observer made the following ± errors: .32 v.d., .47 v.d., .62 v.d., and .59 v.d. This however was somewhat better than any of his other observers could do.

From this Klünder concludes that the voice is very accurate in reproduction of pitch and he answers his questions in substance as follows:

[2] Previous to this time Scott (17) and Blake (3) had developed phonautographs for registering voice curves.

(1) The vocal cords are held in labial tension by muscular tetanus. (2) The musculature allows from 40 to 170 different tensions in the tetanus. (3) The regulation of the pitch of the voice takes place directly through the sensation of tension in the larynx.

Seashore (19) in 1910 published in a very condensed form the results of experiments carried on in 1905 by himself and E. A. Jenner. Previous to that time, however, much work had been done in perfecting a registering apparatus, the tonoscope, which is fully described by Professor Seashore in the foregoing article in this volume of the Studies. Some preliminary experimenting also was done in 1901-'02 with the help of Edward Bechly, the results of which have never been published. Seashore and Jenner in their work sought to answer two questions: (1) Can we facilitate development of control in the pitch of the voice by using an aid to the ear in training? (2) May the ordinary limits of accuracy be exceeded by training with such an aid? In attacking these problems they used three measurements: (1) accuracy in reproducing a given tone, (2) accuracy in singing a required interval, and (3) the least producible change in the pitch of the voice. The standard or fundamental tone was 100 v.d., produced by a large tuning-fork; the intervals were the major third, the fifth, and the octave above this. The least producible change was determined for each of these four tones (1) in the least producible sharp and (2) in the least producible flat from the note as actually sung. Each period of practice consisted of one hundred and sixty trials, which took about forty-five minutes. The tests continued twelve days, approximately successive. During the first five days the singer depended entirely on the ear as in ordinary singing: then followed five days of singing with aid, *i.e.*, the observer was informed of the result of each trial immediately after it was made. The records of the eleventh day were taken without aid, while on the twelfth day aid was again given. Six men acted as observers. The conclusions of this investigation are quoted as follows:

"(1). The aid enhances the ability to strike a tone which has been heard. The superiority of the aided series over the unaided amounts to 42 per cent. (2) The aid enhances the ability to sing an interval. The superiority of the aided series over the unaided amounts to 50 per cent. for the major third, 50 per cent. for the fifth, and 60 per cent. for the octave. (3) The voluntary control

of the pitch of the voice is improved by the aid. The average superiority of the aided series over the unaided for all intervals amounts to 26 per cent. (4) There is probably some transfer of gain from the aided training to following unaided singing. (5) There is no evidence of transfer of the gain in the accuracy of the memory image. This is undoubtedly due to the fact we have here to do with memory rather than discrimination and the acquisition of accurate memory images is a slow process—too slow in this short series. (6) The gain in the discriminative control of pitch of the voice is fully transferred. (7) Improvements in the ability to sing a tone or an interval, and the ability to produce a minimal change, are very much more pronounced and more rapid in the aided than in the unaided series. (8) The second question is not answered absolutely by our records, but it seems probable (a) from the radical and immediate improvement of the aided series over the unaided, and, (b) from the introspection showing a tone which without the instrument seemed entirely satisfactory to the ear could be corrected by the ear after the error had been pointed out by the instrument, that a higher degree of accuracy of pitch in singing may be attained by aiding the ear in the training than would be possible to attain without such aid. No matter how keen the ear of a trained musician, it can be shown in a single test that his ear has been "too generous"—too easily satisfied, for when the error is pointed out objectively he can recognize it. We thus find cumulative evidence to show that the singer can not reach the physiological limit of accuracy by the ordinary methods of voice culture, because he has no objective criterion by which he can check up the accuracy of his ear. (9) The major third, the fifth, and the octave are approximately equally difficult intervals to sing. If we express the average error in relative fractions of a tone (1/25 of a tone) instead of in vibrations, the ratio is 1.4, 1.5, and 1.4, for the three intervals named above. The average error expressed in terms of vibrations shows that the difficulty of a natural interval varies approximately with the magnitude of the interval. (10) The minimal change is a relatively constant fraction of a tone within the octave. This is true for both the aided and the unaided series. If we reduce the records from vibrations to twenty-fifths of a tone, the minimal change is 3.1, 3.1, 3.6, 3.3, for the fundamental, the major third, the fifth, and the octave respectively. This is surprising,

because within this part of the tonal range the pitch discrimination is normally measured by a constant vibration frequency instead of by a constant part of a tone."

Cameron (4) 1907, varied somewhat the conditions of the experiment as performed by Klünder. In the first series the subject was asked to sing any tone of medium pitch, a second tone of low pitch, and a third of high pitch, and to sustain the pitch selected in each case as uniformly as possible throughout the singing. The second series was like the first except that each tone was interrupted by the insertion of short pauses of .3 second duration. In a third series, somewhat longer than those previously mentioned, the ability of one observer to imitate organ tones in the range 94 v.d. to 303 v.d. was tested. The tones were reproduced in sequence and, in chance order, partly simultaneously with the standards and partly by singing the tones immediately after the organ had ceased sounding. In a fourth series various distracting tones, (1) harmonious or inharmonious with the standard tone; (2) of greater or less interval from the standard; and (3) higher or lower than the standard, were introduced either at the beginning or just preceding the beginning of the reproduction by the observer. The more important results of the study are here summarized:

"(1) In the singing of a tone a sudden marked rise in pitch usually occurs near the beginning of the tone. This rise in pitch is so general as to seem to indicate a universal tendency. (2) No tone is sung entirely uniformly. It oscillates in pitch from period to period throughout its length in a somewhat irregular rhythmical fashion. (3) Very marked differences exist in different individuals with regard to their ability to imitate a standard tone. The subjects tested varied in degrees of accuracy in imitation of standard tones of different pitch from a small fraction of 1 per cent. to 13 per cent. of error. (4) There is manifest throughout a tendency to sing a tone higher than it should be sung. Thus the end of a tone is usually higher than the beginning and the sung tone (as a whole) is almost invariably higher than the standard tone. (5) Distractions when causing disturbances may affect the whole of the sung tone or only the beginning of the tone. In either case the effect of the distraction may be to cause the sung tone to vary from the standard (a) in the direction of the distracting tone; or (b) in the opposite direction from the distracting tone. (6) Sung tones vary-

ing from the standard under the effect or distraction are usually harmonious with the distracting tone. When the distracting tone is inharmonious with the standard tone, distraction is more likely to occur than when the two tones form a harmony. (7) A person may more or less closely imitate a tone which he has heard when his attention was engrossed in singing another tone of a standard pitch."

An important contribution to the general problem of control of the pitch of the voice in singing was made by *Berlage* (2) in 1910. During the summer of 1907 Berlage carried on a series of experiments in which definite time intervals were inserted between the breaking off of the standard tone and the beginning of the reproduction by the observer.[3] These intervals were of the following values stated in seconds: 1, 2, 3, 4, 5, 7, 10, 15, 20, 25, and 30. The tones were all sounded as *"a"* ('a' in 'ah'). This series is an amplification of the methods of Klünder and Cameron, and was undertaken for the purpose of finding the time interval most favorable for the imitation of tones, which when found became one of the conditions of further experimentation.

In the winter of 1907-'08 Berlage's general problem was to determine the influence of articulation and hearing in the vocal reproduction of tones. In this series (second) as in the third series by Berlage the standard tones to be imitated are voice tones. The variation of conditions consisted in having the standard tones sung part of the time by the observer and part of the time by the experimenter thus showing the immediate influence of hearing and of loud articulation in tone-reproduction. It seemed desirable to determine to what extent the influence of articulation is due to the larynx, and to the mouth cavity. For this purpose, in a third series of experiments, all the standard tones were sung by the observers, the vowel quality being varied under control. The standard and reproduction were sung, sometimes to the same vowel as *"i"*, *"i"*, or *"u"*, *"u"*, and at other times to different vowels as to *"i"*, and *"u"* or *"a"* and *"u"*. The chief conclusions reached from Berlage's experiments are the following:

(1) "Accuracy in the reproduction of a "strange" voice tone decreases rather regularly with the increasing time interval of from 1 to 30 seconds. Accuracy is greatest with an interval of from

[3] Berlage designates these tones as 'foretone' and 'aftertone'. 'Standard' and 'reproduction' are used throughout this study.

1 to 2 seconds. The values found here, for the variable average error, in the case of the observers amounted to only .5 v.d. and .6 v.d. (2) Observers reproduced their own voice tones more accurately than those of another (time interval 3 seconds). (3) The increase of precision shows itself chiefly in a decrease of the constant error. In the reproduction of outside standards and especially when they are near the boundaries of the voice range there is a tendency toward a constant error near the middle of the voice range. (4) In the reproduction of one's own tones vowel change works a disadvantage upon precision. With the standard tone sung as "*u*" and the reproduction as "*i*" there is a tendency for the latter to be lower, and vice versa when the vowels are changed. (5) In the reproduction of an outside standard the variable average error expressed in vibration frequency becomes larger with rising pitch, while if expressed in per cent. of vibration frequency it diminishes. (6) In the reproduction of one's own tones the variable average error expressed in vibration frequency remains rather constant with rising pitch. (7) The amount of departure of the individual tone sections (measured off in .1 second periods) from the general average of the reproduction shows no tendency, in the variations carried out in these experiments, to change according to the ordinal number of the tone sections in the course. (8) Only in the first .1 second is the reproduction regularly lower than the rest of the tone course. (9) Reproductions after the time intervals of from 3 to 10 seconds, in the case of two observers, show a sudden raising or lowering of the tone after the tone has progressed some .4 to 1.2 seconds. (10) The average departure of individual tone sections from the average for the tone is greatest in the reproduction of one's own tones. (11) The total amount of departure, expressed in vibration frequency grows with rising pitch so that—not considering rather marked irregularities with the individual observers—the amount of variation expressed in per cent. of a tone remains about constant."

The latest published study of this general problem to come to our attention is that of *Sokolowsky* (22) 1911. His apparatus consisted of a combination of the Einthovan string-galvanometer and the Weiss phonoscope. The organ tones, which were used for standards, acted on the string-galvanometer and the sung tones on the phonoscope. Both tones were registered in a convenient way for comparison by means of the Blix-Sandström photokymograph.

Sokolowsky secured the coöperation of seven professional opera singers, three men and four women. The observers were allowed to choose the vowel to which they sang the tones. The musical *"a"* was chosen most frequently. There were three short series of experiments: (1) singing a given tone simultaneously with the sounding of the tone by the organ (unison); (2) allowing a time interval between the organ tone and its reproduction. (The intervals used were 30, 60, and 120 seconds, during which the observers were instructed not to hum or sing to themselves); and (3) singing a specified interval from a simultaneously sounding organ tone. The musical intervals selected were the third, fourth, fifth, sixth and octave.

The results from these three series of experiments may be summarized as follows: (1) Curves for 8 tones were secured in series I. The average pitch was 251 v.d. (range 165 to 296 v.d.), the average error was ± 0.44 per cent. The average pitches for men and women respectively were 197 and 286 v.d., with average ± errors of 0.51 and 0.40 per cent. (2) The introduction of a time interval increases the average error to ± 0.99 per cent. as compared with ± 0.44 of the previous series. Errors are usually larger with an interval of 60 seconds than with 30 seconds. (3) The average error in series III is ± 1.51 per cent. The largest errors, average ± 3.28 per cent. are on the fifth, while the smallest, average ± 0.78 per cent. are on the third. (4) Of the entire number of tones counted (46) 36 are sung flat and 10 sharp. The errors on the side of sharping are divided among three women and one man; those on the side of flatting between three men and three women.

Guttmann (6) 1912, in his consideration of the psychophysics of singing gives some attention to the problem of accuracy in reproducing pitch and states that recently he has been engaged in an extensive research in this field. The results are to be published shortly in one of the psychological journals, but in a preliminary way he says that they agree in general with those secured by Klünder and Sokolowsky, but he thinks that the results of the latter (unison curves) are "too good".

Other investigators, among them *Hensen* (10) and more recently *Marbe* (14), *Grützner* (5) and *Scripture* (18) have developed methods for recording the pitch of the voice, but these seem not to have been used in gathering data on our problem.

THE TONOSCOPE

In the investigations of Klünder, Cameron and Berlage the vibration frequency of the tones was recorded in tracings on smoked paper. Sokolowsky photographed his records; after these had been rendered permanent the waves were counted and the pitch determined by comparison with a time or standard line. This method, commonly known as "graphic recording" has been used with various refinements by many investigators in the field of phonetics. While reliable, it is at best indirect and very laborious.

Seashore and Jenner in their research made use of an early model (20) of the tonoscope. This instrument as lately improved was used by the author in the present experiments.[4] It has several advantages which recommended it as an instrument for the measurement of the pitch of tones. In the first place readings are made quickly and directly. The instant a tone is sounded the vibration frequency is indicated by a row of dots. The experimenter has simply to note the number of this row and to record it. He is, therefore, enabled to secure a large number of observations in a relatively short time. It is not difficult to take two hundred records in thirty minutes. In the second place the experimenter has the advantage of knowing how the test is progressing. If a preliminary practice series is desired to acquaint the observer with some procedure we have in the direct readings from the tonoscope an index to the observer's understanding of the test. The observer must be kept actively trying throughout the experiment. In psychological tests, such as the imitation of tones by singing, there is so much repetition in the program for the observer that his attention easily wanders. Large and unnatural errors are therefore likely to be found in the records. Here the tonoscope as a recording instrument has an advantage over other methods as it provides for detecting these errors as soon as they occur. The experimenter as he takes each reading notes roughly the attack, the steadiness, and the degree of success with which the reproduction approaches the standard. He thus easily becomes acquainted with the unusual range of variability and forms an estimate of the observer's power to control his voice. When a

[4] The instrument is fully described in the preceding article in this volume of Studies, "The Tonoscope", by Professor Seashore. A reading of that article is essential for an understanding of the present report.

tone of unusual divirgence is given he therefore immediately recognizes it and can take cognizance of it, asking for introspection or for a new trial, and all with scarcely any loss of time. He may thus check up and to some extent control the observer,—keep him at his best. Furthermore the possibility of encouraging the observer or even of giving him full information regarding the success or failure of each trial is in itself a most important asset.

The tonoscope has been criticised as giving only an approximate result, because the pitch of the singing voice is not uniform and it is therefore necessary in reading the instrument to select the predominating pitch. This criticism stands or falls according to the needs of the problem to be attacked. If one were studying the oscillations of the voice, or the variations of the individual sections of a tone, as for example the difference in pitch between the first tenth and the fifth tenth of a second of a tone, it would be better to use a graphic method. But even in such problems as these the tonoscope is not without its possibilities. The characteristics of tonal attack in singing are easily discernible in the configurations on the screen. With many of the problems which lie in our field there is no need for so detailed a record. The predominant or modal pitch of a tone of from one to two seconds in length is all that is needed for much of the work in the psychology of pitch singing. The tonoscope can of course meet this condition admirably, as it is this modal pitch which stands out clear and distinct, forcing itself upon the attention of the experimenter.

Tonoscope reading test.—The method of reading the tonoscope, and the various sources of error having been fully treated by Professor Seashore in the accompanying article, there is no need to repeat them here.

In order to determine the degree of accuracy in the reading of the tonoscope the following experiment was performed. A set of ten large, movable-disc, tuning forks ranging from 128 to 131 v.d. was so tuned that no two forks had a pitch difference of over 3 v.d. and in the great majority of cases the differences were much smaller. A revolving shutter, rotated by the tonoscope shaft, was so arranged as to expose the mouth of a resonator connected with the sensitive light for the following time intervals: .25, .50, .75 and 1.00 second. In this way a tone sounded before the shutter was registered by the tonoscope for just the period during which the

resonator was exposed.[5] The presentation of the tones and the recording of the observations were in charge of two helpers. The experimenter did nothing but watch the moving screen and call out the readings. He had no way of knowing the real reading in any case. Five trials were given on each fork with each exposure interval. The order of the forks was determined approximately by chance. There was an interval of about five seconds between tones.

After the fifty trials with the .75 second exposures were finished, the pitch of each fork was carefully determined with the tonoscope, counts being made by the stop-watch during periods of from 6 to 15 seconds. These records formed the basis from which to compute the errors in the first test. The assistant then changed the pitch of all the forks and the above procedure was repeated with a .50 second exposure. Again the forks were changed and the same procedure was followed for the 1.00 second and the .25 second exposures in turn. Thus fifty records were obtained for each of the four exposure periods and the conditions were such that the reader could have no accessory clue. The record is summarized in Table I.

To test the reading ability for tones one octave higher, *i.e.* 256 v.d., where it will be recalled the tonoscope reading, and hence the errors, must be doubled, a set of seven small forks was provided. These were weighted so that no pitch difference between any two forks was greater than 3 v.d. The test was made with the exposure interval of .75 second.

In making the pitch difference between the forks come within a range of 3 v.d. we approximate the condition presented when working with voice tones that require accuracy in reading. If an observer is asked to reproduce a tone or to sing an interval the experimenter knows approximately the point on the scale where the reading should occur. He is watching this point. Should the reproduction be nearly correct and the tone fairly constant for, say .50 second, he can read according to our result (see Table I) within an error of less than $\pm$ .2 v.d. If however the reproduction goes wide of the mark, for example to the extent of 6 v.d. there is no need of reading in fractions smaller than halves.

[5] This arrangement is not ideal in that, as the tone is turned on and cut off by the disc, slightly disturbing waves are set up and show on the screen. In test No. 4 where the tone sounded for .25 second this was felt to be very disturbing. The real time given for the reading of the tones in all these tests was thus slightly less than that represented by the several discs.

TABLE I. The degree of accuracy in the reading of the tonoscope

Exposure	1.00	sec.		Ave.	error	.12	v.d.	;	m.v.	.10	v.d.
"	.75	"	(128 v.d.)	"	"	.15	"	;	"	.12	"
"	.75	"	(256 v.d.)	"	"	.19	"	;	"	.17	"
"	.50	"	(128 v.d.)	"	"	.18	"	;	"	.11	"
"	.25	"	(128 v.d.)	"	"	.65	"	;	"	.27	"

EXPERIMENTS SERIES I: ACCURACY AND THE VOICE RANGE

In the first five series of experiments the purpose was to answer questions concerning some factors which must be considered in any adequate test of voice control. (1) How does accuracy of control vary with the range of the voice? (2) How does the intensity of the standard tone affect the pitch reproduction? (3) What is the relation of voice volume to voice control? (4) Are the reproductions affected by the timbre of the standard tones? (5) Do vowel changes (timbre changes) in the reproductions cause changes in the pitch of the reproduction? The sixth series represents an effort to combine into a single test the results of our previous experiments, together with those of other investigators, and to give this test to a sufficiently large group that we might be enabled to determine from the results some of the norms of voice control.[6]

Seventeen men with splendid enthusiasm gave their services as observers in the experiments of Series I. From among this number several were selected to serve as observers in Series II, III, IV, and V. The observers were all of mature age and more than half their number had had some training in the methods of experimental psychology. P, the only professional musician in the group, is a teacher of "Voice" and a thoroughly trained tenor soloist. H, a baritone of extensive special training, has for some time been the leader of a large choir. He is a soloist of ability. Ma., W, and V. Z. have all had special training in singing, and much experience in solo, quartette and glee club work. S, C. Mi., Ro., An., Wi., and V. H. all have had considerable experience in general singing but are without special training. Ri., Ab., Mc., Br., D, and Bh. very seldom sing in public but they enjoy music.

[6] Gutzmann (7) and Sokolowsky (22) suggest some of the above problems, especially Nos. 1 and 5 as being important. These articles and suggestions however did not come to the attention of the writer until the experimentation was completed.

For Series I the standard tones were provided by a set of twenty tuning forks ranging approximately by the chromatic scale from C_1, 64 v.d., to and including a′, 426 v.d. The first fourteen forks beginning with 64 v.d. were large and carried discs. All the tones were of good quality and their duration of tone was more than ample. Some of the forks were of different vibration frequency than that indicated by the notes of the chromatic scale; for example, the pitch of the fork that corresponded to G was 182 v.d. in place of 192 v.d. These differences were made in order to check the observers from judging and singing the various steps as musical intervals.

An independent selection of five forks was made for each observer after a preliminary determination of his voice range. These forks covered approximately one and one-half octaves in the middle of the range and were fairly distributed. In giving the test the experimenter presented the tones to the ear of the observer, who, after listening for 1.5 seconds and allowing a time interval of 1 second, reproduced the pitch of the tones as accurately as possible. Proceeding from the lowest to the highest and then in reverse order back to the lowest, each tone was given twice in succession, the test consisting of twenty trials on each standard tone.

The results of these experiments are present in Table II. O denotes the observer; P, the pitch of the standard tone; E, average error; m.v., mean variation; and C.E., constant error. These five successive columns give the record of the respective standards for each observer. The footings in the table show the averages of the figures above stated, first in terms of vibration (absolute) and second, in terms of percentage of a tone (relative) at the respective levels. The average C. E. in the footing is the average of C. E. regardless of sign; in the second the sign is taken into account giving group tendency of the constant error, or group constant error (G. C. E.). These footings are represented graphically in Fig. 1.

Taken as a whole these records show that accuracy in the reproduction of the pitch of a tone, as measured by the average error (E) with its mean variation (m.v.) the average of the constant errors (C. E.) and the general tendency of the constant errors (G. C. E.), tends to be a constant in terms of vibration frequency. This is shown in Fig. 1 (A) by the fact that the four curves, for

the absolute variation tend to remain horizontal lines whereas the four curves for the relative variation (B) tend to fall in inverse ratio to the rise of the pitch. The slight tendency to deviate from the constant in terms of vibrations is in the direction of decrease in accuracy with rising pitch. This, in the case of the highest tone,

TABLE II. Accuracy and the voice range

O	P	E. m.v. C.E.	P	E. m.v. C.E.	P	E. m.v. C.E.	P	E. m.v. C.E.	P	E. m.v. C.E.
		1.6		1.2		.9		1.9		1.2
P.	128	1.1	160	.6	182	.8	256	1.6	320	1.2
		—1.5		—1.2		+ .4		—1.6		+ .4
		1.4		3.1		.7		2.9		3.7
H.	95	.6	120	1.1	160	.7	213	1.4	286	1.4
		+1.4		+3.1		— .1		—2.9		—3.7
		1.5		.8		3.0		3.7		3.1
Ma.	95	.7	120	.8	160	1.1	240	1.0	286	1.3
		—1.5		+ .4		+2.9		+3.7		+3.0
		.9		1.0		1.7		2.5		1.8
W.	120	.9	144	.8	182	1.6	256	1.5	320	1.7
		+ .2		+ .4		— .2		+1.9		— .1
		3.1		1.6		1.7		1.1		2.7
V.Z.	120	1.1	144	.7	182	.9	256	1.1	341	1.4
		+3.1		+1.6		+1.4		+ .3		+1.7
		2.6		1.8		1.0		2.1		7.1
S.	120	.8	144	.9	182	.8	240	1.2	286	1.8
		—2.4		—1.5		— .3		— .9		+7.1
		.9		.7		.8		1.3		1.7
C.	95	1.2	144	.7	182	.6	256	1.9	341	1.5
		+ .2		— .0		— .3		+ .8		— .9
		2.9		1.1		1.6		2.7		3.5
Mi.	86	2.3	120	.8	182	1.9	256	2.7	341	1.8
		+4.0		+1.0		— .1		+1.0		+3.4
		2.3		2.9		2.1		4.5		2.5
Ro.	95	4.5	144	2.2	182	2.0	240	1.8	320	2.0
		+ .1		+2.5		+1.3		+4.3		+ .2
		5.0		1.6		2.5		4.5		4.6
An.	95	2.3	120	1.5	160	2.8	213	2.6	256	3.5
		+3.7		+ .8		—1.0		—4.1		—3.1
		3.5		4.2		2.0		1.2		4.2
V.H.	107	1.7	128	1.4	160	1.7	182	1.0	240	2.2
		+2.7		—3.6		—1.0		+ .1		+3.9
		2.3		2.9		2.1		4.5		2.5
Ri.	95	1.9	144	2.3	182	1.9	240	1.5	320	2.1
		+2.5		—1.6		+ .2		— .5		+2.7

TABLE II. (*Continued*)

		3.2		5.5		3.1		1.3		1.4
Ab.	95	1.0	128	1.4	160	.9	182	.8	256	.9
		+3.1		+5.5		+3.1		+1.7		+ .9
		1.7		3.9		1.9		1.0		2.6
Mc.	95	.5	120	1.3	160	.8	213	.9	256	1.2
		—1.6		+3.8		—1.7		+ .4		+2.0
		2.9		5.6		6.4		3.7		2.7
Br.	95	2.2	144	1.0	182	1.1	256	1.0	384	2.8
		+2.7		+5.2		+6.4		+3.6		+1.3
		1.7		3.3		5.9		6.3		11.2
D.	95	.9	128	3.1	160	2.4	182	6.6	256	5.4
		—1.4		— .3		—5.9		—3.4		—6.5
		1.6		2.4		3.0		5.1		2.5
Bh.	120	1.1	144	1.6	182	1.1	240	1.8	286	1.4
		—1.3		+ .9		+3.0		+5.1		+1.5
P	103.0		135.0		172.9		230.6		299.4	
Average in v.d.	E	2.3		2.6		2.4		2.8		3.5
	m.v.	1.5		1.3		1.3		1.8		2.0
	C.E.	2.0		2.0		1.7		2.1		2.5
	G.C.E.	+ .8		+1.0		+ .5		+ .5		+ .8
Ave. in per cent. of a tone	E	.18		.15		.11		.09		.10
	m.v.	.11		.08		.06		.06		.05
	C.E.	.15		.12		.08		.07		.06
	G.C.E.	+.06		+.06		+.02		+.02		+.02

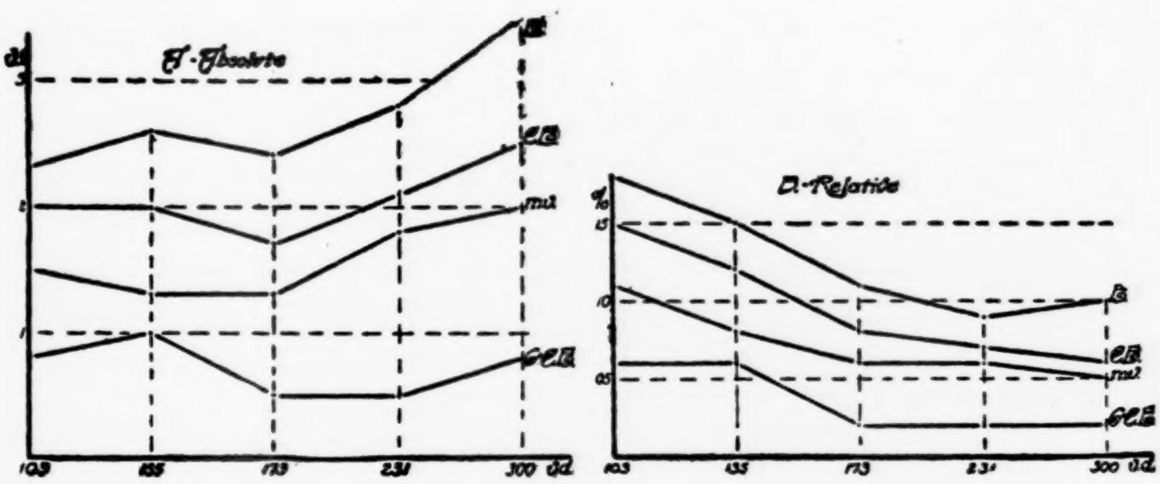

Fig. 1. Accuracy and the voice range. (Table II)

is to be accounted for mainly by the fact that some observers were erratic on this tone, probably because the tone was higher than the observer commonly sings. As a matter of fact only half of the observers, nearly all of whom would be classed as bass or baritone in their range, show any tendency to decrease in accuracy at this

point; five show the tendency to increase in accuracy and the remaining four tend to remain constant.

This result is in harmony with results found by *Preyer* (16), *Luft* (13), *Meyer* (15) and *Vance* (26) on the sensory side, that pitch discrimination is approximately constant in terms of vibration frequency within this range. It is in harmony with the finding of Berlage (2) as quoted above: item 5 (second part), that average error diminishes with rising pitch if expressed in per cent. of vibration frequency; and item 6, with reference to the reproduction of one's own tones.

It is interesting to compare and to contrast these records with those of Seashore and Jenner (19), item 9, showing that the average error in the singing of a natural interval (third, fifth, and octave) varies approximately with the magnitude of the interval; (see also Sokolowsky's results above) and, item 10, showing that the minimal change is a relatively constant fraction of a tone within the octave.

The tendency for the C. E. to be in one direction (+) will be considered in a later section in connection with the constant errors in our other series of experiments.

EXPERIMENTS SERIES II: INTENSITY OF STANDARD

In the experiments of Series I, as stated above, two successive trials were made on each fork. Occasionally upon the presentation of the tones for the second trial at reproduction the observer would say "Let me hear that again; it sounds higher (or lower) than before", or "Is that the same fork?" Such remarks by careful observers led to this consideration of the intensity factor.

The same forks were used with the respective observers as in Series I. The tones were presented to the ear by the experimenter as before. But with half of the trials the standards were made about as strong as possible by striking the forks a heavy blow and presenting them near the ear. The other standards were made as weak as could be heard with distinctness. The observers were encouraged to sing with a medium volume of voice and not to imitate that of the forks, as is the natural tendency. Twenty records were made with each fork, ten on the "weak" and ten on the "strong" in the double fatigue order, as regards pitch and intensity. No successive trials were made on the same fork except on the highest and lowest. Having sung the tones from the highest the

observer would sing them in reverse order from highest to lowest; but a short pause was introduced between such successive reproductions. Of the eight observers tested, P, Ma., V. Z., C, An., V.H., S and Mi., the first six had no definite knowledge of the object in view.

The results are shown in Table III and graphically represented in Fig. 2. *"W"* denotes weak and *"S"* strong, while the other notation is the same as that previously used. It will be seen that the intensity of the standard tone has a decided effect upon the accuracy of reproduction.

(1). Increase in intensity causes a lowering in the pitch of the reproduction. The G. C. E. for S on each of the five levels is less than that measure for *W.*, the minimum amount of difference being 1.4 v.d., the maximum 4.1 v.d. and the average for the five pitches, 2.3 v.d. In all the forty individual constant errors with the exception of two (see V. H.'s lowest tone and C.'s highest; in this latter *"W."* and *"S."* are just the same) the reproductions of the "strong" standards are lower than those of the "weak". If we compare these averages (C. E.'s and G. C. E.'s) with those of the previous series of experiments we find not only that the "strong" C. E.'s and G. C. E.'s are lower in the majority of cases than those of Series I, but that these measures for the reproductions made from the "weak" standards are somewhat higher than those of the former series. The effect of intensity in other words, is evident in both "weak" and "strong" standards, the former heightening the seeming natural tendency to sharp and the latter overcoming this tendency with a more powerful one to flat.[7]

(2). Strong standard tones cause general inaccuracy of voice control. Most of the observers stated that they were less sure with the "strong" standards. Others complained that the test made their ears tired. Reference to the mean variations and also to the E.'s and C. E.'s will show that in the majority of cases these amounts are

[7] When the conditions of this experiment (Series II) were explained to P., the professional musician, he remarked off hand as he began the test: "Loud tones would make your nerves more tense and would in general tend to make you sharp." He was asked then and at other times during the test to let any conscious tendency to flat or sharp take care of itself *e.g.* not knowingly to correct for it. At the last P. said: "I am equally satisfied with my reproductions of weak and strong." Cf. P's. record in Table III.

TABLE III. Intensity of standard tones

Ave. P.	105 v.d.		135 v.d.		174 v.d.		237 v.d.		301 v.d.	
	W.	S.	W.	S.	W.	S.	W.	S.	W.	S.
O.	E. m.v. C.E.	E. m.v. C.E.	E. m.v. C.E.	E. m.v. C.E.	E. m.v. C.E.	E. m.v. C.E.	E. m.v. C.E.	E. m.v. C.E.	E. m.v. C.E.	E. m.v. C.E.
	1.3	2.1	.7	1.2	2.3	1.4	1.1	.7	3.0	1.0
P.	1.0	.6	.5	.9	1.2	1.4	.7	.7	1.3	.8
	— .7	—2.1	+ .3	—1.0	+2.3	+ .3	+1.1	+ .2	+2.9	+ .5
	1.1	1.1	1.0	1.4	3.2	1.7	3.1	2.1	2.1	1.4
Ma.	.9	.9	1.0	.8	.8	1.7	.6	1.0	.7	1.0
	+1.0	— .7	— .2	—1.4	+3.2	+ .8	+3.1	+2.0	+2.1	—1.4
	3.8	1.6	2.9	1.1	3.2	.7	1.9	4.8	2.0	1.8
V.Z.	.7	.9	1.3	.7	.5	.7	1.5	1.2	1.7	1.3
	+3.8	+1.2	+3.1	+1.0	+3.2	— .2	—1.2	—4.8	+ .3	—2.1
	1.2	.7	1.9	1.1	1.7	.7	3.0	2.4	2.8	2.6
C.	.5	.7	.5	1.1	.8	.7	1.1	.9	2.0	1.0
	+1.2	+ .2	+1.9	+ .3	+1.7	— .1	+3.0	+2.4	+2.6	+2.6
	6.4	5.4	1.9	1.1	1.9	2.3	3.2	7.8	3.7	8.1
An.	6.4	2.8	1.3	1.1	1.9	1.3	1.2	1.7	1.8	1.2
	+6.4	+4.8	+1.9	— .1	— .5	—1.9	—3.1	—7.8	—3.4	—8.1
	3.1	2.9	3.9	3.5	1.0	1.9	2.2	2.1	2.9	8.0
V.H.	1.6	1.7	1.0	1.5	.8	1.8	1.9	2.1	3.0	3.4
	+2.6	+2.9	+3.9	+2.9	— .7	—1.2	+ .8	+ .3	— .5	—8.0
	1.8	4.2	1.0	.7	.8	3.6	2.1	4.7	5.3	5.5
S.	.5	.7	.5	.6	.8	1.0	1.2	.9	1.3	1.4
	—1.8	—4.2	+1.0	— .4	— .2	—3.6	—1.8	—4.4	+5.3	+5.5
	9.3	8.5	1.5	1.7	2.2	5.4	4.0	4.5	7.8	4.7
Mi.	1.9	4.1	.9	.9	2.0	2.9	3.1	4.1	1.9	2.9
	+9.3	+8.5	+1.5	+ .5	— .8	—5.4	+2.4	—2.1	+7.8	—4.6
Av. E.	3.5	3.3	1.9	1.5	2.0	2.2	2.6	3.6	3.7	4.1
Av. m.v.	1.7	1.6	.9	1.0	1.1	1.4	1.4	1.6	1.7	1.6
Av. C.E.	3.4	3.1	1.7	1.0	1.6	1.7	2.1	3.0	3.1	4.1
G.C.E.	+2.7	+1.3	+1.7	+ .3	+1.0	—1.4	— .5	—1.8	+2.1	—2.0

larger with strong standards, thus indicating conditions that operate against the best vocal control.

The matter of intensity has been considered in the field of pitch discrimination, where it must really be worked out. Seashore (21) makes the following statements concerning it:

"Extensive experiments show (1) that both trained and untrained observers may be influenced by intensity in their pitch judgment; (2) that although there is a tendency among the untrained, especially the ignorant, to judge the loud tone the higher, it may work either way; (3) that the same individual may show one tendency at one time and the reverse at another; (4) that for trained observers the two tendencies are about equal; and (5) that the tendency is more serious for large than for small intensity differences. Introspection shows that this confusion rests largely on motor tendencies, or motor images. We associate high and strong with strain—the reversal can in some cases be traced to a correction, conscious or unconscious, based on knowledge of this danger.

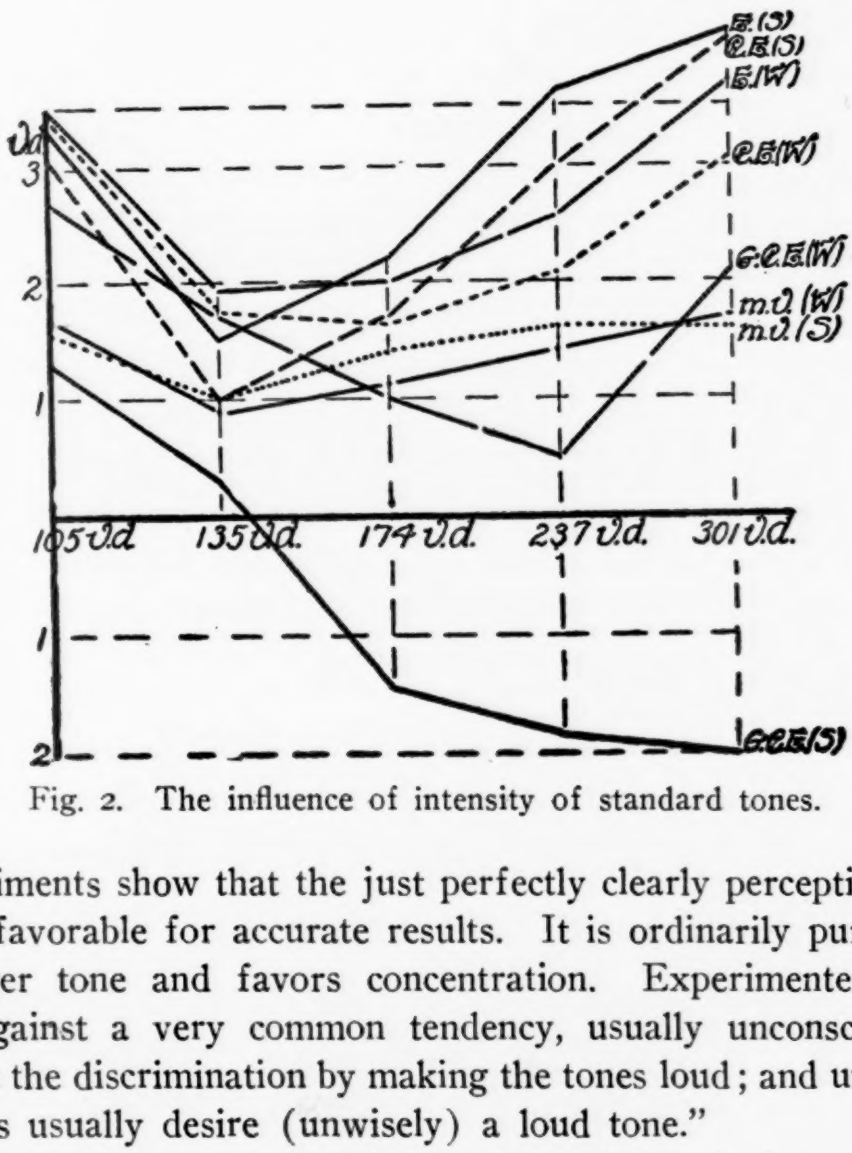

Fig. 2. The influence of intensity of standard tones.

Experiments show that the just perfectly clearly perceptible tone is most favorable for accurate results. It is ordinarily purer than a stronger tone and favors concentration. Experimenters must guard against a very common tendency, usually unconscious, to facilitate the discrimination by making the tones loud; and untrained observers usually desire (unwisely) a loud tone."

These conclusions are found on tests made by Anderson (1) at the level of 435 v.d. Our results just stated led to a re-examination of the effect of intensity on pitch. Hancock (8) found that as

measured in terms of hearing alone the tendency to hear a relatively low strong tone as low is greater than is shown in this series for singing. All these facts make clear that in singing from a standard tone greater care must be exercised to keep the tone constant and at a most favorable strength. We have no adequate quantitative data to show what strength is best but the facts available tend to support the statement made by Seashore (21) that the just perfectly clearly perceptible tone is most favorable for accurate results.[8]

EXPERIMENTS SERIES III: VOLUME OF THE VOICE

The effect produced by varying the intensity of the standard tones suggested a parallel question concerning the relationship of voice volume and accuracy of reproduction. This problem was attacked in the following manner. The forks selected were the same for each observer as in the voice range test, Series I; they were presented to the observer's ear by the experimenter who endeavored to keep the intensity as nearly constant as possible, and the observer was instructed to reproduce the tones in three degrees of voice volume, "loud", "medium" and "weak". Ten trials were made on each fork with each of these three degrees of loudness of voice, the order being as follows: one trial on each fork from lowest to highest and after a pause from highest to lowest with "medium" intensity; from highest to lowest and back to highest with "loud" intensity; from lowest to highest and back to lowest on "weak"; highest to lowest and back on "medium" and so forth until the 150 trials were made.

These records are summarized in Table IV and represented in part in Fig. 3. In this table *L, M* and *W* represent respectively loud, medium, and weak, other notation is the same as in the foregoing tables.

Here we find again, as in the foregoing series, the tendency for accuracy in singing to remain a constant in terms of vibrations, except for the extreme notes, at which there is a decrease in efficiency, especially at the high note. The form of the average error curve (E)

[8] The force of the blow changes the pitch of a fork, (See Winkelmann's Akustik Vol. 2 p. 358) lowering it slightly, but this change in these forks could hardly be detected and certainly fails to acccount for the error in reproduction. See also Seashore (21).

here is entirely analogous to the form of the curve of pitch discrimination referred to above (16, 13, 15 and 26) but it represents a shorter range, as the voice has a shorter range than the ear.

The constant error for men here, as in the foregoing series, is in the direction of sharping. It is a relatively constant fraction of a vibration for all pitches except the highest. The records for the medium and the weak tones practically coincide,[9] and compare very favorably with those of Series I., but there is a uniform tendency to sing the loud reproduction highest. The average difference between the loud and the weak (see the G. C. E.'s) is here .6 v.d. This is not a contradiction, but the reciprocal of the results found in Series II: namely, that the loud (or strong) standard is reproduced low.

It will be remembered that in Series II the standard was made strong, the observer tried to produce a tone that subjectively seemed the same in pitch, and that practically all of his reproductions were flat. This result in the light of Series I, where sharping was the rule, seemed to warrant the conclusion that the strong tone is judged low. Now in Series III we have a confirmation of this; here the standard is of medium intensity while the reproductions are varied: loud, medium and weak. It seems therefore that the instant the observer commences his loud reproduction he is subject to the same error in judgment as was revealed in Series II, and that to make his reproduction subjectively equal in pitch to the standard, he thinks it necessary to raise. This brings about abnormal sharping: the average G. C. E. of Series III is +1.3 v.d. as against +.7 v.d. for Series I, where intensity differences were at a minimum.

The agreement of the errors (G. C. E.'s) in these two series (II and III) at once offers an explanation for them: the error is primarily one of hearing which is basal and the chief cause for the error in singing.[10] This is in harmony with the contention of Klünder

[9] Medium and weak tended to be confused by the observers who would frequently have to be reminded that they were not making sufficient difference between them. This would imply that they each seemed more natural and less distinct than the loud, which is borne out by the fact that the average G. C. E. for weak (Series III) is identical with that for Series I, *i.e.* +.7 v.d. However it should be noted that the curves for weak are less regular than in Series I.

[10] It is interesting to note than in Series II the high strong is flattened most, while in Series III the high loud is sharped most.

TABLE IV. *Volume of the voice and accuracy*

Ave. P.		109 v.d.			137 v.d.			176 v.d.			241 v.d.			308 v.d.		
		L	M	W	L	M	W	L	M	W	L	M	W	L	M	W
O.	E. / m.v. / C.E.															
P.	E.	3.0	2.0	1.3	2.8	1.4	.9	[illegible]	.9	1.8	1.9	1.3	1.5	1.2	2.7	4.1
	m.v.	.6	.8	.7	.5	1.2	.5	.8	.9	.9	1.1	1.3	.6	1.1	1.6	.8
	C.E.	—3.1	—1.9	—1.2	—2.7	— .9	+ .1	— .3	— .2	+1.6	—1.7	— .5	+1.5	+ .8	+2.4	+1.4
Ma.	E.	2.0	1.8	1.3	2.0	1.4	1.7	2.5	3.0	3.9	2.2	1.0	1.1	1.4	1.7	2.4
	m.v.	.7	1.0	.5	1.6	1.4	1.5	1.3	1.3	.9	1.0	1.0	1.1	1.2	1.5	1.3
	C.E.	—2.0	—1.5	—1.3	—1.5	+ .1	— .5	+2.4	+3.0	+3.9	—2.2	+ .5	+1.1	—1.2	—1.2	+2.3
V.Z.	E.	3.9	3.5	3.2	2.9	2.5	2.6	2.3	2.0	1.3	2.2	2.5	2.1	1.0	1.2	1.3
	m.v.	1.0	1.1	1.4	1.1	1.2	1.4	1.6	1.9	.7	1.2	1.0	.7	1.0	1.2	1.0
	C.E.	+3.9	+3.5	+3.2	+2.8	+2.5	+2.3	+2.0	+1.8	+1.0	—1.7	+2.3	—2.0	— .5	— .4	+1.2
S.	E.	2.3	2.3	2.7	1.2	.9	1.1	3.2	2.4	2.6	.9	1.4	.9	7.6	5.2	5.2
	m.v.	.8	.4	.5	1.2	.8	1.0	1.1	.6	.7	.8	.9	.8	1.9	1.4	1.5
	C.E.	—2.2	—2.2	—2.6	+ .1	— .5	+ .6	+3.1	—2.4	—2.6	+4.4	—1.0	— .3	+7.5	+5.2	+5.2
C.	E.	1.4	1.2	1.0	1.9	1.4	1.7	1.5	.4	.7	3.0	3.6	2.4	5.6	3.9	2.8
	m.v.	.5	.6	1.0	1.1	.6	.8	1.5	.4	.8	.9	.4	1.1	1.5	1.6	1.1
	C.E.	+1.4	+1.2	— .6	+1.3	+1.4	+1.7	— .5	— .4	+ .4	+3.0	+3.5	+2.3	+5.5	+3.8	+2.8
V.H.	E.	9.0	4.9	2.5	2.1	1.7	1.6	2.9	2.2	4.3	1.6	2.2	2.0	2.2	4.2	3.0
	m.v.	1.8	2.1	1.3	1.4	1.5	1.6	2.3	2.0	3.6	1.7	2.8	2.0	1.5	2.4	2.8
	C.E.	+8.9	+4.9	+2.5	+2.1	— .8	+2.0	—2.1	—1.2	—2.5	+ .4	+ .8	— .4	—1.3	—4.2	—2.0
Mi.	E.	2.3	1.6	1.7	2.7	2.6	2.7	3.1	2.3	2.0	3.1	1.3	2.6	9.5	3.5	3.0
	m.v.	1.6	1.5	1.2	1.5	1.6	2.0	2.4	2.3	1.6	2.3	1.4	2.2	3.5	1.0	2.6
	C.E.	+2.2	+1.2	[illegible] 1.3	+2.7	+1.9	+2.3	+1.5	+ .3	—1.2	+3.1	+ .7	+1.8	+9.5	+3.5	+1.2
Av. E.		3.4	2.5	2.0	2.2	1.7	1.8	2.3	1.9	2.4	2.1	1.9	1.8	4.1	3.2	3.1
Av. m.v.		1.0	1.1	.9	1.2	1.2	1.3	1.6	1.3	1.3	1.3	1.3	1.2	1.7	1.5	1.6
Av. C.E.		3.4	2.3	1.8	1.9	1.2	1.3	1.7	1.3	1.9	2.3	1.3	1.3	3.8	3.0	2.3
C.G.E.		+1.3	+ .7	+ .2	+ .7	+ .5	+1.0	+ .9	+ .1	+ .1	+ .8	+ .9	+ .6	+2.9	+1.3	+1.7

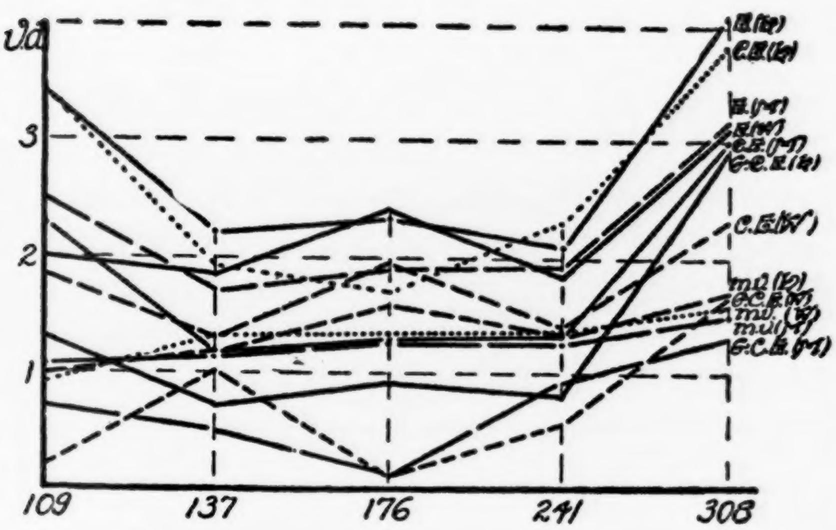

Fig. 3. Volume of the voice and accuracy (Table IV).

(12) that the ear is the chief criterion for regulating the voice. But the result quoted from Hancock (8): that the hearing error is greater than the singing error, (when dealing with a low, strong tone), together with the fact (Series III) that there is relatively more flatting with a strong standard than there is sharping with a loud reproduction, would conform to the conclusion reached by Stern (24) that the kinaesthetic sense of the singer is also an important factor.

One would expect a larger mean variation (m.v.) for the tone that has the largest error, but the table shows the mean variation to be practically equal for all three intensities of sound. This may be taken as a mark of the relative constancy of the motive for the intensity error.

The agreement and the remarkable uniformity in these two laws as shown in Series II and III would indicate that we are here dealing with an important factor of which we must take cognizance, both in the hearing and the producing of musical tones.

EXPERIMENTS SERIES IV: TIMBRE OF STANDARD TONES

Klünder (11 and 12), Cameron (4) and Sokolowsky (22) in their researches used organ tones for standards, while Berlage (2) made use of tones from the voice. Having ourselves used tuning forks it seemed advisable to ascertain if timbre differences in the standards affect the accuracy of reproduction.

The standards selected for the test were: a large disc tuning fork (144 v.d.) sounded before a resonator, the dichord (137.5 v.d.) energized by bowing, and an organ pipe blown by mouth. In using the latter, because of the variability of the blow and hence the uncertainty of the pitch sounded, the vibration frequency of each standard tone was read on the tonoscope and entered in a parallel column with the reproductions. The tones were so far as possible of uniform intensity, they were sounded for approximately 2 seconds and after the interval of 1 second reproduced on *a*, as in "law" with medium volume of voice. Twenty trials were made on each standard, and because the effect of timbre was the point of interest, the reprodu^tions were in groups of five successive trials, the standard of course being sounded before each attempt.

TABLE V. Timbre of standard tones and accuracy

	Fork 144 v.d			String 137.5 v.d			Pipe Av. 150 v.d.		
O.	E.	m.v.	C.E.	E.	m.v.	C.E.	E.	m.v.	C.E.
P.	2.5	.6	+2.5	1.8	.5	—1.9	.9	.4	— .1
S.	1.0	.9	— .8	1.9	.8	—1.9	1.1	.5	— .3
Ma.	1.2	1.3	+1.2	1.2	.5	+1.2	1.3	.8	+ .6
V.Z.	1.3	.5	+1.2	2.1	.4	—1.9	.6	.5	— .5
Mi.	2.0	.6	+2.0	.6	.5	+ .3	.5	.4	— .2
Av. E.	1.6			1.5			.9		
Av. m.v.		.8			.5			.5	
Av. C.E.			1.5			1.4			.3
G.C.E.			+1.2			— .8			— .1

The results of this series of experiments are summarized in Table V. Judging by the magnitude of the average error and the constant error, the record is in favor of the organ pipe. This is probably due to the fact that this tone is most nearly like that of the human voice in tone-color, or timbre. The introspections of our observers, all of whom have good musical ability and were practiced in observation, are however not in accord with this. Four of the five stated that the string was the easiest standard to imitate. P, the one professional musician in the group, felt that he did best on the fork. But reference to the table shows that it was here that he made his largest errors and even the largest made by any observer on that standard. S. stated that the string was by far the best as a standard but made his smallest errors, and the smallest made by anyone, on the fork. It must be noted also that S. has had more practice with forks than any other member of the group. Practice

is undoubtedly a factor and the value of it for a particular observer depends chiefly on what associations are awakened by a given tone-color. Purity, for example, may be thought of as thinness, and secondarily as highness of tone. While tuning forks, being relatively pure and free from over tones, are at a disadvantage on the side of richness, it is also true that in most groups the observers are about equally unpracticed in singing with forks, which is an advantage from the standpoint of measurement. The forks also are decidedly more constant in pitch than any other type of standard tone. Two of the observers noticed a tendency to imitate the timber of the standards.

From the above observations it seems fair to conclude that richness of tone favors accuracy in the reproduction of any particular standard.[11]

EXPERIMENTS SERIES V: VOWEL QUALITY AND ACCURACY

Berlage (2) introduced the problem of the influence of vowel quality (or change in the timbre of the singing voice) upon accuracy in imitating pitch, and made measurements on this point for the purpose of determining the effect of mouth resonance upon the pitch of the reproductions. In considering the problem here there is no thought of discrediting the results found by Berlage. The tonoscope method of recording has enabled us to take many more records than were used by him in computing his results and the matter is of such far reaching importance that it seemed worth while to include in our study a series on this factor, limiting our measurements to the following vowels:

u as oo in "toot"
o as o in "no"
a as in "ah"
e as e in "there"
i as i in "machine"

In addition to these, humming the tone was introduced in the test as the "hum" seemed to have no marked vowel quality.

[11] Starch (23 p. 52) in his conclusions on the effect of timbre in the localization of sound makes this statement: "The richer and more complex a sound the more accurately it can be localized."

TABLE VI. Vowel quality of the voice and accuracy

O.		u	o	a	e	i	"h"	u	o	a	e	i	"h"	u	o	a	e	i	"h"
		144 v.d.						182 v.d.						240 v.d.					
P.	E.	.6	.6	.4	.8	.6	.5	.7	1.6	1.4	1.5	.5	.4	2.6	1.9	2.0	1.2	4.0	2.4
	m.v.	.4	.6	.4	.5	.7	.6	.8	1.0	.8	.8	.5	.4	1.2	1.5	1.0	.7	.4	.8
	C.E.	− .2	+ .1	− .2	0	+ .4	− .3	− .5	−1.4	−1.4	−1.5	0	− .4	+2.6	+1.4	+2.0	+1.2	+4.0	+2.4
S.	E.	1.0	.8	.8	1.1	.9	.9	.9	.9	1.0	.9	1.3	1.0	1.4	1.4	1.4	1.5	1.4	1.3
	m.v.	.7	.8	.5	.8	.6	.8	.7	.9	.7	.8	.8	.8	1.2	1.0	1.2	1.2	1.0	1.1
	C.E.	− .9	− .1	+ .6	+ .8	+ .4	+ .7	− .9	− .8	− .7	− .6	−1.2	−1.3	− .7	− .6	−1.1	− .1	+ .7	+ .6
C.	E.	.7	.9	.9	.8	2.2	1.3	.7	.9	1.1	1.0	1.0	.7	2.5	2.3	2.5	2.6	3.3	2.4
	m.v.	.6	.4	.7	.8	.5	.6	.7	.7	.9	.9	.6	.7	1.2	1.3	.9	1.1	1.4	1.2
	C.E.	+ .4	+ .6	+ .5	+ .3	+2.2	+1.1	− .2	− .8	− .6	− .4	+ .9	+ .1	+2.2	+2.1	+2.4	+2.4	+3.0	+2.2
V.Z.	E.	.9	1.0	.5	.4	1.5	.4	.8	1.1	.6	.7	1.5	1.0	2.3	3.3	2.5	2.7	4.0	3.1
	m.v.	1.1	.7	.5	.3	.4	.4	.4	.9	.5	.6	1.1	.8	1.1	1.7	1.1	1.3	1.6	1.0
	C.E.	+ .9	− .4	+ .2	+ .3	+1.5	+ .2	− .5	−1.0	− .4	+ .1	+ .8	− .3	+2.0	+2.8	+2.1	+2.4	+4.0	+3.1
Wi.	E.	2.4	1.6	1.9	2.1	4.7	4.2	1.4	1.1	1.3	1.2	1.0	1.0	2.1	1.4	1.1	.9	1.6	.7
	m.v.	1.0	1.2	.8	.9	.7	.8	1.1	1.0	.9	.6	1.0	.9	1.5	1.4	.9	.8	1.3	.8
	C.E.	+2.1	+1.5	+1.9	+2.1	+4.7	+4.2	+1.1	− .5	−1.2	+1.2	+ .2	+ .4	+1.4	+ .2	− .6	− .4	+1.4	− .2
Mi.	E.	2.3	1.6	1.3	1.7	1.7	1.8	1.2	1.3	1.2	1.1	1.5	1.1	3.2	2.2	4.5	3.8	4.8	3.7
	m.v.	1.7	1.5	1.2	1.7	1.6	1.9	1.2	1.3	1.3	1.1	1.5	1.1	2.1	1.5	2.1	1.9	1.8	2.0
	C.E.	−1.8	− .9	+ .4	+ .3	+ .6	− .7	− .7	− .8	− .3	− .3	+ .5	− .4	+2.6	+2.2	+4.5	+3.8	+4.8	+3.7
Av.	E.	1.3	1.1	1.0	1.1	1.9	1.5	1.0	1.1	1.1	1.1	1.1	.9	2.4	2.1	2.3	2.1	3.2	2.3
Av.	m.v.	.9	.9	.7	.8	.8	.9	.8	1.0	.8	.8	.9	.8	1.4	1.2	1.2	1.2	1.2	1.2
Av.	C.E.	1.0	.6	.6	.6	1.6	.9	.6	.9	.8	.7	.6	.5	1.9	1.5	2.1	1.7	3.0	2.0
	G.C.E.	+ .1	+ .1	+ .6	+ .6	+1.6	+ .9	− 3	− .9	− .8	− .7	+ .2	− .3	+1.7	+1.0	+1.5	+1.5	+3.0	+2.0

Three forks of the large disc variety were used as standards, the pitches being: 144, 182 and 240 v.d. and each of these three tones was reproduced to the five vowels and the "hum" twenty times, a total of 360 trials for the individual observer. The test was divided between two equal periods. The order of reproducing was two trials on each fork to each vowel in the double fatigue order, illustrated as follows: 144 to *u*, 182 to *u*, 240 to *u*, pause, 240 to *u*, 182 to *u*, and 144 to *u*; then 240 to *o*, 182 to *o*, etc., followed by 144 to *a*, 182 to *a*, and so on throughout the test, the order of the vowels being *u*, *o*, *a*, *e*, *i*, and "hum." All standards were presented to the ear for a duration of approximately 2 seconds and an interval of 1 second was allowed before the singing.

That the vowel quality is a factor influencing the accuracy of reproduction is borne out by the results of the series as shown in Table VI. The average error (E) and the mean variation (m.v.) are given merely for an index to the reliability of the record. They are both large as compared with the constant error (C.E.) which is the factor in terms of which we desire to measure the effect of vowel quality on reproduction.

Although there are characteristic differences for the three pitch levels and for the different individual observers, the results in Table VI may be fairly represented by a single curve (Fig. 4). This

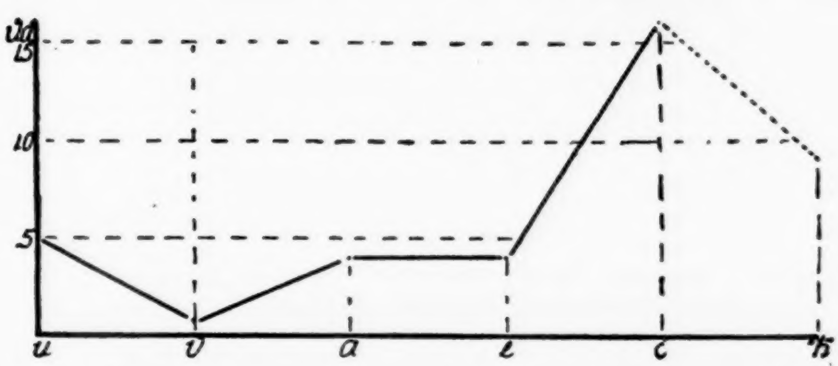

Fig. 4. Vowel quality of the voice and accuracy. (Table VI.)

shows graphically the algebraic average of the records (G. C. E.) for each of the vowels and for the three levels, 144 v.d., 182 v.d. and 240 v.d. There is a tendency for the vowels to fall into three groups: namely, (1) *o* sung the lowest, (2) *a*, *e* and possibly *u* sung moderately sharp and (3) *i* sung decidedly sharp. These facts would seem to point to the general conclusion that the higher the dominating overtone in a vowel clang, the higher that vowel will be sung. In Fig. 4, *u* offers the single exception to that rule.

The hum was supposed to be neutral as it was moderately weak and the record was made from the nasal breath. This assumption is confirmed by the record which gives the hum a middle place with *a* and *e*.

It must be remembered that there is, in the foregoing records which were sung on *a*, a tendency to sharp by about the amount of sharp for *a* here. That tendency is probably due to some other cause than timbre. It may therefore be suggested that *a* and *e*, the vowels usually sung when one is free, are fairly neutral; *o* (and possibly *u*) are sung relatively flat and *i* relatively sharp. This view, it will be observed, is confirmed by the hum.

Our results seem to differ radically from those of Berlage (4), second part of item 4, in the observations which are common to both. But our method also was radically different; moreover, his conclusion (item 4) is somewhat modified when we read in his article p. 76 where the results of the vowel experiments are discussed. "Accordingly one may look upon a slight increase in the variable error as probable with vowel change" (*i.e.* when the observer tries to reproduce his own pitch but on a different vowel) . . . "other generalities cannot be deduced for the table . . . "

These results in reference to vowel quality are of so far reaching significance for speech and song that we may not venture further discussion for the matter must be made a special object of investigation for verification of the empirical data and in search of an interpretation. It seems safe however to proceed in our work using *"a"* as the vowel quality for reproductions.

EXPERIMENTS SERIES VI: ACCURACY IN SINGING

Having gained some insight concerning the influence of voice range, standard tone intensity, voice volume, standard tone timbre, and voice timbre, on the accuracy of voice control, we now turn to the main problems of our research. These may be restated as follows: (1) What is the average error of the human voice in reproducing the pitch of a tone? (2) What is the average minimal producible change of the voice? (3) Is there any general tendency to sing sharp or flat? (4) How does the average performance of men and women compare on the above three points? All the studies referred to in the historical account contain results which cast light on some of these problems. But in almost every case these

results and problems are secondary to the main interest of the study; and moreover the number of observers and observations is usually quite limited. In Series VI therefore, we have made these problems the central issue on a large group of persons to give our results significance as norms.

Apparatus and method

Standards. With the aid of the tonoscope, eleven large disc forks were tuned to the following pitches: 128, 128.5, 129, 130, 131, 133, 136, 140, 145, 151, and 158 v.d. The series of pitch increments between the forks was therefore: .5, 1, 2, 3, 5, 8, 12, 17, 23, and 30 v.d. as measured from 128 v.d. This series of tones was used for men. For the women a second set was provided on 256 v.d. as a basis, namely, 256, 256.5, 257, 258, 259, 261, 264, 268, 273, 279, and 286 v.d. In this second set it will be noted that the same pitch increment (absolute) were used as in the 128 v.d. set instead of the relatively equal increments. In this respect the procedure was based upon the conclusions reached in Series I.

Koenig resonators were provided for each set of forks. As the increments were small it was found that one resonator would speak sufficiently well to several tones. In the case of the 128 set three resonators were used: first, 128 v.d. to and including 136; second, 140 and 145 v.d.; and third, 151 and 158 v.d. For the higher set two resonators were found sufficient: first, 256 v.d. to and including 268 v.d.; and second, 273, 279, and 286 v.d. Both series of forks as reinforced by the resonators gave tones of pleasing quality and medium intensity.

Observers. Two hundred and one individuals, ninety-four men and one hundred and seven women, took the test which is about to be described. This number comprised those enrolled in the elementary psychology courses in the University of Iowa, 1912-1913. Of these about one hundred fifteen were sophomores; the remaining were upperclassmen. None of them had had any practice in this test. Among them were some excellent vocalists and some others who claimed never even to hum or whistle and to have difficulty in recognizing old and familiar tunes if unaccompanied by words. No one was excused because of his inability and no one was selected because of ability, for it was desired in so far as possible to secure what might be considered an average group. A previous lecture on

the measurement of musical capacity had successfully aroused the interest of the observers so that they entered into the experiment with zest, many of them desiring to secure their individual results.

The charge. The instructions were given by word of mouth to each person, although the appointments were so arranged that one observer was present while another was taking the test and so became familiar with the procedure before he actually entered upon it. Supposing the observer to be a man the instructions would be as follows:

"Mr. ———, we have here a series of eleven tuning forks. This one (striking the 128 v.d. and presenting it before the resonator) is c below "middle c", it is a tone of 128 v.d., the lowest tone in the series; we will call it "o". This one (striking and presenting the increment fork 30, 158 v.d.) is considerably higher than o as you easily notice, and is the highest one in the group. These other forks all represent pitches between the two which we have sounded. The test to-day consists in singing these eleven tones one after the other as they are given. They will be presented in pairs. First we will sound the o, the lowest one of the tones; you will listen carefully to it and then sing a tone of the same pitch. Immediately after your singing, the highest tone in the group (30, 158 v.d.) will be sounded; you will listen and sing that one. Then the o will be sounded again and, after you sing it, there will come the next to the highest tone (23, 151 v.d.); and so on we will come down one step at a time always reproducing the o before each of the interval forks. When you have tried all the tones in the series you will go back over them in the reverse order. Simply imitate as nearly as possible the pitch of each tone as it is given, always remembering that the o is the lowest one in the series. Sing all the tones with a natural voice volume and use the vowel *"a"* (a as in "ah") and whenever you feel dissatisfied with any trial ask for a repetition."

Following these instructions, in order to put the observer at ease and to satisfy his curiosity, the experimenter gave a brief explanation and demonstration of reading on the tonoscope.

The test. The forks were presented to the resonator by a helper who gave his attention solely to the task of sounding the tones in the right order and with as nearly uniform intensity and duration as possible. The tones were sounded with medium intensity varying towards the "weak." The observer sat on a high stool or stood at

the side of the instrument in a position which kept him from seeing his own record. He sang the tones into a metal speaking-tube placing the lips lightly against the fingers of the hand which grasped the mouthpiece. The arm was supported by an adjustable rest and, so far as could be, strain and unnaturalness were avoided.

A few preliminary trials were given on increment 0-30 in order that the observer might find himself becoming somewhat familiar, not only with the tonal range covered by the standards but with the experience of taking pitch from a tuning fork. The series was then given in pairs in the following order: 0-30, 0-23, 0-17, 0-12, 0-8, 0-5, 0-2, 0-1, 0-.5; 0-.5, 0-1, 0-2, etc. back to 0-30. The complete test consisted in singing the series thus five times. This gave one hundred reproductions of the 0, and ten on each of the increment tones, a total of two hundred tones for each observer. This series therefore contains forty thousand records. The test as outlined could not be performed with care in less than 30 minutes. In some cases and especially with non-musical persons a much longer time was required than this.

Throughout the test we endeavored to keep the observer seriously trying to sing the exact pitch of the forks. To this end it was deemed desirable to offer some encouragement, especially during the first fifth of the experiment, no matter how poor or good the record. It was observed that encouragement did not cause the singers to be self-satisfied or careless but rather served to make them try the harder. It helped moreover to create an atmosphere of ease and naturalness. But while there was encouragement there was also some criticism. If, for example, the observer was singing the 0 flat 5 or 6 v.d. regularly he was told to listen more carefully to the standard and to make sure that he had the right pitch but no intimation was given as to the character of the error. Little rest periods of twenty seconds were rather frequent and were found to be of much service. Many times it was noted that after such a period the errors were decidedly smaller than before.

A few questions concerning the observer's musical education, voice range, and ability to play and sing were asked during or following the experiment and the answers together with some comments regarding his performance of the test were made matters of record.

Justification of procedure

Before considering the results of this series it remains to justify the form of procedure as outlined above in the light of the sources of error revealed by our previous experiments and by those of other investigators.

Voice level of test. Our experiments (Series I) on the accuracy of pitch singing within the voice range demonstrated that the errors are relatively smaller on the higher tones. Unpracticed observers however, will much more readily try a tone that is medium or low than one that is high. It therefore seemed best for general testing to choose a voice level which all would recognize as being well within range. The selection of 128 to 158 v.d. for men and of 256 to 286 v.d. for women is thus the result of considerable experience in testing groups of individuals, and seems further justifiable on the grounds of pitch discrimination as previously stated.

Forks for standards. Tuning forks were retained for standards even though the records of Series IV indicate that the organ pipe and dichord can be imitated more accurately. Forks are very simple, easily manipulated, of practically constant timbre, and at the same time reliable in pitch. And if, as in our test, a series of tones differing from each other by slight degrees of pitch is desired to be sounded in rapid succession, tuning forks are the most reliable apparatus. Furthermore they are used so little for general musical purposes that in testing with them no group of observers is given undue advantage.

Many standards *vs.* one standard. Berlage (2) found that his observers could reproduce their own voice tones more accurately than tones given by some one else, the increase of precision showing itself chiefly in a decrease of the constant error. We have frequently noticed a tendency, which is a corrolary of Berlage's conclusion. Observers when making successive trials on the same standard very often reproduce their own reproductions rather than make new efforts at imitating the real standard. The observer finds it much easier to reproduce his own previous tone, duplicating the muscle tension and mouth resonance which he experienced at that time and felt to be satisfactory. Indeed, even though he conscientiously work against this tendency, he can not overcome it entirely if engaged in making successive trials where the pauses between are brief. This is confirmed by the fact that frequently when observers

for some cause or other have been dissatisfied with attempts and desired new trials giving them immediately they would in the new trials unconsciously repeat the identical pitch given before. This same tendency has sometimes been evident even in large and unusual errors which the experimenter might rule out, asking for new trials. In view of these considerations it seemed best in our general test to adopt the principle of many standard tones and no successive trials on the same tone.

The increments between the forks were made small and of varying magnitudes for two reasons: first, in using these small increments we do not complicate our work with the factor of musical intervals, and second, in using a series of small increments we make possible the measurement also of the ability to make faint shadings (sharp or flat) in the pitch of the voice. The selection of increments is arbitrary. These particular steps were chosen because they have been found satisfactory in work with pitch discrimination (21) at the level of 435 v.d. and, as stated before, extensive research by Vance (26) and others shows that pitch discrimination is practically constant in terms of vibration frequency in the middle range of tonal hearing here covered. This is also the ground for making the increments for the women the same number of vibrations instead of the relative parts of a tone, in which case they would have been doubled.

Sounding the two tones. Seashore and Jenner (19) employed the method of "least producible, or minimal, change". The observer sang the standard or a tone at a given interval from it and then reproduced his own reproduction, save that he made it "the least possible" sharp or flat according as the experimenter might direct. While this will undoubtedly become a standard method in extensive work with an observer it is not suited to tests of a single sitting, first, because ability is rapidly improved by practice and, second, because the observer tends to be easily satisfied with his effort. The better way is not to rely on the changing subjective standard of the observer but to provide a series of constant objective increments and give him the opportunity to find his own level as by the method of constant stimuli in lifted weights or pitch discrimination. Such a series has been provided in the standards and increments mentioned above.

Order of standards. Manifestly the standards might be presented

to the observer in any one of a number of different orders. After trying out the matter thoroughly with the help of three good observers we selected the order of presentation above described for the following reasons: (1) to give the tones in pairs (0-30, 0-23, etc.) takes direct advantage of all the latitude which the series provides. Most observers can easily detect the difference 0-30, while many (theoretically about 25 per cent.) would be baffled to find a difference between 23 and 30; (2) to begin with the largest increment and work towards the smallest has the double advantage of establishing confidence in the attitude of the subject and of stimulating effort; (3) to give the increments in a series and in double fatigue order rests the voice from the unusual strain of making the least producible change, and (4) to explain definitely at the beginning of the test that all the increments are in one direction, *i.e.* above the 0, simplifies the problem and puts it more definitely under control than if uncertainty as to change of direction in standards were allowed. The test is therefore not to measure the judgment for direction of pitch difference but the judgment and expression of the amount of pitch difference between two tones. In pitch discrimination it is well known that much depends upon the direction of the expectant attention.[12] And should we present the standards of our test in a chance order we would complicate it exceedingly at the critical point of the smallest increments.

Time intervals. At the very beginning of the test the intention was to allow an interval of 1 second between the breaking off of the standard tone and the singing by the observer. But the method

[12] An idea of the influence of this same source of error operating in the field of singing may be gained from the following illustration. The author in instructing a very fine observer thoughtlessly said, (the error was altogether unintentional); "We have here two forks, the first, 128 v.d., and the other one 3 v.d. higher, 131 v.d. You will please sing them one after the other. I will give the lower one first." Then the forks were presented and reproduced as directed. When we came to the twelfth trial the observer remarked: "I seem to feel strain to bring the 131 v.d. up". In the moment of reflection following this remark the writer recognized that he had made a mistake in instructing the subject, as the so-called "131 v.d." was really 3 v.d. lower than 128 v.d., or 125 v.d. We find in the twelve trials made that the average reproduction of 128 v.d. is 123.6 v.d. while the average pitch given for the supposed 131 v.d. (really 125 v.d.) is 124 v.d. The misunderstanding and therefore expectant attention changed the direction of the reproductions, and brought in much larger constant errors than are usual for this individual. It should also be noted that the errors are minus.

was soon given up as, in this case, cumbersome and unpractical, and furthermore we did not care to complicate our test with the factor of tone memory. (See Berlage (2) and Sokolowsky (22)). The observers in their usual singing with musical instruments make no such perceptible time intervals. They sing with the tones of the instrument, perhaps holding them somewhat longer than is done by the instrument. When the standard has been sounded the attention is centered, the muscles of the larynx almost involuntarily assume a particular tension and it is unnatural to wait for the beating of a metronome or some other signal to begin singing. If the unpracticed observer is told to make his own interval, unless checked up diligently, he will very soon be making intervals that are exceedingly short, if indeed he is not singing simultaneously with the standard tones. The method followed therefore was to sound the forks for approximately 1 second, encouraging the observer to begin his tone during the sounding of the fork and to hold it longer than the fork.

It may be objected that one might sing fairly accurately judging simply on the secondary criterion of beats between his voice and the standard tone. Helmholtz indeed (9 p. 326) suggests this as a convenient method for the singer to use for checking his own accuracy in practice exercises. While it would be possible for a highly practiced observer it can hardly have much influence in our test. The author made it a point to question frequently regarding the way observers judged of their success in reproducing tones and was not able to find any one who knowingly made use of this criterion. It is however quite possible that the roughness of 6 or 8 or more beats per second may occasionally have caused some observer to be dissatisfied with his attempt. But the tonal fluctuations and adjustments which are necessary to bring about a lessening of the frequency of beats between the voice and an outside standard are easily recognized with the tonoscope; no such "finding" process was observed.

Another time interval which must be considered is that between the 0 (128 or 256 v.d.) and the increment fork of any particular pair. In order that the standards for minimal change of voice may have their greatest value the interval indicated must be as short as possible admitting of a quick, direct comparison of tones; otherwise the test practically resolves itself into the singing of a single

tone. Hence the presentation of the increment fork followed immediately upon the close of the observer's reproduction of o, the subject being encouraged to make the reproduction about 1 second in length. The increment forks were struck while the o was being reproduced. This was, however, no distraction as only a slight blow on the practically noiseless sounder was necessary, and the forks could not be heard until presented before the resonators. Following the reproduction of each increment fork there was a period of about 2.5 seconds before the next sounding of o.

Other factors. In the matter of intensity of standard and intensity and vowel quality of the voice we took direct advantage of our previous work and adopted such conditions as would give the most normal results according to those findings. By the use of resonators at a considerable distance from the observer's ear we found a satisfactory means of controlling the intensity of the standards,[13] while the intensity of the voice had to be judged subjectively and watched by the experimenter. And in the selection of *"ä"* we are using that vowel quality which according to Berlage and our own results affects least the constant error of the reproductions.

Tables of data

The constant error (C. E.) and mean variation (m.v.) were found for the ten trials on each fork of the ten pairs given in the test. These twenty C. E.'s and twenty m.v.'s for each individual tested are embodied in Table VII, which has been divided into two parts, A. and B., for the men and women respectively. In the first column of the table, at the left, are given the numbers which stand for the individual observers. This numbering is in no sense a ranking, but simply for convenience in handling the data and aid in identification. Odd numbers are used throughout to refer to women and even numbers to men. The second column from the left shows the C. E. and m.v. (the latter is under the former) for the ten trials on o. when used in the pair o-30. The same measures for the ten trials on variant (or interval) tone 30 are given in column three, and each of the successive smaller increments are represented in the same manner. The arithmetic averages for the constant error

[13] It is possible that the pitch of a standard is not only varied by its intensity but also by its position when held near the ear.

Obs. 0-30 0-23 0-17 0-12 0-8 0-5 0-3 0-2
128 158 128 151 128 145 128 140 128 136 128 133 128 131 128

Obs.	0-30		0-23		0-17		0-12		0-8		0-5		0-3		0-2
	128	158	128	151	128	145	128	140	128	136	128	133	128	131	128
2	−0.2	+4.0	−0.3	+2.7	+1.0	+4.6	−0.1	+8.9	+0.1	+7.6	−0.2	+11.4	−0.1	+9.5	+0.5
	1.8	2.2	2.1	2.5	1.0	1.8	1.5	2.3	1.7	4.0	1.6	2.8	2.1	2.1	1.9
4	−1.9	−6.6	−2.9	−2.3	−1.3	−0.3	−3.0	−1.5	−4.8	+4.6	−2.4	+6.0	−3.5	+7.7	−3.8
	2.7	4.8	1.9	5.5	1.5	7.7	2.8	6.9	1.4	6.8	3.0	6.8	1.7	5.7	1.6
6	−0.1	+1.7	0	+4.5	−0.2	+4.7	+0.1	+5.0	0	+6.0	+0.1	+4.5	−0.2	+4.2	−0.2
	1.1	.7	.8	1.9	1.0	3.1	1.1	4.0	.4	3.4	.9	2.5	.6	1.2	.6
8	+1.0	+3.6	+1.1	+2.9	+1.3	+5.2	+1.2	+5.5	+0.9	+3.9	+0.8	+4.7	+1.1	+5.4	+0.9
	.4	1.2	.7	1.3	.9	3.	.4	1.9	.3	1.7	.4	1.7	.5	2.6	.7
10	+0.3	−0.1	+0.3	+0.7	+0.1	−0.1	0.0	+2.8	0.0	+4.5	+0.1	+4.8	−0.2	+4.6	−0.2
	.7	.5	.7	1.1	.5	2.3	.4	1.0	.4	.9	.1	1.2	.6	1.4	.8
12	−0.1	+1.2	+0.4	+0.8	+0.3	+3.0	+0.3	+2.3	+0.4	+2.3	+0.2	+3.0	−0.2	+3.3	+0.1
	.7	3.0	.6	2.2	.7	3.8	.7	2.7	.6	1.9	.8	1.2	.2	1.1	.5
14	+4.0	+17.3	+2.0	+21.8	+1.3	+25.0	+4.0	+26.5	+3.5	+28.8	+0.3	+36.0	−0.2	+34.5	+0.5
	6.0	9.0	2.0	4.1	2.0	6.8	5.0	6.5	5.1	5.1	4.0	5.3	3.1	4.5	2.5
16	+0.4	−0.3	0.0	+0.3	−0.2	+2.0	0.0	+3.8	−0.5	+4.1	−0.5	+5.0	−1.1	+5.3	−1.0
	.8	1.4	.9	2.1	1.0	1.0	.8	1.6	1.0	1.7	.6	1.9	.5	1.2	.8
18	−0.7	+2.8	−1.1	+2.4	−0.9	+5.4	−0.5	+5.9	−0.9	+7.9	−0.4	+8.9	0.0	+10.6	+0.7
	1.3	2.2	1.7	4.5	1.7	2.5	2.0	4.3	1.3	4.9	1.4	3.7	1.3	5.3	1.1
20	−1.0	+4.4	−1.1	+8.1	−2.9	+10.9	−2.3	+7.0	−1.5	+9.4	−1.6	+9.5	−3.2	+3.5	−2.5
	1.6	2.5	1.9	2.1	2.5	3.4	2.1	3.6	2.1	4.7	2.2	4.3	2.2	4.7	3.4
22	−2.2	−3.4	−0.5	−0.9	−0.7	+0.8	−0.7	+6.1	−3.3	+7.2	−3.7	+5.5	−2.8	+2.3	−4.9
	2.8	2.7	2.4	5.5	4.1	6.8	2.8	7.1	2.6	9.5	1.2	8.0	2.8	2.8	1.4
24	−0.8	−1.1	−1.6	+1.3	−1.9	+5.1	−1.0	+7.5	−1.6	+9.3	−1.8	+11.1	−2.4	+11.3	−2.6
	2.4	2.3	2.3	3.6	1.1	4.1	1.6	5.0	2.2	4.9	1.8	4.9	2.3	5.4	2.2
26	−2.4	−1.9	−2.2	−4.2	−2.4	−2.6	−2.6	−0.1	−2.8	+0.1	−2.9	+1.4	−3.2	+1.0	−3.8
	1.6	1.9	1.4	4.2	1.5	2.4	1.9	2.1	1.4	1.5	.9	2.1	.8	3.2	1.0
28	+0.1	2.2	+0.4	+0.6	−0.2	+2.4	−0.1	+4.8	0.0	+5.2	−0.4	+5.0	+0.2	+5.2	−0.1
	.7	1.2	.7	1.3	.4	2.0	.9	1.4	.6	2.8	.7	2.6	.8	1.8	.9
30	+0.7	+0.6	+0.3	+1.6	+1.0	+4.5	+0.8	+6.4	−0.2	+8.5	+0.9	+9.2	+1.1	+9.6	+1.0
	1.2	1.9	1.0	3.9	1.6	4.2	1.8	3.7	1.5	5.0	.9	3.1	1.3	3.3	.6
32	−2.3	−0.5	−2.9	−1.6	−3.0	+0.4	−2.9	+4.4	−3.6	+4.3	−3.6	+4.0	−3.3	+5.5	−2.8
	2.2	4.1	1.2	6.0	1.4	4.7	1.3	4.9	1.4	5.1	1.4	5.8	1.9	4.7	2.2
34	−0.8	+3.6	0	+6.8	+0.4	+8.5	−1.1	+9.9	0	+10.1	0.7	+10.4	+0.6	+9.7	+1.1
	1.0	3.2	1.0	1.6	1.1	3.6	2.1	3.5	1.6	4.3	.7	4.2	.9	3.9	1.3
36	−1.8	+1.4	−1.9	+4.0	−2.1	+6.7	−2.8	+7.6	−2.7	+9.5	−2.0	+8.0	−2.2	+7.7	−2.7
	.8	1.9	1.1	4.2	.7	6.0	.8	6.3	1.4	3.4	.6	4.6	1.2	6.8	.9
38	+1.1	+5.9	+1.0	+7.3	+0.9	+9.3	+0.7	+9.9	+0.4	+8.6	−0.2	+8.1	−0.1	+5.5	0
	.9	2.3	.8	2.0	1.1	1.9	1.1	3.9	.8	2.4	.7	2.1	6.0	1.9	.9
40	+3.2	+2.2	+3.1	+7.7	+3.0	+7.6	+2.9	+9.0	+3.3	+2.4	+3.0	+5.6	+3.3	+2.0	+4.3
	1.6	.6	1.9	1.6	1.8	8.4	1.9	11.8	1.9	8.5	1.8	8.5	2.2	1.2	1.6
42	−2.2	−2.0	−2.5	+1.8	−2.6	+6.2	−4.0	+4.4	−3.9	+5.7	−2.8	+4.5	−3.5	+3.2	−3.8
	3.0	9.4	3.0	10.5	3.2	11.4	2.5	12.0	2.7	12.3	3.7	13.7	3.5	12.5	3.4
44	−2.9	−3.8	−3.4	−1.2	−3.0	+5.5	−1.7	+11.8	−4.7	+17.1	−3.9	+16.0	−3.7	+17.0	+0.5
	2.1	4.7	2.6	5.3	2.2	4.3	2.7	4.2	2.3	4.8	4.3	6.0	4.1	6.6	5.6
46	−4.3	−1.0	−3.7	+0.6	−3.3	+5.9	−4.7	+7.0	−3.7	+4.8	−1.5	+6.4	−1.3	+6.9	−2.8
	3.9	7.8	4.7	3.7	4.7	3.7	4.4	2.2	4.0	6.2	3.7	6.5	4.2	6.3	4.2
48	−1.3	+8.5	−2.0	+3.5	−1.3	−0.5	−1.4	+1.3	−2.1	+1.2	−1.1	+3.2	−0.2	+3.1	−0.5
	1.7	3.9	1.6	5.0	2.3	3.9	2.6	2.6	1.7	3.2	1.9	3.4	1.9	3.9	1.6
50	+1.3	+2.6	+1.1	+1.4	+1.3	+4.4	+1.1	+5.4	+1.0	+5.6	+1.1	+6.0	+1.0	+5.8	+1.2
	.5	1.6	.7	1.4	.3	1.6	.5	2.4	.4	2.8	.9	2.0	.8	2.2	.8
52	+2.2	+3.6	+2.6	+0.6	+2.1	+2.8	+2.3	+5.9	+2.6	+7.2	+2.6	+5.8	+2.4	+6.7	+2.3
	.6	.8	.8	2.4	.7	2.0	.7	1.1	1.0	1.6	.4	2.4	.6	1.9	1.1
54	+0.1	+0.3	−0.2	+3.8	−1.0	+7.6	−0.1	+0.4	−0.6	+16.7	−0.3	+16.3	+0.6	+16.6	+0.4
	2.1	3.5	1.8	2.0	2.2	2.0	1.9	3.6	2.0	2.1	1.5	1.9	2.0	3.6	1.2
56	−1.1	+0.8	−1.7	+4.4	−2.3	+6.7	−1.7	+5.4	−0.5	+5.1	−1.1	+6.2	−0.5	+6.0	−0.6
	1.7	2.0	1.7	2.2	2.1	3.9	1.3	4.2	1.7	2.3	2.1	2.6	2.3	3.8	1.4
58	−0.4	+1.0	−0.3	+2.4	−0.6	+7.9	−0.3	+8.9	+0.2	+11.7	+0.2	+11.1	+0.9	+10.0	+0.9
	1.8	4.6	1.9	3.8	1.8	3.9	2.1	2.9	1.4	3.3	2.2	8.9	1.7	7.4	1.9
60	−1.6	+1.1	−1.4	+1.5	−1.8	+3.7	−1.6	+4.6	−1.5	+6.8	−2.1	+6.9	−2.2	+8.6	−2.2
	.8	2.1	1.4	4.5	1.2	5.5	1.4	5.2	.7	3.6	.7	4.3	1.2	3.2	1.0
62	+0.1	+3.7	+0.5	+1.1	+0.1	+3.0	+0.1	+1.7	−0.8	+1.1	−1.0	0.0	−1.0	+0.6	−1.4
	1.1	1.5	1.1	2.9	1.1	2.4	1.1	3.1	1.4	1.7	.8	1.6	1.0	2.2	1.2
64	−0.9	−3.6	−0.8	−1.1	−0.5	+0.6	−0.4	+2.6	−1.1	+4.7	−0.8	+6.6	−0.7	+8.2	−0.7
	[illegible]	[illegible]	.8	[illegible]	[illegible]	2.8	1.0	1.8	1.2	2.1	1.2	3.6	1.1	[illegible]	1.1

en

0-2		0-1		0-.5		Average Arithmetic		Average Algebraic	
						O	V	O	V
128	130	128	129	128	128.5	128	138	128	138

Average
0-2 0-1 0-.5 Arithmetic Algebraic
O V O V
128 130 128 129 128 128.5 128 138 128 138

0-2		0-1		0-.5		Arith. O	Arith. V	Alg. O	Alg. V
+0.5	+8.2	+0.8	+5.7	+1.3	+4.3	.46	6.69	+.29	+6.69
1.9	1.8	2.2	2.5	1.5	1.4	1.74	2.34		
−3.8	+5.9	−2.3	+10.5	−3.3	+5.5	2.92	5.09	−2.92	+2.90
1.6	6.3	1.5	8.3	2.1	4.8	2.02	6.36		
−0.2	+3.4	−0.3	+3.5	+0.1	+3.0	.12	4.05	−.06	+4.05
.6	1.4	.8	1.3	.5	1.4	.78	2.06		
+0.9	+5.6	+0.7	+5.1	+0.9	+5.1	.99	4.70	+.99	+4.70
.7	1.8	.5	1.7	.3	1.4	.51	1.83		
−0.2	+3.9	+0.1	+2.6	0.0	+1.7	.13	2.58	+.05	+2.54
.8	1.5	.5	1.2	.2	1.2	.49	1.23		
+0.1	+3.2	+0.1	+3.2	+0.3	+2.3	.24	2.46	+.18	+2.46
.5	1.4	.7	1.8	.5	1.0	.60	2.02		
+0.5	+27.3	+1.1	+33.0	+0.5	+28.0	1.74	27.0	+1.7	+27.8
2.5	11.6	5.1	8.0	3.1	3.8	3.25	6.47		
−1.0	+5.8	−1.3	+4.5	−1.7	+3.7	.67	3.48	−.57	+3.48
.8	1.8	1.4	1.4	1.3	1.6	.91	1.55		
+0.7	+8.9	+0.3	+7.6	0.0	+5.8	.55	6.62	−.35	+6.62
1.1	4.9	1.4	3.6	1.2	2.8	1.43	3.97		
−2.5	+2.1	−2.8	+2.1	−2.0	−0.1	2.09	5.71	−2.09	+5.71
3.4	6.1	2.6	3.9	3.4	5.0	2.40	4.03		
−4.9	+6.3	−3.9	+1.2	−3.9	+0.2	2.66	3.39	−2.66	+2.53
1.4	4.3	2.1	2.7	3.1	4.2	2.52	5.36		
−2.6	+7.4	−2.2	+7.5	−4.2	+6.8	2.01	6.84	−2.01	+6.62
2.2	6.0	2.2	4.9	2.2	4.7	2.03	4.58		
−3.8	−0.4	−3.3	−2.1	−3.4	−2.5	2.90	1.63	−2.90	−1.23
1.0	3.2	1.5	3.3	1.3	1.9	1.33	2.58		
−0.1	+3.1	−0.8	+2.8	−0.1	+1.4	.24	3.27	−.10	+3.27
.9	1.5	.6	1.4	.7	1.3	.76	1.73		
+1.0	+7.7	+2.0	+8.8	+1.5	+6.5	.95	6.34	+.91	+6.34
.6	2.9	1.2	3.6	.9	1.8	1.20	3.34		
−2.8	+4.7	−3.1	+4.2	−3.7	+4.2	3.12	3.38	−3.12	+2.96
2.2	1.8	1.9	1.4	2.3	3.4	1.73	4.19		
+1.1	+8.5	+1.0	+5.1	+0.5	+5.7	.62	7.83	+.24	+7.83
1.3	4.6	.8	2.1	1.3	3.0	1.18	3.40		
−2.7	+1.8	−2.7	+0.6	−2.3	+0.9	2.32	4.82	−2.32	+4.82
.9	4.6	.7	4.2	1.0	3.6	.92	4.56		
0	+5.8	−0.8	+4.8	−0.2	+4.1	.54	6.93	+.28	+6.93
.9	1.2	.6	1.8	.5	1.7	.80	2.12		
+4.3	+4.7	+4.2	+4.1	+4.4	+4.2	3.47	4.95	+3.47	+4.95
1.6	4.5	1.4	1.5	1.3	1.6	1.74	4.82		
−3.8	+5.0	−2.8	+4.1	−2.3	−0.5	3.04	3.74	−3.04	+3.24
3.4	12.0	3.4	13.2	2.7	7.7	3.11	11.2		
+0.5	+16.6	−0.5	+17.5	−2.5	+20.1	2.68	12.6	−2.58	+11.7
5.6	4.8	6.5	3.7	5.6	3.7	3.90	4.82		
−2.8	+1.7	−1.7	−0.9	−1.1	−0.3	2.81	3.55	−2.81	+3.11
4.2	6.4	4.7	5.7	4.7	5.0	4.32	5.35		
−0.5	+1.9	+0.7	−0.1	−1.2	−0.3	1.18	2.36	−1.18	+2.16
1.6	4.3	1.1	1.7	1.6	2.7	1.80	3.47		
+1.2	+4.7	+0.6	+3.7	+0.9	+1.4	1.06	4.10	+1.06	+4.10
.8	2.1	.8	2.5	1.1	1.7	.68	2.03		
+2.3	+5.6	+2.6	+5.1	+2.0	+4.5	2.37	4.78	+2.37	+4.78
1.1	1.6	.8	1.1	.8	1.4	.75	1.63		
+0.4	+16.8	+0.7	+17.0	+0.7	+15.7	.38	11.1	+.12	+11.1
1.2	3.0	1.5	3.0	1.3	3.2	1.75	2.89		
−0.6	+8.8	0	+9.1	−1.3	+9.1	1.08	6.16	−1.08	+6.16
1.4	4.2	2.4	4.3	.7	4.4	1.74	3.39		
+0.9	+9.2	+0.8	+4.4	+2.0	+3.7	.66	7.03	+.34	+7.03
1.9	5.8	1.6	2.8	1.8	2.8	.82	4.60		
−2.2	+8.1	−2.1	+6.4	−1.1	+4.9	1.72	5.26	−1.76	+5.26
1.0	2.7	1.1	1.6	1.3	2.4	1.08	3.51		
−1.4	−0.6	−1.8	−1.0	−0.9	−0.2	.77	1.30	−.61	+.94
1.2	1.2	.8	1.2	.9	1.5	1.05	1.93		
−0.7	+8.2	−1.0	+7.2	−0.9	+5.9	.78	4.87	−.78	+3.93
1.1	4.7	.6	2.2	.0	1.6	.94	2.63		

Obs.	0-30		0-23		
	256	286	256	279	25

Obs. 0-30 0-23
256 286 256 279 25
Obs. 0-30 0-23
256 286 256 279 25
Obs. 0-30 0-23
256 286 256 279 25

Obs.	0-30 256	0-30 286	0-23 256	0-23 279	25
1	+1.0	+1.0	+0.2	+2.6	+0
	1.0	1.4	.6	2.0	
3	+2.8	+3.6	+2.2	+6.2	+2
	.8	2.8	.6	2.4	1
5	+0.6	+6.0	+1.6	+5.0	+1
	1.4	1.6	1.6	2.0	2
7	+1.4	+0.8	+1.8	+2.8	+0
	1.0	2.0	1.0	1.4	1
9	+0.6	+1.6	+0.2	−0.6	+0
	1.0	1.2	1.4	2.0	
11	+4.4	+4.4	+4.6	+6.4	+4
	1.6	2.4	1.6	4.0	1
13	+1.6	+1.0	+2.2	+2.8	+2
	1.1	1.0	.9	1.0	1
15	−0.8	−0.6	−0.8	−0.2	−0
	1.0	1.0	1.0	2.4	1
17	+1.2	+8.0	+0.2	+10.0	0
	1.4	2.8	1.8	4.4	
19	+2.2	+0.8	+2.2	+2.0	+1
	.6	2.2	1.0	2.4	1
21	+1.0	+0.6	+0.8	+0.8	+1
	1.1	2.0	1.2	1.4	1
23	+2.0	+0.8	+2.8	+2.0	+2
	1.0	1.0	1.4	1.4	1
25	+1.6	+2.0	+1.4	+3.8	+1
	1.2	.4	2.2	2.4	1
27	+2.2	−3.6	+2.0	−1.4	+2
	2.2	2.0	2.4	2.8	2
29	+0.7	0.0	+1.6	+2.8	+1
	1.4	2.0	1.2	2.2	1
31	+1.8	−0.6	+2.2	−0.2	+1
	1.1	1.6	1.0	2.0	
33	−0.4	+0.4	−1.0	+0.2	−1
	1.0	2.0	1.8	1.8	
35	+3.0	−0.4	+3.9	+0.4	+3
	1.8	2.0	1.3	2.4	
37	+0.8	−1.0	+0.1	+0.8	+0
	.9	1.4	.7	2.6	
39	+1.6	+2.6	+0.2	+4.0	+1
	.8	1.4	1.8	2.2	1
41	+2.0	+3.6	+1.8	+4.4	+2
	.8	.8	1.0	1.4	1
43	+3.2	+3.6	+2.0	+5.0	+3
	2.8	1.6	1.6	6.0	1
45	−0.6	−1.0	−1.4	+0.8	−1
	.6	1.4	.6	1.8	
47	−0.6	+0.6	−1.2	+4.6	+0
	1.4	2.6	.8	1.6	1
49	+2.0	+1.8	+2.4	+6.0	+2
	3.6	1.4	2.4	2.6	2
51	+2.0	+1.2	+2.8	+2.4	+2
	1.6	1.6	1.6	3.4	1
53	+1.2	−0.4	−0.4	+2.8	0
	1.6	1.6	2.0	2.2	2
211	+2.8	+2.4	+2.6	+6.6	+2
	1.2	1.6	1.0	2.4	1
213	+1.2	+2.0	+1.8	+1.8	+0
	.8	1.2	.6	1.2	1

Average Arithmetic O 128	V 138	Average Algebraic O 128	V 138
.46	6.69	+.29	+6.69
1.74	2.34		
2.92	5.09	—2.92	+2.90
2.02	6.36		
.12	4.05	—.06	+4.05
.78	2.06		
.99	4.70	+.99	+4.70
.51	1.83		
.13	2.58	+.05	+2.54
.49	1.23		
.24	2.46	+.18	+2.46
.60	2.02		
1.74	27.0	+1.7	+27.8
3.25	6.47		
.67	3.48	—.57	+3.48
.91	1.55		
.55	6.62	—.35	+6.62
1.43	3.97		
2.09	5.71	—2.09	+5.71
2.40	4.03		
2.66	3.39	—2.66	+2.53
2.52	5.36		
2.01	6.84	—2.01	+6.62
2.03	4.58		
2.90	1.63	—2.90	—1.23
1.33	2.58		
.24	3.27	—.10	+3.27
.76	1.73		
.95	6.34	+.91	+6.34
1.20	3.34		
3.12	3.38	—3.12	+2.96
1.73	4.19		
.62	7.83	+.24	+7.83
1.18	3.40		
2.32	4.82	—2.32	+4.82
.92	4.56		
.54	6.93	+.28	+6.93
.80	2.12		
3.47	4.95	+3.47	+4.95
1.74	4.82		
3.04	3.74	—3.04	+3.24
3.11	11.2		
2.68	12.6	—2.58	+11.7
3.90	4.82		
2.81	3.55	—2.81	+3.11
4.32	5.35		
1.18	2.36	—1.18	+2.16
1.80	3.47		
1.06	4.10	+1.06	+4.10
.68	2.03		
2.37	4.78	+2.37	+4.78
.75	1.63		
.38	11.1	+.12	+11.1
1.75	2.89		
1.08	6.16	—1.08	+6.16
1.74	3.39		
.66	7.03	+.34	+7.03
.82	4.60		
1.72	5.26	—1.76	+5.26
1.08	3.51		
.77	1.30	—.61	+.94
1.05	1.93		
.78	4.87	—.78	+3.93
.94	2.63		

Average
Arithmetic Algebraic
O V O V
128 138 128 138

TABLE VII, B.

Obs.	0-30 256	286	0-23 256	279	0-17 256	273	0-12 256	268	0-8 256	264
1	+1.0	+1.0	+0.2	+2.6	+0.2	+2.0	+0.6	+4.2	+0.6	+6.0
	1.0	1.4	.6	2.0	.6	1.8	1.4	1.8	1.0	2.4
3	+2.8	+3.6	+2.2	+6.2	+2.2	+5.8	+2.0	+4.4	+1.8	+3.6
	.8	2.8	.6	2.4	1.0	4.0	1.2	2.8	1.4	2.4
5	+0.6	+6.0	+1.6	+5.0	+1.0	+5.0	+1.0	+8.0	+0.8	+7.2
	1.4	1.6	1.6	2.0	2.2	2.0	1.8	2.0	1.2	1.6
7	+1.4	+0.8	+1.8	+2.8	+0.8	+4.8	+1.4	+6.4	+1.2	+6.4
	1.0	2.0	1.0	1.4	1.2	2.2	1.0	2.8	.8	2.4
9	+0.6	+1.6	+0.2	—0.6	+0.5	+0.4	—0.6	0.0	—1.0	+1.2
	1.0	1.2	1.4	2.0	.9	2.4	1.4	1.2	1.8	1.6
11	+4.4	+4.4	+4.6	+6.4	+4.8	+8.0	+5.4	+8.8	+5.4	+9.8
	1.6	2.4	1.6	4.0	1.2	3.0	1.6	1.6	1.4	3.4
13	+1.6	+1.0	+2.2	+2.8	+2.2	+4.6	+1.4	+5.2	+2.0	+6.6
	1.1	1.0	.9	1.0	1.0	1.2	1.2	1.4	.8	3.0
15	—0.8	—0.6	—0.8	—0.2	—0.8	+1.2	—1.0	+1.8	—1.2	+3.2
	1.0	1.0	1.0	2.4	1.4	3.0	.9	1.8	1.2	1.8
17	+1.2	+8.0	+0.2	+10.0	0.0	+10.6	—0.4	+8.6	—0.4	+6.2
	1.4	2.8	1.8	4.4	.8	6.0	1.2	4.2	1.2	7.0
19	+2.2	+0.8	+2.2	+2.0	+1.2	+2.2	+0.4	+2.8	0.0	+5.6
	.6	2.2	1.0	2.4	1.2	2.2	1.2	1.6	.8	2.8
21	+1.0	+0.6	+0.8	+0.8	+1.4	+2.6	+0.8	+4.8	+1.5	+5.8
	1.1	2.0	1.2	1.4	1.0	1.6	1.2	2.0	1.5	2.3
23	+2.0	+0.8	+2.8	+2.0	+2.6	+2.4	+2.6	+4.8	+2.8	+6.8
	1.0	1.0	1.4	1.4	1.6	1.4	.8	1.0	1.0	1.6
25	+1.6	+2.0	+1.4	+3.8	+1.4	+3.4	+1.0	+4.4	+1.2	+6.6
	1.2	.4	2.2	2.4	1.6	1.2	1.6	1.2	2.1	1.4
27	+2.2	—3.6	+2.0	—1.4	+2.0	+1.6	+1.4	+4.0	+1.0	—7.2
	2.2	2.0	2.4	2.8	2.0	4.4	2.0	4.8	2.2	5.0
29	+0.7	0.0	+1.6	+2.8	+1.0	+5.6	+0.6	+6.0	+0.6	+8.4
	1.4	2.0	1.2	2.2	1.0	1.8	1.6	2.0	1.6	1.6
31	+1.8	—0.6	+2.2	—0.2	+1.9	+1.8	+2.7	+4.0	+2.8	+4.8
	1.1	1.6	1.0	2.0	.9	3.0	1.1	2.4	1.4	2.0
33	—0.4	+0.4	—1.0	+0.2	—1.4	+0.6	—0.8	+2.6	—0.8	+4.0
	1.0	2.0	1.8	1.8	.9	2.4	1.2	2.6	1.5	2.5
35	+3.0	—0.4	+3.9	+0.4	+3.6	+2.8	+3.0	+6.4	+2.8	+7.2
	1.8	2.0	1.3	2.4	.8	3.0	1.4	4.0	1.6	3.8
37	+0.8	—1.0	+0.1	+0.8	+0.4	+1.0	—0.3	—0.6	—0.8	—0.8
	.9	1.4	.7	2.6	.8	2.0	1.5	3.2	1.4	2.8
39	+1.6	+2.6	+0.2	+4.0	+1.2	+2.6	+1.2	+4.0	+0.8	+6.2
	.8	1.4	1.8	2.2	1.6	2.0	1.6	1.6	1.6	1.8
41	+2.0	+3.6	+1.8	+4.4	+2.6	+5.4	+2.8	+7.4	+3.4	+9.2
	.8	.8	1.0	1.4	1.0	2.0	.8	2.2	1.0	2.4
43	+3.2	+3.6	+2.0	+5.0	+3.0	+9.0	+2.4	+5.4	+2.4	+3.8
	2.8	1.6	1.6	6.0	1.8	5.2	2.0	7.4	1.2	5.0
45	—0.6	—1.0	—1.4	+0.8	—1.8	+1.0	—0.6	+3.6	—1.0	+4.2
	.6	1.4	.6	1.8	.6	3.2	1.8	4.0	1.0	1.8
47	—0.6	+0.6	—1.2	+4.6	+0.4	+5.0	—0.6	+5.4	—0.4	+6.2
	1.4	2.6	.8	1.6	1.6	1.6	2.2	2.2	1.2	2.2
49	+2.0	+1.8	+2.4	+6.0	+2.2	+8.8	+1.4	+9.2	+1.6	+13.4
	3.6	1.4	2.4	2.6	2.6	2.2	2.2	5.2	2.4	5.0
51	+2.0	+1.2	+2.8	+2.4	+2.8	+1.8	+2.6	+1.0	+2.0	+3.8
	1.6	1.6	1.6	3.4	1.6	2.0	1.0	1.4	.8	1.8
53	+1.2	—0.4	—0.4	+2.8	0.0	+7.6	+0.2	+10.8	+0.4	+9.0
	1.6	1.6	2.0	2.2	2.0	1.8	1.8	3.2	1.6	5.4
211	+2.8	+2.4	+2.6	+6.6	+2.0	+8.2	+1.8	+11.2	+1.6	+11.8
	[illegible]	[illegible]	1.0	2.4	1.2	2.4	1.9	[illegible]	1.2	2.2

Obs. 0-30 0-23 0-17 0-12 0-8
256 286 256 279 256 273 256 268 256 264
Obs. 0-30 0-23 0-17 0-12 0-8
256 286 256 279 256 273 256 268 256 264
Obs. 0-30 0-23 0-17 0-12 0-8
256 286 256 279 256 273 256 268 256 264

VII, B. *Accuracy in singing: women*

-8 | 0-5 | 0-3 | 0-2 | 0-1 | 0-.5 | Average: Arithmetic O V, Algebraic O V
264 256 261 256 259 256 258 256 257 256 256.5 256 266 256 266

-8 | 0-5 | 0-3 | 0-2 | 0-1 | 0-.5 | Average: Arithmetic O V, Algebraic O V
264 256 261 256 259 256 258 256 257 256 256.5 256 266 256 266

-8 | 0-5 | 0-3 | 0-2 | 0-1 | 0-.5 | Average: Arithmetic O V, Algebraic O V
264 256 261 256 259 256 258 256 257 256 256.5 256 266 256 266

-8	0-5		0-3		0-2		0-1		0-.5		Average Arithmetic		Average Algebraic	
											O	V	O	V
264	256	261	256	259	256	258	256	257	256	256.5	256	266	256	266
+6.0	+0.6	+6.0	+1.2	+4.6	+1.4	+5.2	+1.6	+3.2	+1.2	+2.4	.86	3.72	+.86	+3.72
2.4	1.0	2.2	1.2	1.2	1.0	2.4	.4	.6	.8	.8	.90	1.66		
+3.6	+2.4	+3.0	+1.8	+3.2	+1.6	+2.0	+2.0	+2.8	+1.4	+3.0	2.02	3.76	+2.02	+3.76
2.4	1.2	1.2	1.4	1.0	1.6	1.2	.4	1.2	1.0	1.0	1.06	2.00		
+7.2	+1.0	+8.2	+1.0	+8.2	+0.6	+8.2	+0.2	+7.4	+0.4	+4.5	.82	6.77	+.82	+6.77
1.6	1.8	2.0	1.4	1.6	1.0	1.8	1.4	.8	1.2	1.4	1.50	1.68		
+6.4	+1.0	+6.2	+1.8	+5.8	+1.2	+5.0	+3.2	+4.2	+1.0	+3.5	1.48	4.59	+1.48	+4.59
2.4	1.0	2.4	.6	2.4	.8	1.4	2.8	1.6	1.8	1.6	1.20	2.02		
+1.2	+0.2	+3.0	0.0	+2.2	+0.2	+2.6	−0.2	+1.6	+0.6	+1.5	.41	1.47	+.05	+1.35
1.6	1.6	2.2	1.2	2.3	1.0	1.6	1.7	2.6	1.4	1.3	1.34	1.84		
+9.8	+5.6	+9.6	+5.4	+8.6	+5.0	+9.2	+5.8	+8.6	+5.0	+8.4	5.14	8.18	+5.14	+8.18
3.4	1.2	2.8	2.0	2.0	1.0	1.6	1.4	1.6	1.0	1.6	1.40	2.40		
+6.6	+3.0	+4.8	+2.6	+3.6	+2.2	+3.4	+2.4	+3.4	+2.2	+2.6	2.18	3.80	+2.18	+3.80
3.0	.9	2.2	1.1	1.4	1.0	1.6	.8	2.0	.7	.9	.95	1.57		
+3.2	−0.4	+4.2	−1.4	+1.6	−0.8	+1.6	−0.6	+0.4	−1.8	−0.6	.96	1.54	−.96	+1.26
1.8	1.2	1.6	1.6	2.8	.9	2.0	1.2	1.6	.5	.8	1.09	1.98		
+6.2	+0.2	+4.2	−1.2	+4.6	−0.4	+5.4	−0.8	+3.2	−0.4	+5.0	.52	6.58	−.20	+6.58
7.0	1.4	5.6	1.8	6.0	1.6	4.6	2.2	3.2	1.6	3.8	1.50	4.76		
+5.6	−0.2	+4.6	+0.8	+3.8	−0.4	+2.6	−0.8	+1.8	+0.4	+1.0	.86	2.72	+.58	+2.72
2.8	1.0	3.2	1.4	2.4	1.2	1.8	1.4	1.6	1.6	1.5	1.14	2.17		
+5.8	−1.6	+7.0	+1.4	+6.2	+2.0	+6.6	+1.8	+5.2	+1.6	+4.2	1.39	4.38	+1.07	+4.34
2.3	1.2	1.6	1.4	2.3	.8	2.2	.6	.6	.9	1.6	1.09	1.76		
+6.8	+2.6	+7.0	+2.4	+8.4	+2.0	+7.4	+2.8	+6.2	+2.4	+5.2	2.50	5.10	+2.50	+5.10
1.6	1.2	2.4	.4	1.4	0	1.6	1.2	1.9	.5	.8	.91	1.45		
+6.6	+1.8	+8.8	+2.4	+6.6	+2.0	+7.6	+2.2	+6.0	+2.4	+4.5	1.74	5.37	+1.74	+5.37
1.4	1.4	1.8	1.6	3.6	1.9	1.6	2.2	2.0	1.6	1.9	1.74	1.75		
−7.2	−0.5	+7.8	+1.0	+9.4	+1.2	+6.2	+0.2	+4.4	0.0	+1.6	1.15	4.72	+1.05	+3.72
5.0	1.7	6.0	2.0	4.4	1.4	3.8	1.4	3.6	2.0	4.0	1.93	4.08		
+8.4	+0.6	+10.8	+1.6	+10.6	+0.9	+10.2	+0.6	+7.2	+0.8	+6.4	.90	6.80	+.90	+6.80
1.6	1.8	1.4	1.2	2.0	1.0	1.0	1.8	2.2	1.6	1.3	1.42	1.75		
+4.8	+2.0	+3.6	+1.2	+2.8	+1.6	+4.0	+2.0	+3.6	+2.6	+4.1	2.08	2.95	+2.08	+2.79
2.0	1.2	2.0	1.4	1.0	2.4	1.2	.8	1.3	1.8	1.9	1.31	1.84		
+4.0	−0.8	+3.8	−0.5	+3.6	−1.8	+3.8	−1.8	+2.8	−1.2	+1.6	1.05	2.34	−1.05	+2.34
2.5	1.2	2.2	1.7	2.0	1.0	2.2	1.4	2.4	2.0	1.3	1.37	2.14		
+7.2	+3.0	+8.0	+3.2	+7.8	+2.6	+8.4	+3.0	+8.8	+2.4	+7.6	3.05	5.78	+3.05	+5.70
3.8	1.2	3.2	1.0	3.0	1.4	3.2	1.4	2.6	.8	2.8	1.27	3.00		
−0.8	−1.4	−1.2	−2.0	−1.6	−1.4	−1.4	−2.2	−0.8	−2.0	−1.2	1.14	1.04	−.88	−.68
2.8	1.4	3.3	1.2	2.2	1.4	1.4	1.8	1.2	1.3	1.3	1.24	2.14		
+6.2	0.0	+6.8	0.0	+6.0	−0.2	+6.2	0.0	+5.0	0.0	+4.8	.52	4.82	+.48	+4.80
1.8	2.0	2.2	2.0	1.8	1.4	2.6	2.0	2.8	1.6	2.4	1.64	2.08		
+9.2	+3.0	+10.2	+2.0	+10.0	+2.8	+8.6	+2.6	+7.6	+2.6	+5.6	2.56	7.20	+2.56	+7.20
2.4	1.0	2.8	.8	3.0	1.2	2.6	1.0	1.4	1.0	1.6	.96	2.02		
+3.8	+2.2	+0.8	+2.4	+1.6	+2.0	+0.6	+1.8	+1.6	+1.4	+0.4	2.28	3.18	+2.28	+3.18
5.0	1.8	3.0	1.2	2.6	1.2	1.4	1.4	1.9	1.8	2.8	1.68	3.69		
+4.2	−1.0	+4.2	−0.8	+4.2	−1.3	+3.8	−1.6	+3.2	−2.0	+2.0	1.21	2.80	−1.21	+2.60
1.8	1.4	2.0	1.6	3.6	1.9	3.4	2.0	2.6	1.2	2.0	1.27	2.58		
+6.2	0.0	+4.8	+0.2	+3.2	−0.4	+2.8	−0.6	+1.6	−0.7	0.0	.51	3.42	−.39	+3.42
2.2	.8	3.4	.6	3.0	.8	2.4	1.2	1.8	1.1	1.3	1.17	2.21		
+13.4	+1.6	+9.0	+1.2	+14.2	+2.8	+13.0	+1.8	+10.4	+2.4	+5.8	1.94	9.16	+1.94	+9.16
5.0	2.0	7.0	3.6	4.4	3.2	4.6	3.4	5.4	2.4	4.6	2.78	4.24		
+3.8	+1.2	+3.8	+2.8	+4.0	+3.0	+3.8	+2.2	+4.2	+3.6	+4.4	2.50	3.04	+2.50	+3.04
1.8	1.2	1.6	.8	1.4	1.0	1.0	1.0	.8	.8	1.4	1.14	1.64		
+9.0	+0.8	+6.0	+0.8	+4.0	−0.8	+6.0	−0.4	+1.8	+0.6	+1.2	.56	4.96	+.24	+4.88
5.4	1.2	5.4	1.6	2.6	1.6	6.4	2.8	4.8	2.6	[illegible]	[illegible]	[illegible]		
+11.8	+1.4	+11.6	+1.0	+10.6	+0.8	+10.8	+1.4	+9.6	+0.6	+8.2	1.60	9.10	+1.60	+9.10
2.2	1.0	[illegible]	1.4	3.6	1.6	4.4	1.0	3.0	.6	3.4	1.12	2.91		

66	0.0	+1.9	—0.3	+2.9	+0.2	+0.9	+0.2	+4.8	+0.1	+4.2	—0.1	+4.7	—0.6	[illegible]	[illegible]	[illegible]
	1.2	.9	1.3	3.1	.8	3.1	1.4	3.2	1.1	1.8	1.5	1.7	1.4	1.1	1.6	2.8
68	+1.4	+3.8	+1.2	+3.2	+1.3	+2.8	+1.8	+4.5	+1.2	+6.5	+0.8	+8.5	+0.6	+8.1	+0.8	+8.3
	1.2	1.0	.4	2.2	.3	1.8	.8	.7	.6	1.1	.4	1.1	.6	1.3	.8	1.9
70	+1.4	—4.3	+1.4	—3.8	+1.3	0.0	+2.0	+1.8	+2.2	+3.3	+2.8	+7.5	+2.8	+8.7	+2.2	+8.5
	2.4	3.7	2.4	5.4	2.5	4.8	3.0	4.6	2.0	3.7	2.4	2.3	2.4	5.1	2.0	4.7
72	+2.3	—1.0	+1.4	+0.6	+1.5	+1.0	+1.7	+1.8	+1.4	+2.3	+0.9	+1.7	+1.0	+3.4	+0.8	+2.9
	1.1	.8	1.2	2.0	.9	.8	.9	1.6	.8	1.7	1.3	1.5	.8	1.4	1.2	1.1
74	—2.6	—2.2	—2.4	+2.7	—1.6	+6.5	—2.3	+10.3	—1.4	+14.3	—1.5	+15.6	0.0	+14.7	+0.6	+13.8
	4.0	2.4	4.2	2.9	3.4	2.9	3.1	2.1	3.0	3.1	3.5	3.0	2.6	5.5	2.4	6.2
76	—3.6	+4.5	—3.5	+4.9	—3.7	+5.7	—2.9	2.1	—3.9	+4.6	—2.5	+3.6	—2.5	+4.0	—2.6	+4.6
	1.0	.7	1.1	1.7	1.7	2.1	3.1	2.9	1.9	4.2	2.9	2.8	2.3	3.2	3.4	3.0
78	—2.0	—0.2	—2.0	+4.4	—2.4	+0.6	—2.5	+3.5	—2.1	+5.5	—1.8	+8.3	—2.1	+7.3	—2.3	+7.4
	.8	.8	1.0	1.0	.8	2.8	.9	2.1	.5	2.5	.8	1.5	.5	2.5	.5	2.2
80	+0.1	—0.2	0.0	+0.3	—1.1	+3.0	—1.6	+5.7	—1.6	+6.0	—1.6	+7.7	—2.1	+7.4	—1.8	+8.9
	2.5	1.4	2.8	2.1	2.5	1.8	3.0	3.1	1.8	2.0	2.2	3.5	2.3	3.4	1.6	2.1
82	—1.1	+1.5	—1.2	+2.9	—1.2	+4.1	—1.3	+4.5	—1.3	+5.5	—1.0	+4.3	—1.2	+2.8	—2.3	+0.6
	1.5	1.9	1.2	1.3	1.4	2.1	1.1	2.5	.9	1.3	1.0	2.5	1.0	2.2	.9	1.4
84	—1.0	+4.1	—1.0	+4.0	—1.4	+3.5	—1.0	+3.0	—1.7	+3.1	—1.9	+3.4	—2.0	+3.9	—2.2	+4.5
	1.2	2.1	1.4	2.0	1.2	2.7	2.0	2.0	1.9	2.1	1.7	2.2	1.2	1.7	1.6	1.5
86	—1.2	+0.5	—1.2	+3.9	—0.9	+6.5	—0.9	+6.7	—0.9	+8.9	—1.3	+9.2	—1.3	+9.8	—1.3	+9.5
	2.4	1.1	2.0	1.3	1.5	3.9	1.9	4.9	1.9	2.3	2.1	1.0	2.1	2.2	1.7	2.3
88	—0.1	+3.5	—0.5	+2.6	+0.2	+5.4	+0.5	+6.0	—0.6	+6.1	—0.8	+4.4	—0.5	+4.6	—0.8	+4.4
	1.3	1.5	.7	1.4	1.0	1.6	.9	1.6	1.0	2.1	1.0	.8	1.3	1.6	1.2	1.6
90	—1.7	+0.7	—2.5	+2.9	—2.3	+4.4	—2.6	+5.4	—2.0	+5.4	—2.3	+7.3	—1.7	+8.9	—1.8	+7.2
	1.7	2.5	1.7	3.1	1.5	3.0	1.8	4.4	1.8	2.2	1.3	2.3	1.3	1.9	1.0	1.4
92	+2.5	+2.5	+3.2	+6.3	+3.6	+3.5	+3.2	+6.7	+3.6	+9.5	+4.3	+9.4	+3.0	+10.0	+4.2	+10.1
	1.9	1.9	1.0	3.7	2.0	2.3	1.4	2.7	2.6	2.3	4.7	4.4	1.6	2.6	2.8	3.1
94	+0.6	+3.3	+0.9	+5.9	+1.9	+5.8	+1.1	+7.0	+0.9	+8.4	+1.1	+7.9	+0.6	+6.0	+0.5	+4.8
	1.0	.7	.9	1.9	1.5	1.0	.7	2.2	1.1	3.8	1.1	2.7	.8	2.2	.9	2.4
96	+0.2	+0.4	—0.4	+2.5	—0.5	+3.8	0	+2.6	—0.2	+3.4	+0.9	+4.2	+0.1	+4.7	+0.2	+4.9
	1.0	2.0	1.0	2.3	.7	3.4	.6	4.4	1.2	3.0	.9	3.0	.9	3.8	.8	3.1
98	+1.5	+3.3	+1.7	+4.0	+2.0	+5.1	+1.9	+7.3	+1.7	+7.8	+1.7	+7.9	+1.9	+7.5	+1.9	+3.4
	.5	.7	.7	2.0	.6	2.7	.5	1.7	.5	2.2	.7	2.1	1.1	2.3	.7	1.6
100	—1.5	+1.7	—1.5	+6.7	—1.7	+6.6	—1.4	+6.2	—1.6	+6.9	—2.4	+5.8	—2.8	+7.7	—2.5	+8.2
	1.5	.7	1.7	1.5	1.3	4.0	1.6	3.4	2.0	3.1	2.0	4.0	2.0	3.9	1.9	2.6
102	—0.2	+1.3	+0.1	+3.1	+0.1	+2.4	+0.5	+1.4	0.0	+3.0	+0.1	+4.2	0.0	+6.0	+0.3	+6.2
	.6	.9	1.1	2.1	1.1	3.0	.7	1.1	.8	1.0	.7	1.4	1.2	1.2	.7	1.2
104	—1.0	+1.0	—1.5	+4.7	—1.1	+5.1	—1.1	+4.6	—1.1	+7.4	—1.4	+8.7	—1.2	+7.4	—0.9	+5.9
	1.2	1.4	.9	1.5	1.5	1.9	1.1	1.9	1.3	3.2	1.6	1.9	1.6	2.6	1.7	2.1
106	—0.4	+3.7	—0.4	+2.9	—0.2	+5.8	—0.2	+6.7	+0.2	+6.9	—0.3	+4.4	+0.1	+4.2	—0.8	+4.5
	.4	1.1	1.0	3.3	.8	2.0	1.0	2.7	.4	2.9	.5	2.4	1.7	2.4	.8	2.3
108	—1.3	+1.2	—1.0	+5.3	—1.6	+6.8	—1.7	+6.4	—1.7	+7.3	—0.3	+8.6	—0.6	+7.8	—1.3	+6.6
	1.9	2.2	2.2	2.1	2.1	3.0	1.1	2.4	1.3	3.6	1.9	2.6	3.0	3.4	1.5	4.2
110	+1.7	+1.4	+1.4	+3.8	+1.7	+4.7	+1.3	+6.2	+1.5	+6.0	+1.6	+6.5	+1.6	+5.4	+1.4	+3.7
	.9	1.0	.6	1.4	.7	1.5	.7	1.6	.9	1.6	1.2	1.7	.8	1.2	.8	1.5
112	+0.6	—4.5	+0.6	—2.2	—1.1	+5.1	—0.2	+8.1	+0.2	+11.9	—0.6	+11.4	+0.6	+12.8	+0.9	+13.4
	1.6	3.9	3.2	3.8	1.7	4.7	1.4	3.3	2.0	3.1	1.2	1.8	1.9	3.4	1.5	1.4
114	—0.3	+1.6	—0.8	+1.3	—0.2	+3.8	—0.1	+3.5	—0.4	+4.6	—0.8	+2.8	—0.7	+4.4	—0.9	+3.8
	.9	1.0	.8	.9	1.0	2.0	1.3	1.9	1.0	2.6	.6	1.0	.9	1.6	1.3	1.4
116	—0.5	+1.8	—1.5	+2.5	—2.3	+3.6	—2.5	+3.6	—2.9	+7.0	—2.8	+9.1	—2.0	+9.0	—2.4	+9.7
	1.1	2.8	1.3	5.5	1.3	4.8	1.3	4.4	1.5	3.8	1.6	3.1	2.0	2.0	.8	2.9
118	—0.6	—0.1	—0.5	+0.9	—0.9	+4.7	—1.1	+4.3	—0.1	+8.1	—0.5	+7.3	—1.0	+8.2	—1.4	+7.1
	1.0	2.3	1.1	4.1	1.3	4.5	1.7	2.1	1.9	2.1	1.9	4.3	2.2	4.8	2.2	4.1
120	+03	+1.9	+0.6	+3.4	+0.3	+4.2	+0.1	+6.8	+0.3	+7.4	+0.3	+6.5	—0.1	+6.1	—0.2	+5.3
	.7	.7	1.2	1.4	.9	2.0	.7	2.4	1.3	1.6	1.1	1.7	.9	1.7	.8	1.7
122	—2.5	—0.7	—3.4	+2.3	—3.9	+5.6	—2.9	+7.3	—2.7	+10.5	—1.2	+12.0	—1.9	+13.2	—3.8	+12.2
	1.7	3.6	1.1	3.8	1.8	3.8	1.3	4.3	.8	2.7	2.3	3.8	1.8	4.2	1.5	3.0
124	—1.2	—1.1	—1.3	+3.6	—1.3	+6.7	—1.1	+9.4	—1.2	+12.3	—1.1	+11.9	—1.2	+12.5	—1.8	+16.6
	1.2	3.5	1.1	3.0	1.5	3.1	1.3	4.6	1.4	6.4	1.5	8.3	1.8	8.1	1.4	8.2
126	—0.5	+0.9	—0.5	+2.1	—0.8	+2.0	—0.7	+3.1	—0.9	+3.6	—0.8	+2.8	—0.7	+3.8	—0.9	+3.5
	.5	.3	.7	1.1	.6	1.4	.3	.9	.5	1.6	.4	1.2	.5	1.0	.3	1.1
128	+0.1	+0.7	0.0	+3.4	—0.6	+3.2	—0.5	+4.6	—0.6	+5.9	—0.4	+5.5	—0.3	+4.5	0.0	+3.8
	.7	1.5	1.0	1.8	.8	2.2	.5	2.0	.6	.7	.6	1.5	.5	2.1	.8	2.4
130	—0.3	—1.2	—0.8	+3.1	—1.1	—0.6	—0.8	+2.2	—1.2	+6.3	—0.8	+8.1	—0.8	+9.2	—0.6	+9.5
	.5	1.8	1.0	1.5	.7	2.8	.4	2.2	.8	2.5	1.0	2.5	.8	1.8	.8	1.5
132	—1.5	—3.2	—2.5	—2.7	—2.7	+1.8	—2.0	+5.0	—2.3	+7.6	—1.7	+10.3	—0.4	+9.8	—1.3	+11.6
	2.0	5.1	2.5	3.6	2.8	2.8	2.5	2.0	2.5	2.1	3.1	2.1	2.6	3.6	2.2	2.3
134	—2.6	+0.7	—2.7	+3.1	—3.0	+3.0	—2.7	+3.7	—3.1	+5.9	—3.9	+5.9	—3.2	+6.4	—3.7	+7.2
	2.2	6.7	2.5	4.1	2.6	4.4	2.7	2.9	2.7	3.1	2.9	4.5	2.8	3.8	2.9	3.6
136	—1.7	+0.3	—0.8	—1.9	—1.2	0.0	—1.2	+4.8	—1.4	+6.7	—1.7	+4.4	—1.5	+5.4	—1.5	+5.1
	.5	2.1	.8	2.9	.6	1.6	.6	2.6	.4	3.1	.3	2.6	.5	2.4	.5	1.3
138	—2.5	—16.0	—7.0	—8.1	—1.4	—3.4	—6.2	+4.1	—1.2	+2.8	+1.5	+8.6	—3.7	+4.6	—1.5	+8.6
	8.3	6.1	7.3	6.1	3.6	3.8	7.8	12.1	3.1	1.8	4.1	6.0	10.1	12.1	5.6	8.1
140	0.0	+3.2	0.0	+4.2	—0.2	+8.3	—0.2	+8.4	+0.2	+8.8	—0.5	+2.8	—0.2	+2.1	—0.6	+0.1
	1.0	3.6	.8	1.8	.6	2.7	.6	5.2	1.0	4.8	1.5	3.6	1.4	4.9	1.6	2.5
142	—0.2	+0.7	+0.8	+1.7	+0.1	+5.8	+1.2	+7.3	+0.5	+4.6	+0.1	5.7	+1.3	+2.8	+1.6	+2.7
	.6	2.3	1.4	2.1	1.5	2.8	2.2	3.7	1.9	4.8	1.3	4.7	.9	1.6	.8	[illegible]
144	+0.5	—0.6	+0.4	+1.2	+1.0	+1.1	+0.6	+2.0	+0.5	+3.2	+0.3	+3.8	[illegible]	[illegible]	—0.2	+4.1
	.7	1.4	.8	1.8	.4	1.3	.6	2.2	.5	1.8	.5	1.6	[illegible]	[illegible]	[illegible]	[illegible]

+3.0	[illegible]	+3.7	−0.3	+5.1	.31	4.09	−.21	+4.09
2.8	1.3	2.3	1.3	2.6	1.29	2.26		
+8.3	+0.7	+6.9	+1.1	+5.0	1.09	5.76	+1.09	+5.76
1.9	.9	1.9	.5	1.1	.65	1.41		
+8.5	−1.9	+7.2	+1.2	+5.5	1.92	5.06	+1.64	+3.44
4.7	1.9	4.4	1.0	1.6	2.20	4.03		
+2.9	+0.7	+3.7	+0.8	+3.3	1.25	2.08	+1.25	+2.08
1.1	1.1	.8	1.0	1.0	1.03	1.27		
+13.8	+1.9	+17.0	+1.8	+15.6	1.61	11.3	−.75	+10.8
6.2	2.5	6.0	2.6	7.7	3.03	4.18		
+4.6	−3.2	+2.7	−2.9	+1.5	3.13	3.82	−3.13	+3.82
3.0	2.8	3.5	2.3	3.4	2.15	2.85		
+7.4	−2.2	+6.8	−1.8	+5.2	2.12	4.92	−2.12	+4.90
2.2	.8	3.4	1.0	3.5	.76	2.23		
+8.9	−2.6	+8.3	−2.2	+8.4	1.47	5.59	−1.45	+5.57
2.1	1.8	2.3	2.0	1.7	2.25	2.34		
+0.6	−2.5	+0.8	−2.2	−1.3	1.53	2.83	−1.53	+2.57
1.4	1.1	2.2	.8	.6	1.09	1.80		
+4.5	−2.7	+2.1	−2.5	+0.2	1.74	3.18	−1.74	+3.18
1.5	1.5	3.1	1.1	2.3	1.48	2.17		
+9.5	−1.9	+7.8	−1.0	+8.4	1.19	7.10	−1.19	+7.10
2.3	1.5	2.6	2.8	2.1	1.99	2.47		
+4.4	−1.1	+3.1	−0.4	+1.8	.55	4.19	−.41	+4.19
1.6	1.3	2.3	1.0	2.1	1.07	1.66		
+7.2	−2.2	+7.2	−1.9	+6.7	2.10	5.61	−2.10	+5.61
1.4	.8	2.8	.9	2.8	1.38	2.64		
+10.1	+4.9	+10.8	+4.4	+10.8	3.69	7.96	+3.69	+7.96
3.1	2.9	2.4	3.0	3.1	2.19	2.85		
+4.8	+0.5	+4.6	+0.7	+4.2	.88	5.79	+.88	+5.79
2.4	1.1	2.2	.9	2.3	1.00	2.14		
+4.9	+0.3	+5.6	+0.8	+5.9	.36	3.80	+.14	+3.80
3.1	1.1	3.6	.8	3.2	.90	3.18		
+3.4	+1.6	+3.1	+1.3	+2.3	1.72	5.17	+1.72	+5.17
1.6	.6	1.1	.5	.6	.64	1.70		
+8.2	−2.9	+5.9	−2.3	+4.6	2.06	6.03	−2.06	+6.03
2.6	2.1	3.1	1.7	3.3	1.78	2.96		
+6.2	+0.4	+7.0	+0.4	+7.2	.21	4.18	+.17	+4.18
1.2	1.4	1.4	1.0	1.7	.93	1.50		
+5.9	+0.1	+3.4	−0.4	+1.1	.96	4.93	−.96	+4.93
2.1	1.1	1.2	1.0	1.6	1.30	1.83		
+4.5	−0.2	+3.8	−0.1	+4.1	.29	4.70	−.23	+4.70
2.3	.8	1.8	.7	1.8	.71	2.27		
+6.6	−0.9	+3.5	−1.2	+3.9	1.16	5.74	−1.16	+5.74
4.2	1.7	3.5	1.4	3.0	1.71	3.00		
+3.7	+1.4	+3.6	+1.3	+3.0	1.49	4.43	+1.49	+4.43
1.5	.8	1.6	.7	.9	.81	1.40		
+13.4	+0.1	+14.4	−0.5	+12.6	.54	9.64	+.06	+8.30
1.4	1.5	3.6	1.7	5.3	1.72	3.43		
+3.8	−1.2	+3.3	−1.3	+2.9	.67	3.20	−.67	+3.20
1.4	1.2	1.1	1.3	.8	1.03	1.43		
+9.7	−2.9	+8.7	−2.7	+9.3	2.25	6.43	−2.25	+6.43
2.9	1.1	2.7	1.3	3.0	1.33	3.50		
+7.1	−1.4	+6.6	−2.1	+2.0	.96	4.93	−.96	+4.91
4.1	2.8	4.6	2.5	4.7	1.86	3.76		
+5.3	−0.3	+4.2	−0.1	+3.4	.26	4.92	+.12	+4.92
1.7	.9	1.0	.9	.5	.94	1.47		
+12.2	−2.8	+12.2	−3.5	+13.5	2.86	8.95	−2.86	+8.81
3.0	2.0	2.7	1.5	3.7	1.58	3.66		
+16.6	−1.5	+15.6	−1.1	+17.4	1.28	10.7	−1.28	+10.5
8.2	1.5	8.0	1.7	8.7	1.44	6.24		
+3.5	−1.0	+2.3	−0.9	+0.9	.77	2.50	−.77	+2.50
1.1	.4	.7	.5	.4	.47	.97		
+3.8	−0.1	+3.6	+0.1	+2.7	.27	3.79	−.23	+3.79
2.4	.9	2.3	.5	1.6	.63	1.80		
+9.5	−0.6	+7.4	−0.9	+6.6	.79	5.42	−.79	+5.08
1.5	.6	2.0	1.1	2.7	.77	2.13		
+11.6	−0.4	+9.5	−0.3	+10.2	1.51	7.17	−1.51	+5.99
2.3	3.1	3.7	2.0	2.0	2.53	2.93		
+7.2	−4.0	+3.0	−3.5	+1.7	3.24	4.06	−3.24	+4.06
3.6	2.6	5.0	2.5	5.0	2.37	4.31		
+5.1	−1.6	+3.1	−1.2	+2.1	1.38	3.38	−1.38	+3.00
1.3	.6	1.9	.6	2.0	.54	2.25		
+8.6	−2.5	+4.1	−5.2	+1.6	3.27	6.19	−2.97	+.69
8.1	5.0	6.8	5.5	12.1	6.04	7.50		
+0.1	−0.6	0.0	−0.9	−0.4	.34	3.83	−.34	+3.75
2.5	1.6	2.6	1.1	2.1	1.12	3.38		
+2.7	+1.8	+2.9	+0.7	+1.9	.83	3.61	+.79	+3.61
[illegible]	1.4	1.7	.7	1.0	1.27	2.70		
+4.1	−0.2	+3.8	+0.2	+3.3	.40	2.74	+.32	+2.62

55	+1.8	−0.2	+1.8	+1.0	+2.8	+2.2
	1.0	1.4	.6	1.6	1.2	2.8
57	+2.0	+2.4	+0.8	+4.6	+1.2	+6.8
	2.4	1.2	2.8	2.2	2.8	2.
59	+4.4	+2.2	+3.2	+2.8	+2.6	+2.
	.4	1.0	1.6	1.0	.6	1.
61	+2.2	+2.2	+1.6	−0.4	+2.2	+1.
	1.0	1.0	1.6	3.0	1.4	1.
63	+0.2	−1.4	−0.4	+0.4	−0.8	−1.
	.2	1.0	.4	2.2	1.2	3.
65	−0.8	−2.2	−2.6	−1.2	−2.6	−1.
	1.2	1.0	1.0	1.8	1.4	2.
67	−1.6	0.0	−1.4	+2.6	−0.8	+5.
	3.2	3.6	2.2	2.4	1.6	3.
69	+3.0	−1.0	+2.8	+1.6	+2.0	+5.
	1.6	1.6	1.4	2.2	2.0	2.
71	+1.4	+0.6	+1.6	+1.0	+2.8	+4.
	1.8	1.4	1.6	1.4	1.6	2.
73	−3.2	+1.0	+3.2	+1.0	−3.2	+1.
	1.2	1.4	2.0	1.8	1.2	1.
75	−1.6	+3.4	−1.0	+4.4	−1.8	+4
	1.2	3.0	1.4	1.8	1.0	2
77	−1.0	−0.8	−0.8	+0.8	−1.2	+4
	1.0	2.0	.8	2.6	.8	2
79	+1.2	+1.2	+1.6	+1.0	+2.6	+5
	1.6	2.4	1.6	1.6	1.0	2
81	−1.4	+1.6	−1.4	+4.2	−1.6	+6
	1.4	1.2	1.4	1.2	.8	2
83	+0.8	+2.4	+0.8	+4.2	+0.6	+2
	1.6	2.0	1.2	2.4	1.0	1
85	−1.4	+1.6	+2.0	+4.0	+1.2	+3
	.6	1.6	.4	1.4	1.2	2
87	+1.6	+1.6	+1.4	+3.0	+0.8	+3
	1.2	1.6	1.8	2.4	2.0	2
89	+1.0	+3.0	+0.8	+6.6	+0.6	+7
	1.4	1.8	2.0	1.2	1.0	2
91	−4.0	+1.0	−3.5	+2.4	−3.0	+
	2.0	2.4	2.0	.4	2.0	
93	+0.4	−0.3	+0.4	+0.4	−0.2	+
	.4	.9	.8	1.1	1.4	
95	−0.4	+2.4	−0.4	+5.6	+3.6	+
	1.6	1.2	1.2	3.0	1.2	
97	+1.2	+1.6	+1.0	+2.8	+1.4	+
	1.2	.8	1.0	1.4	.6	
99	+2.2	+1.0	+3.0	+2.6	+2.8	+
	1.0	1.4	1.8	2.0	1.6	
101	−0.8	+0.6	−0.6	+2.0	−0.6	+
	.8	1.4	1.4	3.4	1.0	
103	+1.8	0.0	+1.0	+3.6	+1.4	+
	.6	1.2	1.8	1.8	1.0	
105	−0.2	+1.2	−0.2	+2.0	−0.4	+
	.2	1.6	.6	2.2	.4	
107	+1.6	+1.6	+1.0	+3.2	+1.4	+
	.8	1.2	1.0	1.8	.6	
109	+0.6	+1.6	+0.2	+4.6	−0.6	+
	1.8	2.8	1.0	2.0	1.8	
111	−0.4	+1.2	+0.2	+2.0	−0.8	+
	1.6	.8	1.4	1.4	1.2	
113	+1.6	+1.2	+1.8	+0.6	+1.4	+
	.4	.8	1.4	2.0	1.0	
115	+2.2	+3.4	+2.4	+6.2	+3.0	+
	1.0	1.4	.4	1.6	1.0	
117	0.0	−0.6	0.0	+2.2	−0.6	+
	.4	.6	1.2	1.6	1.0	
119	+1.8	+2.8	+2.2	+10.2	+1.6	+1
	1.4	1.6	1.4	2.4	2.0	
121	−0.4	+1.4	−0.6	+3.2	−0.6	+
	1.6	1.4	1.4	2.2	1.4	
123	0.0	−0.6	+1.0	+1.0	+1.8	+
	.8	1.8	1.0	2.8	.6	
125	−1.0	−1.0	−0.4	+1.6	+0.4	+
	1.4	1.4	.4	1.8	.8	
127	−0.2	+1.6	−0.4	+2.0	+0.2	+
	1.0	.8	1.6	2.2	1.0	
129	−5.4	+4.0	−6.8	+5.0	−7.0	+
	4.2	3.6	2.6	3.2	2.0	
131	−1.6	+1.0	−1.6	+2.8	−1.6	+
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
133	+3.4	+3.2	+4.0	+7.8	+4.2	+1
					1.8	

5.7	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	+4.09
2.3	1.3	2.6	1.29	2.26		
+6.9	+1.1	+5.0	1.09	5.76	+1.09	+5.76
1.9	.5	1.1	.65	1.41		
7.2	+1.2	+5.5	1.92	5.06	+1.64	+3.44
4.4	1.0	1.6	2.20	4.03		
3.7	+0.8	+3.3	1.25	2.08	+1.25	+2.08
.8	1.0	1.0	1.03	1.27		
7.0	+1.8	+15.6	1.61	11.3	—.75	+10.8
6.0	2.6	7.7	3.03	4.18		
2.7	—2.9	+1.5	3.13	3.82	—3.13	+3.82
3.5	2.3	3.4	2.15	2.85		
6.8	—1.8	+5.2	2.12	4.92	—2.12	+4.90
3.4	1.0	3.5	.76	2.23		
8.3	—2.2	+8.4	1.47	5.59	—1.45	+5.57
2.3	2.0	1.7	2.25	2.34		
0.8	—2.2	—1.3	1.53	2.83	—1.53	+2.57
2.2	.8	.6	1.09	1.80		
2.1	—2.5	+0.2	1.74	3.18	—1.74	+3.18
3.1	1.1	2.3	1.48	2.17		
7.8	—1.0	+8.4	1.19	7.10	—1.19	+7.10
2.6	2.8	2.1	1.99	2.47		
3.1	—0.4	+1.8	.55	4.19	—.41	+4.19
2.3	1.0	2.1	1.07	1.66		
7.2	—1.9	+6.7	2.10	5.61	—2.10	+5.61
2.8	.9	2.8	1.38	2.64		
0.8	+4.4	+10.8	3.69	7.96	+3.69	+7.96
2.4	3.0	3.1	2.19	2.85		
4.6	+0.7	+4.2	.88	5.79	+.88	+5.79
2.2	.9	2.3	1.00	2.14		
5.6	+0.8	+5.9	.36	3.80	+.14	+3.80
3.6	.8	3.2	.90	3.18		
3.1	+1.3	+2.3	1.72	5.17	+1.72	+5.17
.1	.5	.6	.64	1.70		
5.9	—2.3	+4.6	2.06	6.03	—2.06	+6.03
3.1	1.7	3.3	1.78	2.96		
7.0	+0.4	+7.2	.21	4.18	+.17	+4.18
.4	1.0	1.7	.93	1.50		
3.4	—0.4	+1.1	.96	4.93	—.96	+4.93
.2	1.0	1.6	1.30	1.83		
3.8	—0.1	+4.1	.29	4.70	—.23	+4.70
.8	.7	1.8	.71	2.27		
3.5	—1.2	+3.9	1.16	5.74	—1.16	+5.74
3.5	1.4	3.0	1.71	3.00		
.6	+1.3	+3.0	1.49	4.43	+1.49	+4.43
.6	.7	.9	.81	1.40		
.4	—0.5	+12.6	.54	9.64	+.06	+8.30
.6	1.7	5.3	1.72	3.43		
.3	—1.3	+2.9	.67	3.20	—.67	+3.20
.1	1.3	.8	1.03	1.43		
.7	—2.7	+9.3	2.25	6.43	—2.25	+6.43
.7	1.3	3.0	1.33	3.50		
.6	—2.1	+2.0	.96	4.93	—.96	+4.91
.6	2.5	4.7	1.86	3.76		
.2	—0.1	+3.4	.26	4.92	+.12	+4.92
.0	.9	.5	.94	1.47		
.2	—3.5	+13.5	2.86	8.95	—2.86	+8.81
.7	1.5	3.7	1.58	3.66		
.6	—1.1	+17.4	1.28	10.7	—1.28	+10.5
.0	1.7	8.7	1.44	6.24		
.3	—0.9	+0.9	.77	2.50	—.77	+2.50
.7	.5	.4	.47	.97		
.6	+0.1	+2.7	.27	3.79	—.23	+3.79
.3	.5	1.6	.63	1.80		
.4	—0.9	+6.6	.79	5.42	—.79	+5.08
.0	1.1	2.7	.77	2.13		
.5	—0.3	+10.2	1.51	7.17	—1.51	+5.99
.7	2.0	2.0	2.53	2.93		
.0	—3.5	+1.7	3.24	4.06	—3.24	+4.06
.0	2.5	5.0	2.37	4.31		
.1	—1.2	+2.1	1.38	3.38	—1.38	+3.00
.9	.6	2.0	.54	2.25		
.1	—5.2	+1.6	3.27	6.19	—2.97	+.69
.8	5.5	12.1	6.04	7.50		
.0	—0.9	—0.4	.34	3.83	—.34	+3.75
.6	1.1	2.1	1.12	3.38		
.9	+0.7	+1.9	.83	3.61	+.79	+3.61
.7	.7	1.0	1.27	2.70		
.8	+0.2	+3.3	.40	2.74	+.32	+2.62
.4	.8	.8	.68	1.51		
.9	—1.3	+2.5	.89	2.98	—.89	+2.60
.7	.9	2.0	1.13	1.81		
.0	—0.5	—0.1	1.11	2.27	—.75	+2.15
.8	.9	1.0	.97	2.14		
.0	+1.0	+5.7	2.14	3.97	+2.14	+3.81
.2	.4	2.6	1.32	3.56		
.6	—0.7	+0.9	.90	3.16	—.64	+2.14

[illegible]	[illegible]	—0.4	—0.4	+2.8	0.0	+7.6	+0.2	+10.8
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.8	1.8	3.2
55	+1.8	—0.2	+1.8	+1.0	+2.8	+2.2	+2.6	+9.2
	1.0	1.4	.6	1.6	1.2	2.8	.6	2.0
57	+2.0	+2.4	+0.8	+4.6	+1.2	+6.8	+1.2	+8.0
	2.4	1.2	2.8	2.2	2.8	2.2	2.4	1.6
59	+4.4	+2.2	+3.2	+2.8	+2.6	+2.6	+2.4	+5.4
	.4	1.0	1.6	1.0	.6	1.6	.8	2.2
61	+2.2	+2.2	+1.6	—0.4	+2.2	+1.6	+1.0	+4.0
	1.0	1.0	1.6	3.0	1.4	1.4	1.4	1.2
63	+0.2	—1.4	—0.4	+0.4	—0.8	—1.6	—0.2	+1.0
	.2	1.0	.4	2.2	1.2	3.0	.6	1.4
65	—0.8	—2.2	—2.6	—1.2	—2.6	—1.2	—2.8	+1.0
	1.2	1.0	1.0	1.8	1.4	2.6	1.6	2.2
67	—1.6	0.0	—1.4	+2.6	—0.8	+5.0	—0.8	+5.4
	3.2	3.6	2.2	2.4	1.6	3.2	1.6	2.6
69	+3.0	—1.0	+2.8	+1.6	+2.0	+5.4	+1.8	+6.8
	1.6	1.6	1.4	2.2	2.0	2.4	1.8	2.6
71	+1.4	+0.6	+1.6	+1.0	+2.8	+4.8	+3.0	+5.4
	1.8	1.4	1.6	1.4	1.6	2.2	1.0	1.8
73	—3.2	+1.0	+3.2	+1.0	—3.2	+1.8	—3.6	+2.2
	1.2	1.4	2.0	1.8	1.2	1.6	1.2	1.4
75	—1.6	+3.4	—1.0	+4.4	—1.8	+4.2	—1.2	+4.8
	1.2	3.0	1.4	1.8	1.0	2.4	1.2	2.8
77	—1.0	—0.8	—0.8	+0.8	—1.2	+4.6	—1.0	+6.0
	1.0	2.0	.8	2.6	.8	2.0	1.4	2.0
79	+1.2	+1.2	+1.6	+1.0	+2.6	+5.0	+3.2	+6.0
	1.6	2.4	1.6	1.6	1.0	2.0	1.2	2.0
81	—1.4	+1.6	—1.4	+4.2	—1.6	+6.2	—0.8	+9.6
	1.4	1.2	1.4	1.2	.8	2.4	1.6	2.8
83	+0.8	+2.4	+0.8	+4.2	+0.6	+2.8	+0.4	+5.6
	1.6	2.0	1.2	2.4	1.0	1.8	1.6	1.6
85	—1.4	+1.6	+2.0	+4.0	+1.2	+3.8	+1.0	+4.8
	.6	1.6	.4	1.4	1.2	2.2	1.0	3.2
87	+1.6	+1.6	+1.4	+3.0	+0.8	+3.8	—0.2	+3.8
	1.2	1.6	1.8	2.4	2.0	2.8	1.4	1.8
89	+1.0	+3.0	+0.8	+6.6	+0.6	+7.0	+0.8	+5.6
	1.4	1.8	2.0	1.2	1.0	2.0	1.2	2.0
91	—4.0	+1.0	—3.5	+2.4	—3.0	+4.0	—3.5	+7.2
	2.0	2.4	2.0	.4	2.0	1.6	2.0	1.4
93	+0.4	—0.3	+0.4	+0.4	—0.2	+2.0	+0.4	+3.2
	.4	.9	.8	1.1	1.4	3.0	.8	1.6
95	—0.4	+2.4	—0.4	+5.6	+3.6	+5.0	—2.0	+5.8
	1.6	1.2	1.2	3.0	1.2	1.6	.8	2.2
97	+1.2	+1.6	+1.0	+2.8	+1.4	+3.2	+1.4	+4.4
	1.2	.8	1.0	1.4	.6	1.0	.6	1.2
99	+2.2	+1.0	+3.0	+2.6	+2.8	+5.0	+3.0	+7.4
	1.0	1.4	1.8	2.0	1.6	2.0	1.8	2.2
101	—0.8	+0.6	—0.6	+2.0	—0.6	+3.2	—0.8	+3.4
	.8	1.4	1.4	3.4	1.0	4.2	1.2	3.0
103	+1.8	0.0	+1.0	+3.6	+1.4	+6.8	+1.8	+7.6
	.6	1.2	1.8	1.8	1.0	2.2	.6	2.4
105	—0.2	+1.2	—0.2	+2.0	—0.4	+2.8	—0.2	+3.6
	.2	1.6	.6	2.2	.4	1.0	1.0	1.2
107	+1.6	+1.6	+1.0	+3.2	+1.4	+4.0	+0.8	+4.6
	.8	1.2	1.0	1.8	.6	1.4	.8	1.4
109	+0.6	+1.6	+0.2	+4.6	—0.6	+6.0	—0.6	+9.0
	1.8	2.8	1.0	2.0	1.8	2.6	1.4	2.2
111	—0.4	+1.2	+0.2	+2.0	—0.8	+2.4	—0.8	+4.0
	1.6	.8	1.4	1.4	1.2	1.4	1.2	.8
113	+1.6	+1.2	+1.8	+0.6	+1.4	+2.2	+1.2	+4.8
	.4	.8	1.4	2.0	1.0	1.2	1.2	1.6
115	+2.2	+3.4	+2.4	+6.2	+3.0	+4.6	+2.4	+7.8
	1.0	1.4	.4	1.6	1.0	1.6	.4	1.8
117	0.0	—0.6	0.0	+2.2	—0.6	+3.2	—0.8	+4.2
	.4	.6	1.2	1.6	1.0	1.0	.8	1.8
119	+1.8	+2.8	+2.2	+10.2	+1.6	+12.2	+2.2	+14.8
	1.4	1.6	1.4	2.4	2.0	3.2	2.6	3.2
121	—0.4	+1.4	—0.6	+3.2	—0.6	+4.2	—1.0	+5.2
	1.6	1.4	1.4	2.2	1.4	3.2	1.4	4.4
123	0.0	—0.6	+1.0	+1.0	+1.8	+2.8	+2.2	+5.0
	.8	1.8	1.0	2.8	.6	1.4	.2	2.2
125	—1.0	—1.0	—0.4	+1.6	+0.4	+2.0	—1.2	+2.6
	1.4	1.4	.4	1.8	.8	3.0	1.2	3.0
127	—0.2	+1.6	—0.4	+2.0	+0.2	+3.6	—0.6	+4.4
	1.0	.8	1.6	2.2	1.0	1.8	.6	1.6
129	—5.4	+4.0	—6.8	+5.0	—7.0	+7.6	—7.6	+10.1
	4.2	3.6	2.6	3.2	2.0	2.6	1.2	3.6
131	—1.6	+1.0	—1.6	+2.8	—1.6	+3.6	—2.2	+2.0
	.8	2.6	2.4	3.0	2.0	2.2	1.8	2.4
133	+3.4	+3.2	+4.0	+7.8	+4.2	+10.6	+3.8	+10.0
	1.4	2.8	1.6	2.4	1.8	3.6	1.0	3.2
135	—3.4	0.0	+3.6	+4.0	+4.0	+7.2	+4.0	+8.8
	1.0	1.6	1.2	1.4	1.2	1.4	1.2	2.0
137	+4.6	+3.2	+6.2	+3.6	+5.6	+6.4	+6.0	+9.6
	1.8	2.0	1.8	2.2	1.4	2.0	1.6	1.2
139	+1.0	+2.4	+1.4	+3.0	+1.2	+5.6	+1.0	+6.4
	1.0	.4	1.0	2.0	.8	2.0	1.0	1.6
[illegible]	+2.2	+2.0	+2.4	+6.2	+1.8	+6.6	+1.8	+7.4

+10.8	+0.4	+9.0	+0.8	+6.0	+0.8	+4.0	−0.8	+6.0	−0.4	+1.8	+0.6	+1.2	.56	4.96	+.24	+4.88
3.2	1.6	5.4	1.2	5.4	1.6	2.6	1.6	6.4	2.8	4.8	2.6	2.4	1.88	3.58		
+9.2	+2.0	+5.4	+3.2	+8.0	+2.2	+7.6	+2.2	+7.8	+2.6	+6.2	+2.2	+5.0	2.34	5.20	+2.34	+5.22
2.0	.8	3.0	.8	3.4	1.0	2.6	1.0	2.6	1.4	1.2	1.8	3.0	1.02	2.36		
+8.0	+1.2	+8.8	+1.2	+6.8	+1.0	+3.6	+1.6	+4.0	0.0	+2.0	+0.8	+1.8	1.10	4.88	+1.10	+4.88
1.6	2.4	2.0	2.8	2.2	1.8	3.0	1.6	3.2	2.4	2.2	2.0	2.5	2.34	2.23		
+5.4	+2.6	+7.4	+2.6	+5.8	+2.8	+5.4	+1.8	+5.6	+3.4	+5.2	+3.0	+4.6	2.88	4.70	+2.88	+4.70
2.2	1.0	1.4	.6	1.6	.8	1.2	.6	2.4	1.0	1.9	1.4	2.2	.88	1.65		
+4.0	+1.2	+6.4	+2.2	+7.8	+2.4	+6.8	+2.0	+6.2	+2.0	+6.0	+2.0	+6.0	1.88	4.74	+1.88	+4.66
1.2	1.2	2.0	1.4	2.0	.4	2.6	.8	2.6	.4	1.8	.4	1.6	1.00	1.92		
+1.0	−0.6	+4.4	−0.6	+6.4	−1.4	+4.8	−0.8	+2.6	−1.0	+0.6	−1.4	+0.5	.74	2.37	−.70	+1.77
1.4	1.4	1.6	1.8	1.4	1.0	3.8	1.2	1.8	1.4	2.7	1.0	1.7	1.02	2.06		
+1.0	−2.8	+3.0	−2.6	+1.8	−2.8	+2.6	−3.2	+2.6	−2.8	+3.2	−2.8	+1.8	2.58	2.06	−2.58	+1.14
2.2	1.2	2.2	1.4	1.2	.8	2.0	1.2	3.0	1.2	2.2	1.2	1.8	1.22	2.00		
+5.4	−1.0	+4.8	−1.4	+5.4	−1.6	+5.8	−1.0	+6.6	−2.2	+4.6	−1.6	+4.0	1.34	4.42	−1.34	+4.42
2.6	1.4	2.8	1.8	4.0	1.6	3.2	1.8	4.2	1.8	2.8	2.8	3.6	1.98	3.24		
+6.8	+3.0	+8.4	+2.2	+9.6	+1.2	+8.8	+1.8	+8.8	+1.8	+8.6	+1.4	+3.8	2.10	6.19	+2.10	+6.19
2.6	1.6	2.8	.6	2.2	1.2	1.4	1.0	2.8	.6	2.0	.6	2.2	1.24	2.22		
+5.4	+3.0	+6.8	+2.8	+7.4	+2.8	+7.8	+2.4	+6.0	+1.8	+6.0	+2.0	+6.2	2.36	5.20	+2.36	+5.20
1.8	1.8	2.8	1.6	1.6	1.2	2.4	1.2	.8	1.4	2.2	.8	1.0	1.40	1.76		
+2.2	−2.6	+4.6	−1.4	+6.0	−3.6	+6.2	−4.0	+5.2	−3.8	+5.8	−5.0	+4.8	3.36	3.86	−3.36	+3.86
1.4	3.4	1.4	3.0	2.2	1.6	1.6	1.6	1.6	1.4	1.2	2.2	1.6	1.88	1.58		
+4.8	−1.6	+6.8	+0.2	+8.2	−2.0	+7.4	−2.2	+5.4	−2.8	+5.6	−2.4	+5.2	1.46	5.54	−1.46	+5.54
2.8	.8	2.8	1.4	2.8	1.6	1.6	1.4	1.8	1.6	2.2	1.6	2.4	1.32	2.36		
+6.0	−1.2	+7.4	−0.2	+6.0	−0.6	+6.4	−1.4	+4.0	−1.2	+2.6	−1.0	+2.2	.96	4.08	−.96	+3.92
2.0	1.2	1.8	.6	2.6	.6	3.0	1.0	1.6	.8	1.2	1.0	1.3	.92	2.01		
+6.0	+2.4	+7.2	+2.2	+7.6	+2.4	+8.0	+1.8	+7.0	+1.6	+5.2	+1.4	+5.0	2.04	5.32	+2.04	+5.32
2.0	2.4	2.8	2.2	3.0	3.2	3.8	2.6	2.6	4.0	3.0	2.6	1.1	2.24	2.43		
+9.6	−1.2	+8.2	+1.0	+6.4	+0.4	+4.6	+0.2	+2.4	−0.2	+0.8	+1.0	+1.8	.92	4.58	−.40	+4.58
2.8	1.2	3.0	1.8	4.6	1.2	4.4	.6	3.6	1.0	1.4	1.0	1.4	1.20	2.60		
+5.6	+0.6	+6.0	+0.6	+8.6	+0.4	+5.2	+0.8	+5.0	+1.0	+5.0	+0.6	+4.4	.66	4.72	+.66	+4.72
1.6	1.4	2.0	1.8	2.8	1.2	2.2	1.2	2.2	1.8	1.2	1.4	1.8	1.42	2.00		
+4.8	+0.6	+7.6	+1.2	+9.0	+1.4	+8.8	+1.6	+7.2	+1.2	+8.4	+1.2	+6.6	1.28	6.18	+1.28	+6.18
3.2	1.4	2.4	1.2	1.6	.6	1.8	1.6	2.4	1.2	2.2	1.2	2.2	1.04	2.10		
+3.8	+0.6	+5.6	+0.4	+6.2	0.0	+6.8	−1.0	+6.2	0.0	+4.2	+0.6	+4.0	.66	4.52	+.42	+4.52
1.8	1.0	1.2	.8	2.1	1.2	1.4	1.8	1.8	1.2	2.0	1.8	2.0	1.42	1.90		
+5.6	+0.2	+4.8	−0.6	+6.4	−0.4	+4.6	−0.4	+4.6	−0.4	+4.6	−0.4	+2.2	.56	4.94	+.12	+4.94
2.0	1.4	2.8	1.4	1.4	2.0	3.6	1.6	2.2	1.2	2.4	2.4	2.2	1.56	2.16		
+7.2	−4.4	−2.4	−4.4	−7.0	−4.0	−6.4	−4.6	−6.0	−5.0	−1.5	−4.4	−5.2	4.08	4.31	−4.08	−1.40
1.4	2.2	6.0	2.2	3.6	1.6	2.0	2.0	1.5	2.0	2.0	1.8	1.8	1.98	2.27		
+3.2	−0.2	+5.4	+0.4	+5.4	−0.4	+2.6	−0.6	+0.9	−0.3	+2.2	+0.2	+1.2	.35	2.36	+.01	+2.30
1.6	1.4	1.0	.8	1.2	.8	1.6	1.0	1.9	.9	1.4	1.0	1.5	.93	1.52		
+5.8	−1.2	+7.6	−2.6	+7.4	−2.0	+7.8	−3.4	+7.4	−2.8	+7.2	−1.6	+5.2	2.00	6.14	−1.28	+6.14
2.2	.8	2.0	1.4	3.2	1.2	3.2	1.8	3.0	2.0	2.2	.8	2.4	1.28	2.46		
+4.4	+1.6	+3.8	+1.6	+4.6	+2.0	+4.0	+1.8	+4.4	+2.2	+4.2	+1.6	+3.5	1.58	3.65	+1.58	+3.65
1.2	.8	1.4	.8	.8	.4	1.0	.2	.8	.6	.8	.8	.4	.70	.96		
+7.4	+2.2	+8.2	+3.2	+9.8	+3.2	+9.2	+2.8	+8.6	+2.8	+8.2	+2.6	+6.0	2.78	6.60	+2.78	+6.60
2.2	1.0	2.2	2.4	2.8	1.6	3.0	1.6	3.0	1.6	2.2	1.4	3.2	1.58	2.18		
+3.4	−1.0	+4.0	−1.6	+5.0	−1.0	+4.6	−1.2	+4.8	−1.6	+4.0	−2.2	+2.6	1.14	3.42	−1.14	+3.42
3.0	1.4	2.0	1.6	3.2	1.8	2.4	1.2	1.6	1.6	1.8	1.4	1.8	1.34	2.48		
+7.6	+1.2	+9.0	+2.2	+11.6	+1.6	+9.0	+1.8	+10.6	+1.8	+7.6	+1.8	+5.4	1.64	7.12	+1.64	+7.12
2.4	.8	2.2	1.0	1.4	.8	2.0	1.4	4.2	1.4	2.6	1.0	2.2	1.04	2.02		
+3.6	−0.4	+5.2	−0.2	+5.8	−0.6	+5.8	−0.6	+5.4	−1.2	+4.4	−0.8	+3.2	.48	3.94	−.48	+3.94
1.2	.4	1.2	.6	.8	.6	1.6	1.0	1.4	.8	.6	1.2	1.5	.68	1.31		
+4.6	+1.4	+5.6	+1.2	+6.8	+1.4	+6.4	+1.2	+5.4	+0.6	+5.0	+1.0	+4.8	1.16	4.74	+1.16	+4.74
1.4	1.0	1.6	.8	1.4	.6	1.4	1.2	2.2	1.4	1.6	1.0	1.2	.92	1.52		
+9.0	−0.4	+9.0	−1.2	+9.4	−1.4	+7.0	−1.6	+8.6	+2.2	+8.2	−2.4	+5.4	1.12	6.88	−.96	+6.88
2.2	1.2	2.2	1.6	2.0	1.8	3.6	2.0	3.8	2.2	4.0	2.4	3.0	1.72	2.60		
+4.0	−1.0	+5.6	−0.6	+7.4	−1.0	+7.0	−1.4	+5.2	−1.6	+5.8	−1.4	+4.0	.92	4.46	−.88	+4.46
.8	1.0	2.0	1.8	2.4	1.8	2.0	2.2	2.0	1.6	2.4	1.8	1.6	1.56	1.68		
+4.8	+0.6	+6.8	+1.8	+6.2	0.0	+2.8	−0.6	+2.6	−0.6	+4.4	+0.2	+2.0	.98	3.36	+.74	+3.36
1.6	1.4	1.6	1.0	1.6	.4	2.6	.6	1.4	1.4	1.6	.6	1.1	.94	1.55		
+7.8	+2.6	+11.6	+2.2	+11.6	+2.2	+8.4	+3.4	+6.2	+3.0	+6.4	+2.6	+5.4	2.60	7.16	+2.60	+7.16
1.8	.6	2.0	.2	2.2	.6	2.2	.6	3.0	1.0	2.2	1.0	1.4	.68	1.94		
+4.2	−0.8	+6.6	0.0	+7.0	−0.4	+5.4	−0.4	+4.2	−0.6	+3.4	−0.6	+2.0	.42	3.88	−.42	+3.76
1.8	1.2	2.6	1.2	2.4	.8	2.8	1.2	2.1	1.0	.8	1.0	1.0	.98	1.67		
+14.8	+1.8	+14.4	+1.6	+16.8	+1.6	+16.6	+2.0	+14.0	+1.2	+11.6	+2.0	+10.4	1.80	10.90	+1.80	+10.90
3.2	2.6	4.0	2.8	1.6	2.4	2.8	2.4	4.4	3.2	4.4	3.6	4.0	2.18	3.46		
+5.2	−1.0	+5.6	−0.8	+7.0	−0.8	+6.8	−1.0	+5.4	−1.2	+2.2	−1.2	+1.0	.86	4.20	−.86	+4.20
4.4	1.8	3.6	.8	3.2	1.6	3.8	1.4	3.0	1.2	2.0	2.0	1.8	1.46	2.86		
+5.0	+2.4	+5.6	+2.6	+5.2	+3.0	+5.2	+2.2	+5.0	+1.2	+4.6	+0.6	+1.8	1.70	3.68	+1.70	+3.56
2.2	1.2	2.4	1.0	1.8	1.0	2.6	1.4	2.2	1.2	1.6	1.0	2.6	.94	2.14		
+2.6	−1.0	+3.6	−1.2	+0.8	−0.8	+2.8	−1.0	+1.6	−1.0	+1.8	−1.4	+0.8	.94	1.86	−.94	+1.66
3.0	1.0	2.4	.8	2.4	.8	2.2	1.0	2.0	1.0	2.0	1.0	1.7	.94	2.19		
+4.4	−1.0	+5.6	−1.2	+5.0	−1.0	+2.2	−1.8	+3.2	−1.0	+5.0	−1.4	+2.2	.88	3.48	−.86	+3.48
1.6	1.4	1.6	1.2	.8	1.0	1.6	.6	2.0	1.4	2.0	1.0	1.2	1.08	1.56		
+10.1	−5.0	+12.2	−8.0	+14.0	−8.8	+12.2	−9.6	+10.6	−9.4	+10.4	−7.0	+12.6	7.46	9.86	−7.46	+9.86
3.6	2.0	4.6	2.0	3.6	1.8	2.6	1.6	2.6	2.6	2.0	2.4	4.8	2.24	3.32		
+2.0	−1.0	+3.2	−1.6	+4.2	−1.8	+5.6	−1.8	+2.8	−1.2	+1.4	−2.4	−0.8	1.68	2.74	−1.68	+2.55
2.4	2.2	2.0	1.6	2.4	1.4	1.8	1.4	2.0	.8	2.4	2.4	2.8	1.68	2.36		
+10.0	+3.8	+8.0	+7.0	+9.8	+4.6	+8.4	+4.4	+7.0	+4.8	+7.2	+9.0	+8.4	4.90	8.04	+4.90	+8.04
3.2	1.0	4.4	3.8	3.6	1.4	1.8	1.2	1.4	1.6	1.8	1.0	1.6	1.58	2.66		
+8.8	+4.4	+9.0	+3.6	+9.6	+3.4	+9.2	+4.0	+7.6	+4.2	+8.0	+4.0	+6.4	3.86	6.98	+3.86	+6.98
2.0	.8	3.8	1.2	3.4	.6	3.4	1.2	2.0	.6	2.6	1.2	1.2	1.02	1.94		
+9.6	+6.2	+7.4	+6.4	+10.6	+7.0	+10.4	+6.6	+10.2	+6.0	+10.0	+5.6	+8.6	6.02	8.00	+6.02	+8.00
1.2	1.0	1.8	.8	2.0	1.0	1.0	1.4	1.0	.8	1.0	1.2	1.4	1.28	1.56		
+6.4	+0.4	+7.8	+0.4	+9.6	+1.2	+9.6	+1.8	+9.4	+1.2	+8.8	+0.8	+7.2	1.04	6.98	+1.04	+6.98
1.6	.4	1.8	1.2	1.4	.8	1.8	.6	1.8	1.2	2.6	.8	3.2	.88	1.86		
+7.4	+3.2	+8.2	+2.0	+8.8	+1.4	+8.2	+3.2	+7.2	+3.4	+5.4	+3.2	+5.2	2.06	6.52	+2.06	+6.52

	.6	2.5	1.4	2.1	1.5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
144	+0.5	−0.6	+0.4	+1.2	+1.0	+1.1	+0.6	+2.0	+0.5	+3.2	+0.3	+3.8	+0.1	+4.3	−0.2
	.7	1.4	.8	1.8	.4	1.3	.6	2.2	.5	1.8	.5	1.6	.9	.9	.8
146	−0.8	−1.9	−0.3	+1.2	−0.2	+3.1	−1.0	+2.9	−1.3	+2.6	−1.1	+3.7	−0.5	+4.6	−1.1
	1.0	1.3	.7	2.6	1.0	2.5	1.2	2.5	1.3	2.2	1.1	1.5	1.9	1.2	1.1
148	−1.3	−0.5	−1.3	+3.3	−1.1	+2.9	−1.3	+3.1	−1.6	+4.3	−1.1	+5.1	+1.8	+3.0	−0.5
	.7	1.3	.9	3.3	.9	2.3	.7	1.5	2.0	2.1	.9	3.3	.8	3.4	.9
150	+1.4	+9.2	+2.4	0.0	+1.8	−0.8	+1.6	+5.0	+2.8	+3.4	+1.8	+2.4	+4.8	+5.4	+2.4
	.8	7.0	1.6	4.0	1.4	3.0	1.2	5.2	2.6	3.6	1.4	2.4	1.4	3.6	1.2
152	−0.7	−0.1	−1.3	+3.0	−0.7	+2.6	−0.9	+4.0	−1.4	+7.3	−1.0	+6.5	−1.0	+3.4	+0.4
	1.9	2.5	1.7	3.2	1.3	2.4	1.1	3.8	1.2	4.1	1.4	4.0	1.0	4.0	1.4
154	−0.1	+1.3	0.0	−0.3	−0.4	+0.2	−0.3	+3.2	−0.1	+4.7	−0.1	+5.5	+0.7	+5.7	0.0
	.5	1.3	.4	4.3	1.0	1.4	.7	1.6	.9	2.5	.9	2.7	1.1	.9	1.2
156	+1.1	+1.1	+1.0	+2.8	+0.8	+3.2	+0.8	+4.3	+0.5	+5.7	+0.7	+6.3	+0.7	+5.5	+0.5
	.7	.7	.4	1.6	.3	2.6	.6	1.9	.5	1.3	.3	1.9	.3	2.1	.7
158	−0.1	+2.6	−0.2	+4.1	−0.2	+6.8	−0.6	+9.1	−0.5	+11.3	−0.3	+9.6	−0.6	+8.4	−0.1
	.5	1.8	.8	2.3	1.0	2.0	.4	1.9	.7	1.7	.5	3.0	.4	2.2	.7
160	−0.6	+1.1	−0.4	+4.8	−0.6	0.0	−1.2	+5.0	−1.5	+6.4	−1.0	+7.8	−1.3	+4.4	−0.9
	1.0	1.7	1.0	1.4	1.0	1.4	1.0	2.8	1.3	1.8	.6	1.2	.9	4.6	1.1
162	−1.0	+2.3	−1.0	+0.4	−0.8	+0.8	−1.3	+1.7	−1.2	+5.7	−1.1	+6.7	−1.1	+5.5	−1.4
	1.0	.9	.6	3.0	.6	2.4	.7	1.1	1.4	1.5	1.3	1.3	1.3	2.9	1.8
164	−0.5	+1.5	+0.5	+2.4	−1.0	+2.4	−0.7	+4.6	−1.0	+5.1	−0.9	+5.5	−0.9	+6.3	−1.2
	.9	.9	.9	1.8	.8	1.6	.7	.8	1.0	.7	.9	.7	.7	1.1	.4
166	−1.5	+1.5	−1.8	+0.7	−1.6	+2.2	−1.4	+1.6	−2.3	+2.5	−2.0	+3.6	−2.5	+4.0	−2.4
	1.3	2.1	1.0	2.1	.8	2.0	1.0	3.4	1.3	3.1	1.4	2.0	.9	3.2	1.0
168	−0.8	+5.4	−1.6	+8.7	−0.9	+5.0	−1.4	+6.3	−1.3	+6.4	−1.2	+5.1	−1.9	+3.7	−2.3
	1.0	2.4	1.0	2.5	1.1	4.4	1.2	5.1	1.1	3.8	1.6	2.7	1.3	2.1	1.5
170	−2.2	+4.8	−3.0	+8.0	−2.7	+6.7	−2.7	+13.4	−1.9	+16.8	−1.3	+13.6	−3.4	+16.2	−1.8
	3.2	1.6	3.4	3.2	2.4	5.0	2.9	3.8	2.7	7.4	3.3	3.1	3.0	5.0	2.2
172	−1.0	1.0	−1.2	−0.9	−1.8	+1.0	−1.5	+0.3	−0.5	+2.3	−1.7	+2.9	−0.6	+3.3	−1.3
	1.6	2.4	1.4	3.9	.8	2.6	1.2	2.1	1.2	1.7	1.4	1.9	1.7	2.0	1.6
174	−1.0	+3.6	−0.5	+7.9	0.0	+7.3	+0.4	+4.2	−0.4	+7.7	0.0	+7.5	−0.2	+6.1	−0.1
	2.6	2.4	2.5	3.3	2.6	2.1	2.2	4.6	2.1	3.7	2.2	1.8	2.6	2.5	2.3
176	+1.1	0.0	+1.7	+3.3	+0.9	+2.7	+1.6	+4.5	+1.7	+6.6	+1.3	+7.5	+1.2	+7.4	+1.3
	1.3	.8	.9	2.1	.5	3.1	1.1	2.7	.7	3.0	.9	2.2	.6	1.7	1.1
178	−2.6	−3.9	−3.1	−1.6	−3.3	−1.1	−4.1	0.0	−4.0	+3.9	−4.5	+4.1	−4.4	+5.0	−4.0
	2.3	2.9	2.7	5.1	2.7	3.7	2.7	5.4	3.2	4.7	2.7	3.3	3.3	3.0	3.2
180	+7.7	−2.6	8.0	+4.8	+6.8	+9.1	+7.2	+11.9	+6.2	+13.6	+5.9	+12.7	+6.9	+13.6	+6.3
	2.4	2.1	2.4	3.6	2.2	3.9	2.0	4.3	2.2	4.1	2.1	4.9	2.3	3.6	2.3
182	−6.2	−6.3	−5.8	−2.1	−5.2	+0.5	−5.3	+1.4	−6.0	+5.4	−5.4	+5.0	−5.2	+1.1	−5.5
	3.0	6.2	2.6	7.1	2.4	7.6	2.5	8.8	3.0	8.5	2.6	6.4	2.6	4.7	2.0
184	+5.3	+0.7	+7.9	+4.4	+5.4	+5.7	+4.5	+7.7	+5.4	+10.5	+6.0	+11.5	+6.1	+13.3	+7.0
	4.5	2.4	5.5	2.9	1.7	4.1	5.7	4.5	6.4	3.8	5.0	4.4	5.5	4.0	5.2
186	+1.8	+2.0	+1.9	+4.2	+1.8	+0.7	+2.4	+3.9	+1.9	+5.8	+2.3	+5.0	+1.9	+4.5	+1.8
	1.0	1.4	.5	3.2	.6	2.5	.6	1.3	.5	1.8	.9	2.2	.3	1.5	.4
188	−0.7	+6.2	−1.0	+3.8	−0.5	+7.8	−0.1	+6.6	−0.8	+10.5	−1.3	+11.6	−1.5	+9.4	−2.2
	2.3	4.4	1.6	6.8	1.9	5.2	2.5	5.6	1.6	4.9	1.1	5.4	1.9	6.4	1.6

—0.2	+4.1	[illegible]	+3.8	+0.2	+3.3	.40	2.74	+.32	+2.62
.8	1.9	.8	1.4	.8	.8	.68	1.51		
—1.1	+4.4	—1.3	+2.9	—1.3	+2.5	.89	2.98	—.89	+2.60
1.1	1.6	1.1	.7	.9	2.0	1.13	1.81		
—0.5	+0.4	—0.6	0.0	—0.5	—0.1	1.11	2.27	—.75	+2.15
.9	1.4	1.0	1.8	.9	1.0	.97	2.14		
+2.4	+3.8	+1.4	+4.0	+1.0	+5.7	2.14	3.97	+2.14	+3.81
1.2	3.0	1.2	1.2	.4	2.6	1.32	3.56		
+0.4	+2.2	+0.9	+1.6	—0.7	+0.9	.90	3.16	—.64	+3.14
1.4	3.4	1.1	2.2	.7	2.0	1.28	3.17		
0.0	+4.5	+0.1	+4.3	—0.5	+3.7	.23	3.34	—.09	+3.28
1.2	1.5	1.3	1.9	1.7	1.4	.97	1.95		
+0.5	+4.5	+1.1	+3.2	+1.1	+3.1	.83	3.97	+.83	+3.97
.7	2.1	1.0	1.6	.5	1.2	.43	1.70		
—0.1	+7.0	—0.1	+6.1	—0.3	+3.7	.30	6.92	—.30	+6.92
.7	2.9	.7	2.7	.5	3.0	.62	2.35		
—0.9	+3.7	—1.5	+5.0	—0.8	+1.4	.98	3.95	—.98	+3.96
1.1	4.3	1.5	4.6	1.0	2.1	1.04	2.59		
—1.4	+2.2	—1.1	+2.0	—1.4	+0.9	1.14	2.82	—1.14	+2.82
1.8	3.8	.7	4.6	.8	2.4	1.02	2.37		
—1.2	+3.5	—1.2	+2.5	—1.3	+1.1	.92	3.49	—.82	+3.49
.4	1.1	.6	.9	.9	1.0	.78	1.06		
—2.4	+3.6	—2.3	+2.4	—2.0	+0.4	1.98	2.25	—1.98	+2.25
1.0	3.0	1.5	3.2	1.6	1.3	1.18	2.60		
—2.3	+2.6	—2.6	+2.2	—2.6	+0.5	1.66	4.59	—1.66	+4.59
1.5	2.2	1.6	2.2	1.4	1.6	1.28	2.90		
—1.8	+15.2	—1.9	+15.1	—2.5	+15.5	2.34	12.5	—2.34	+12.5
2.2	5.4	2.5	5.1	2.6	8.8	2.84	4.84		
—1.3	+3.0	—0.4	+3.7	—0.2	+2.7	1.02	2.11	—1.02	+1.93
1.6	2.2	2.1	1.4	1.4	2.0	1.44	2.22		
—0.1	+6.6	+1.5	+4.9	+1.5	+1.9	.56	5.77	+.12	+5.77
2.3	3.4	2.0	2.9	2.4	3.0	2.35	2.97		
+1.3	+6.8	+1.2	+6.2	+0.9	+4.3	1.29	4.93	+1.29	+4.93
1.1	.6	1.2	2.4	.7	1.6	.90	2.12		
—4.0	+4.3	—4.9	+5.9	—5.0	+5.5	3.99	3.53	—3.99	+2.21
3.2	3.0	3.3	3.1	3.0	3.0	2.91	3.72		
+6.3	+12.7	+6.9	+12.9	+3.4	+14.1	6.53	10.8	+6.53	+10.3
2.3	3.1	2.1	3.3	4.8	3.6	2.48	3.65		
—5.5	—2.7	—5.0	+4.1	—5.9	—0.7	5.55	2.93	—5.55	+.57
2.0	3.1	2.6	2.9	1.9	4.0	2.52	5.93		
+7.0	+11.6	+6.4	+10.4	+6.2	+9.8	6.02	8.56	+6.02	+8.56
5.2	2.9	5.2	3.3	5.4	3.2	5.31	3.58		
+1.8	+3.8	+2.0	+2.9	+1.5	+2.7	1.93	3.55	+1.93	+3.55
.4	1.6	1.0	.9	.5	.6	.63	1.70		
—2.2	+7.6	—2.2	+5.3	—1.9	+4.5	1.22	7.33	—1.22	+7.33
1.6	3.4	1.8	5.3	1.5	5.0	1.78	5.24		

133	+3.4	+3.2	+4.0	+7.8
	1.4	2.8	1.6	2.4
135	—3.4	0.0	+3.6	+4.0
	1.0	1.6	1.2	1.4
137	+4.6	+3.2	+6.2	+3.6
	1.8	2.0	1.8	2.2
139	+1.0	+2.4	+1.4	+3.0
	1.0	.4	1.0	2.0
141	+2.2	+2.0	+2.4	+6.2
	.6	1.2	.8	1.6
143	+1.8	+2.6	+2.4	+4.0
	1.0	1.4	1.2	1.0
145	+1.0	+2.0	+1.4	+4.0
	1.0	1.2	.6	1.0
147	+1.2	+1.4	—0.4	+2.2
	1.6	1.8	1.6	2.8
149	+2.4	—0.4	+1.8	+1.2
	1.2	3.2	1.4	2.6
151	—2.4	—4.4	—2.8	—3.8
	3.6	5.6	2.8	8.4
153	+0.6	+1.6	+0.2	+3.6
	1.0	1.3	1.0	1.6
155	—0.6	—1.4	—0.4	+1.8
	1.8	2.2	1.2	1.2
157	+1.8	—0.4	+1.4	+4.0
	.6	2.0	.6	2.6
159	+1.2	+1.2	+1.2	+2.4
	1.2	2.4	.8	1.0
161	+0.8	+2.4	+0.8	+5.4
	1.6	.8	1.0	1.2
163	+3.8	+3.0	+4.0	+5.2
	.6	1.2	1.6	1.8
165	—1.2	—1.4	—1.6	—0.2
	2.0	2.0	1.2	2.4
167	+0.8	—3.4	+0.6	—0.6
	1.2	1.8	1.0	2.0
169	+2.0	+1.0	+1.8	+2.8
	.8	1.8	1.0	3.4
171	+1.0	+0.8	0.0	+3.6
	1.8	2.0	1.2	1.4
173	+3.6	—2.4	+3.8	—0.8
	1.6	2.0	1.4	1.4
175	+0.8	+3.8	+0.6	+4.6
	.8	3.0	.6	1.2
177	+1.4	+1.4	+2.0	+5.0
	1.0	1.0	.8	1.2
179	+0.6	+0.8	+0.8	+5.2
	1.8	1.6	2.0	2.4
181	+1.8	+1.2	+1.6	+1.4
	.2	.8	.4	1.6
183	+1.4	+0.4	+1.6	+2.4
	1.0	1.2	1.2	3.0
185	+2.2	—3.2	+1.8	—4.6
	1.0	2.6	.6	2.8
187	+1.2	+0.2	+0.4	+4.2
	2.4	1.8	2.4	2.0
189	+1.8	+3.2	+1.6	+5.0
	.2	1.2	.4	2.0
191	+1.2	+1.4	+0.8	+2.8
	.8	1.0	1.2	2.6
193	+0.1	+0.4	+0.8	+2.2
	1.0	1.6	1.2	2.2
195	—0.2	—0.2	+0.2	+3.6
	1.0	1.4	.6	1.4
197	+1.2	+0.4	+1.6	+3.2
	.8	1.2	1.2	1.8
199	+1.2	+2.2	+1.0	+4.0
	.8	1.0	1.0	2.6
201	+1.6	+1.0	+1.4	+0.8
	.8	1.8	.6	1.4
203	+0.8	+0.2	+0.8	+4.0
	.8	1.4	.8	1.4
205	+0.6	—0.6	0.0	+2.6
	1.0	1.0	.8	1.6
207	+1.8	+1.0	+1.6	+3.8
	.2	1.0	1.2	1.2
209	+0.8	+3.0	+0.8	+5.2
	.8	1.4	1.2	1.8
211	+2.8	+2.4	+2.6	+6.6
	1.2	1.6	1.0	2.4
213	+1.2	+2.0	+1.8	+1.8
	.8	1.2	.6	1.2

	0-30		0-23	
Ave.	128	158	128	151
M.v. (v.d.)	1.49	2.38	1.53	3.01
% + C.E.	33	68	32	83
% — C.E.	66	32	61	16
% o C.E.	1	0	7	1
C.E. (v.d.)	1.36	2.59	1.53	3.28
G.C.E. (v.d.)	—.38	+1.00	—.47	+2.56
	256	286	256	279
M.v. (v.d.)	1.23	1.63	1.27	2.10
% + C.E.	76	73	74	90
% — C.E.	23	24	23	10
% o C.E.	1	3	3	0
C.E. (v.d.)	1.56	1.69	1.57	3.13
G.C.E. (v.d.)	+.99	+1.04	+.91	+2.86

.3	.40	2.74	+.32	+2.62
.8	.68	1.51		
.5	.89	2.98	—.89	+2.60
.0	1.13	1.81		
.1	1.11	2.27	—.75	+2.15
.0	.97	2.14		
.7	2.14	3.97	+2.14	+3.81
.6	1.32	3.56		
.9	.90	3.16	—.64	+3.14
.0	1.28	3.17		
3.7	.23	3.34	—.09	+3.28
1.4	.97	1.95		
3.1	.83	3.97	+.83	+3.97
1.2	.43	1.70		
3.7	.30	6.92	—.30	+6.92
3.0	.62	2.35		
1.4	.98	3.95	—.98	+3.96
2.1	1.04	2.59		
0.9	1.14	2.82	—1.14	+2.82
2.4	1.02	2.37		
1.1	.92	3.49	—.82	+3.49
1.0	.78	1.06		
0.4	1.98	2.25	—1.98	+2.25
1.3	1.18	2.60		
0.5	1.66	4.59	—1.66	+4.59
1.6	1.28	2.90		
5.5	2.34	12.5	—2.34	+12.5
8.8	2.84	4.84		
2.7	1.02	2.11	—1.02	+1.93
2.0	1.44	2.22		
1.9	.56	5.77	+.12	+5.77
3.0	2.35	2.97		
4.3	1.29	4.93	+1.29	+4.93
1.6	.90	2.12		
5.5	3.99	3.53	—3.99	+2.21
3.0	2.91	3.72		
4.1	6.53	10.8	+6.53	+10.3
3.6	2.48	3.65		
0.7	5.55	2.93	—5.55	+.57
4.0	2.52	5.93		
9.8	6.02	8.56	+6.02	+8.56
3.2	5.31	3.58		
2.7	1.93	3.55	+1.93	+3.55
.6	.63	1.70		
4.5	1.22	7.33	—1.22	+7.33
5.0	1.78	5.24		

133	+3.4	+3.2	+4.0	+7.8	+4.2	+10.6	+3.8	+10.0	+3.8
	1.4	2.8	1.6	2.4	1.8	3.6	1.0	3.2	1.0
135	—3.4	0.0	+3.6	+4.0	+4.0	+7.2	+4.0	+8.8	+4.4
	1.0	1.6	1.2	1.4	1.2	1.4	1.2	2.0	.8
137	+4.6	+3.2	+6.2	+3.6	+5.6	+6.4	+6.0	+9.6	+6.2
	1.8	2.0	1.8	2.2	1.4	2.0	1.6	1.2	1.0
139	+1.0	+2.4	+1.4	+3.0	+1.2	+5.6	+1.0	+6.4	+0.4
	1.0	.4	1.0	2.0	.8	2.0	1.0	1.6	.4
141	+2.2	+2.0	+2.4	+6.2	+1.8	+6.6	+1.8	+7.4	+2.2
	.6	1.2	.8	1.6	1.0	2.4	1.0	1.8	1.4
143	+1.8	+2.6	+2.4	+4.0	+2.4	+6.6	+2.4	+8.2	+2.6
	1.0	1.4	1.2	1.0	1.2	1.6	.8	1.8	1.0
145	+1.0	+2.0	+1.4	+4.0	+1.2	+5.0	+0.8	+7.0	+1.0
	1.0	1.2	.6	1.0	.8	1.2	.8	1.0	1.0
147	+1.2	+1.4	—0.4	+2.2	+1.2	+4.0	+0.4	+7.2	+0.6
	1.6	1.8	1.6	2.8	2.0	3.0	1.6	3.6	2.2
149	+2.4	—0.4	+1.8	+1.2	+1.4	+4.0	+2.0	+5.2	+1.2
	1.2	3.2	1.4	2.6	1.4	3.0	1.2	2.0	1.2
151	—2.4	—4.4	—2.8	—3.8	—4.4	+1.8	—2.8	+1.2	—2.6
	3.6	5.6	2.8	8.4	3.6	6.0	4.4	9.2	2.6
153	+0.6	+1.6	+0.2	+3.6	+0.4	+5.8	0.0	+4.8	—0.8
	1.0	1.3	1.0	1.6	1.2	1.2	.8	2.8	1.2
155	—0.6	—1.4	—0.4	+1.8	—1.2	+2.6	—1.0	+3.6	—1.4
	1.8	2.2	1.2	1.2	.8	1.6	1.0	1.6	1.4
157	+1.8	—0.4	+1.4	+4.0	+2.0	+5.0	+1.8	+6.6	+2.0
	.6	2.0	.6	2.6	.8	3.2	.6	3.8	.8
159	+1.2	+1.2	+1.2	+2.4	+0.6	+3.6	+0.1	+5.2	+1.4
	1.2	2.4	.8	1.0	1.0	2.2	1.0	1.6	1.8
161	+0.8	+2.4	+0.8	+5.4	+1.0	+7.8	+1.0	+5.4	+1.0
	1.6	.8	1.0	1.2	1.7	4.2	2.0	2.6	1.6
163	+3.8	+3.0	+4.0	+5.2	+4.0	+7.0	+4.2	+10.0	+4.4
	.6	1.2	1.6	1.8	1.2	1.2	1.4	1.6	1.2
165	—1.2	—1.4	—1.6	—0.2	—1.7	+0.8	—1.8	+3.2	—1.2
	2.0	2.0	1.2	2.4	1.5	2.6	.8	2.4	1.4
167	+0.8	—3.4	+0.6	—0.6	0.0	+0.8	—0.4	+2.0	—0.8
	1.2	1.8	1.0	2.0	.8	3.0	1.6	3.6	1.2
169	+2.0	+1.0	+1.8	+2.8	+2.0	+1.6	+2.4	+4.4	+1.6
	.8	1.8	1.0	3.4	0.0	2.2	.8	1.6	.8
171	+1.0	+0.8	0.0	+3.6	+1.2	+5.0	0.0	+5.8	+0.4
	1.8	2.0	1.2	1.4	1.2	1.2	.8	2.2	1.2
173	+3.6	—2.4	+3.8	—0.8	+4.6	+0.8	+4.6	+1.6	+4.2
	1.6	2.0	1.4	1.4	2.2	3.0	1.4	2.0	1.0
175	+0.8	+3.8	+0.6	+4.6	+0.4	+6.0	+0.4	+7.0	—0.4
	.8	3.0	.6	1.2	1.6	3.0	1.2	3.4	.8
177	+1.4	+1.4	+2.0	+5.0	+2.0	+1.8	+1.0	+5.4	+1.0
	1.0	1.0	.8	1.2	1.2	2.8	1.0	1.4	1.4
179	+0.6	+0.8	+0.8	+5.2	+0.6	+4.2	0.0	+5.4	—0.4
	1.8	1.6	2.0	2.4	1.8	2.8	1.2	3.0	1.6
181	+1.8	+1.2	+1.6	+1.4	+1.6	+2.2	+1.6	+3.8	+0.8
	.2	.8	.4	1.6	.4	1.6	.4	.6	.8
183	+1.4	+0.4	+1.6	+2.4	+1.6	+4.0	+1.0	+6.0	+1.0
	1.0	1.2	1.2	3.0	1.2	3.8	1.4	3.2	1.4
185	+2.2	—3.2	+1.8	—4.6	+0.4	—3.0	+0.4	+0.2	+1.2
	1.0	2.6	.6	2.8	1.6	2.4	1.6	2.2	.8
187	+1.2	+0.2	+0.4	+4.2	+0.2	+5.6	+0.8	+5.0	+0.4
	2.4	1.8	2.4	2.0	2.6	3.4	2.4	3.4	2.4
189	+1.8	+3.2	+1.6	+5.0	+1.6	+5.8	+2.0	+7.0	+1.4
	.2	1.2	.4	2.0	.8	1.6	0.0	2.6	.6
191	+1.2	+1.4	+0.8	+2.8	+1.6	+1.0	+1.0	+3.6	+1.8
	.8	1.0	1.2	2.6	1.6	2.0	1.0	2.0	1.0
193	+0.1	+0.4	+0.8	+2.2	+0.6	+6.2	+0.8	+7.0	+0.4
	1.0	1.6	1.2	2.2	.6	1.6	1.2	2.6	.8
195	—0.2	—0.2	+0.2	+3.6	0.0	+5.4	—0.4	+7.6	—0.2
	1.0	1.4	.6	1.4	.8	1.6	.4	2.0	1.0
197	+1.2	+0.4	+1.6	+3.2	+2.0	+3.4	+1.0	+5.8	+0.6
	.8	1.2	1.2	1.8	.4	1.6	1.8	1.8	.6
199	+1.2	+2.2	+1.0	+4.0	+1.2	+5.2	+0.8	+6.6	+0.6
	.8	1.0	1.0	2.6	.8	1.4	.8	1.8	1.0
201	+1.6	+1.0	+1.4	+0.8	+1.2	+2.0	+1.4	+3.6	+1.4
	.8	1.8	.6	1.4	.8	1.4	.6	.4	.6
203	+0.8	+0.2	+0.8	+4.0	+0.2	+4.2	—0.6	+5.2	—0.6
	.8	1.4	.8	1.4	1.4	2.4	1.0	1.6	1.0
205	+0.6	—0.6	0.0	+2.6	+1.2	+2.6	+0.8	+4.6	+0.2
	1.0	1.0	.8	1.6	1.2	2.8	1.6	2.6	1.0
207	+1.8	+1.0	+1.6	+3.8	+1.8	+5.6	+1.6	+6.4	+1.6
	.2	1.0	1.2	1.2	.2	1.0	1.2	1.6	.4
209	+0.8	+3.0	+0.8	+5.2	+0.8	+6.8	+0.8	+7.2	+0.6
	.8	1.4	1.2	1.8	.8	1.4	1.2	1.6	.6
211	+2.8	+2.4	+2.6	+6.6	+2.0	+8.2	+1.8	+11.2	+1.0
	1.2	1.6	1.0	2.4	1.2	2.4	1.0	2.0	1.2
213	+1.2	+2.0	+1.8	+1.8	+0.6	+3.2	+1.2	+3.6	+1.0
	.8	1.2	.6	1.2	1.0	1.4	.8	1.6	1.4

	0-30		0-23		0-17		0-12		
Ave.	128	158	128	151	128	145	128	140	128
M.v. (v.d.)	1.49	2.38	1.53	3.01	1.47	3.26	1.61	3.43	1.5
% + C.E.	33	68	32	83	33	88	32	96	3
% − C.E.	66	32	61	16	66	9	65	3	64
% o C.E.	1	0	7	1	1	3	3	1	
C.E. (v.d.)	1.36	2.59	1.53	3.28	1.47	4.18	1.55	5.34	1.5
G.C.E. (v.d.)	—.38	+1.00	—.47	+2.56	—.51	+3.98	—.53	+5.31	—.6
	256	286	256	279	256	273	256	268	25
M.v. (v.d.)	1.23	1.63	1.27	2.10	1.25	2.30	1.25	2.32	1.2
% + C.E.	76	73	74	90	75	97	66	98	6
% − C.E.	23	24	23	10	22	3	31	1	3
% o C.E.	1	3	3	0	3	0	3	1	
C.E. (v.d.)	1.56	1.69	1.57	3.13	1.65	4.23	1.54	5.48	1.5
G.C.E. (v.d.)	+.99	+1.04	+.91	+2.86	+.92	+4.12	+ .74	+5.46	+.7

.0	+3.8	+8.0	+7.0	+9.8	+4.6	+8.4	+4.4	+7.0	+4.8	+7.2	+9.0	+8.4	4.90	8.04	+4.90	+8.04
.2	1.0	4.4	3.8	3.6	1.4	1.8	1.2	1.4	1.6	1.8	1.0	1.6	1.58	2.66		
.8	+4.4	+9.0	+3.6	+9.6	+3.4	+9.2	+4.0	+7.6	+4.2	+8.0	+4.0	+6.4	3.86	6.98	+3.86	+6.98
.0	.8	3.8	1.2	3.4	.6	3.4	1.2	2.0	.6	2.6	1.2	1.2	1.02	1.94		
.6	+6.2	+7.4	+6.4	+10.6	+7.0	+10.4	+6.6	+10.2	+6.0	+10.0	+5.6	+8.6	6.02	8.00	+6.02	+8.00
.2	1.0	1.8	.8	2.0	1.0	1.0	1.4	1.0	.8	1.0	1.2	1.4	1.28	1.56		
.4	+0.4	+7.8	+0.4	+9.6	+1.2	+9.6	+1.8	+9.4	+1.2	+8.8	+0.8	+7.2	1.04	6.98	+1.04	+6.98
.6	.4	1.8	1.2	1.4	.8	1.8	.6	1.8	1.2	2.6	.8	3.2	.88	1.86		
.4	+2.2	+8.2	+2.0	+8.8	+1.4	+8.2	+2.2	+7.2	+2.4	+5.4	+2.2	+5.2	2.06	6.52	+2.06	+6.52
.8	1.4	1.4	.8	1.4	1.0	2.4	.6	2.0	.4	1.2	.2	.8	.78	1.62		
.2	+2.6	+8.4	+2.0	+10.8	+3.0	+10.4	+2.4	+10.2	+2.8	+10.0	+2.8	+8.6	2.46	7.98	+2.46	+7.98
.8	1.0	1.6	1.2	1.8	1.4	1.4	1.2	2.2	1.2	2.2	1.6	2.6	1.18	1.76		
.0	+1.0	+8.6	+1.0	+7.4	+1.4	+7.4	+1.0	+5.0	+1.8	+6.2	+1.4	+4.0	1.20	5.66	+1.25	+5.66
.0	1.0	1.8	1.4	2.8	1.0	1.2	1.0	1.4	1.0	2.4	.6	.8	.92	1.48		
.2	+0.6	+5.6	0.0	+4.4	+1.6	+5.6	+1.2	+6.0	+0.6	+4.8	+2.2	+2.4	.94	4.36	+.86	+4.36
.6	2.2	2.0	1.2	2.2	2.4	1.4	1.6	2.8	1.0	1.4	2.2	2.8	1.74	2.38		
.2	+1.2	+7.4	+2.0	+8.2	+2.6	+8.8	+2.2	+9.2	+2.6	+5.8	+1.6	+2.6	1.98	5.28	+1.98	+5.20
2.0	1.2	1.8	1.2	1.6	1.0	2.6	2.2	2.4	1.4	3.2	1.6	2.6	1.38	2.50		
.2	—2.6	+0.8	—2.2	+2.0	—3.2	+2.4	—3.0	+0.6	—2.8	+1.2	—2.0	—2.2	2.82	2.04	—2.82	—.04
0.2	2.6	8.0	1.8	5.8	2.4	4.2	1.8	4.2	2.0	4.2	2.8	3.0	2.78	5.86		
.8	—0.8	+5.4	—0.8	+4.6	—0.4	+3.0	—1.4	+4.4	+0.4	+2.4	+0.4	+1.2	.46	3.68	—.06	+3.68
2.8	1.2	2.2	.8	2.4	1.2	2.0	1.4	.8	1.2	1.8	.4	1.2	1.02	1.77		
3.6	—1.4	+3.6	—1.2	+5.6	—1.4	+6.0	—1.8	+5.4	—0.6	+5.8	—1.4	+3.4	1.10	3.02	—1.10	+3.64
.6	1.4	2.8	1.2	3.0	1.0	2.6	1.0	2.2	1.4	2.4	1.0	1.0	1.18	2.06		
6.6	+2.0	+4.2	+2.6	+4.2	+3.0	+4.8	+3.2	+4.4	+2.8	+4.2	+3.4	+4.4	2.40	4.22	+2.40	+4.14
3.8	.8	4.2	1.0	3.6	1.8	2.6	1.6	1.6	.8	1.6	1.4	1.2	1.00	2.64		
.2	+1.4	+2.6	+0.4	+3.8	+1.4	+3.8	+1.4	+3.2	+0.8	+3.2	+0.4	+2.0	.89	3.10	+.89	+3.10
.6	1.8	3.4	1.6	1.6	1.0	1.2	1.0	1.2	.8	1.4	.8	.8	1.10	1.68		
5.4	+1.0	+5.4	+1.0	+4.0	+1.4	+3.2	+2.6	+3.6	+2.8	+2.8	+2.6	+3.0	1.50	4.30	+1.50	+4.30
2.6	1.6	3.8	1.4	4.4	1.4	2.5	1.5	1.9	1.6	1.7	2.1	2.1	1.59	2.52		
0.0	+4.4	+10.4	+4.2	+9.8	+3.6	+9.6	+3.4	+8.8	+4.4	+8.6	+3.2	+7.0	3.92	7.94	+3.92	+7.94
.6	1.2	2.8	1.4	4.8	1.2	3.8	1.8	2.4	1.2	3.2	1.2	2.0	1.28	2.53		
3.2	—1.2	+3.4	+0.8	+3.0	—3.8	+2.4	—2.2	—0.4	—3.6	—1.4	—3.0	—1.0	2.09	1.72	—2.01	+.84
2.4	1.4	3.4	2.0	4.0	2.6	4.1	2.2	3.4	1.6	3.2	1.7	3.5	1.70	3.10		
2.0	—0.8	+3.8	+0.6	+4.6	+0.8	+4.0	+1.2	+3.4	—0.2	+3.4	+0.2	+2.4	.56	2.84	+.28	+2.84
3.6	1.2	2.6	2.2	2.0	1.6	1.8	1.6	1.8	2.2	1.6	.6	1.6	1.40	2.18		
4.4	+1.6	+6.8	+2.2	+6.8	+1.8	+7.2	+2.6	+7.4	+2.2	+6.6	+2.4	+4.6	2.10	4.82	+2.10	+4.82
1.6	.8	1.2	1.0	1.4	.2	1.8	.6	1.8	.2	2.0	.8	1.4	.62	1.86		
5.8	+0.4	+4.4	+0.8	+4.6	+0.8	+3.8	+0.4	+3.8	+1.2	+3.8	+1.2	+3.2	.70	3.88	+.70	+3.88
2.2	1.2	3.2	1.6	2.4	1.6	1.6	.4	1.8	1.2	2.8	1.2	2.0	1.22	2.06		
1.6	+4.2	+4.2	+5.2	+4.6	+4.2	+4.6	+4.8	+5.2	+4.6	+5.6	+5.0	+6.0	4.46	3.58	+4.46	+2.94
2.0	1.0	1.4	.8	2.4	1.0	1.2	1.6	1.2	1.4	1.4	1.8	1.6	1.42	1.76		
7.0	—0.4	+6.2	+2.2	+6.6	+1.2	+7.8	+1.4	+9.0	+0.4	+6.2	+0.2	+7.2	.80	6.44	+.72	+6.44
3.4	.8	2.6	1.4	5.2	1.2	3.6	1.8	3.4	2.0	3.2	2.2	2.0	1.36	3.06		
5.4	+1.0	+6.4	+1.2	+7.0	+1.2	+5.6	+1.6	+5.8	+2.0	+4.4	+2.2	+5.2	1.56	4.80	+1.56	+4.80
1.4	1.4	1.2	.8	2.4	1.2	2.2	.8	1.8	.4	1.0	.2	.8	.88	1.58		
5.4	—0.4	+8.0	—1.0	+7.0	—0.6	+6.2	—0.6	+5.4	—0.4	+4.8	—1.0	+4.2	.60	5.12	—.20	+5.12
3.0	1.6	2.0	1.8	2.0	1.8	2.4	1.4	1.8	1.2	1.4	1.8	1.4	1.64	2.08		
3.8	+0.8	+5.4	+0.6	+6.6	+0.4	+5.2	+1.0	+3.8	+1.0	+3.0	+1.6	+2.8	1.20	3.54	+1.20	+3.54
.6	.8	1.4	1.0	.8	1.2	1.8	1.0	2.6	1.4	1.2	.8	1.2	.76	1.36		
6.0	+1.0	+7.2	+1.6	+8.2	+2.2	+6.8	+2.2	+4.8	+1.6	+3.8	+2.0	+2.8	1.62	4.64	+1.62	+4.64
3.2	1.4	3.6	.4	2.8	1.0	3.8	.6	2.8	1.2	2.0	1.2	2.4	.92	2.86		
0.2	+1.2	+1.2	+0.8	+4.6	+2.2	+6.0	+2.2	+6.6	+2.0	+6.4	+1.6	+6.2	1.48	4.20	+1.48	+2.04
2.2	.8	2.0	1.6	1.6	1.8	2.2	1.4	1.8	1.2	2.2	.8	2.6	1.24	2.24		
5.0	+0.4	+6.0	—0.8	+7.2	—.10	+4.8	—1.0	+5.2	—0.4	+6.2	—0.4	+3.8	.66	4.82	—.06	+4.82
3.4	2.4	3.2	2.8	3.8	2.6	1.4	3.0	1.2	2.8	2.4	2.8	1.4	2.62	2.40		
7.0	+1.4	+10.0	+1.6	+9.8	+1.0	+10.0	+1.0	+7.4	+1.4	+6.4	+1.0	+5.0	1.44	6.96	+1.44	+6.96
2.6	.6	1.6	.8	3.2	1.0	5.0	1.0	4.2	1.0	4.2	1.0	3.4	.68	2.90		
3.6	+1.8	+6.4	+1.2	+5.4	+1.4	+4.6	+1.4	+4.8	+1.6	+3.8	+1.2	+3.2	1.32	+3.70	+1.32	+3.70
2.0	1.0	1.6	1.2	1.6	1.0	1.6	1.0	2.0	.8	1.6	.8	.8	1.04	1.52		
7.0	+0.4	+8.0	—0.2	+5.8	0.0	+5.4	+0.6	+5.6	+0.8	+4.2	+0.4	+5.6	.47	5.04	+.43	+5.04
2.6	.8	2.8	1.0	4.4	.4	4.4	.6	3.6	1.2	3.6	1.2	3.2	.92	3.00		
7.6	—0.2	+9.6	—0.6	+9.0	—0.6	+10.0	—0.6	+8.0	—0.2	+7.6	—0.2	+4.6	.32	6.56	—.28	+6.52
2.0	1.0	4.4	1.0	4.4	.6	3.8	1.0	4.0	2.2	4.2	.2	4.6	.88	3.18		
5.8	+0.6	+6.6	+1.6	+7.4	+0.6	+4.2	+1.0	+5.6	+1.4	+3.6	+0.8	+4.2	1.18	4.44	+1.18	+4.44
1.8	.6	1.4	1.6	1.2	1.0	1.6	1.0	2.0	1.4	1.8	1.2	1.8	1.10	1.62		
6.6	+0.6	+8.2	+0.8	+9.0	+0.2	+7.6	+1.0	+8.0	+0.8	+6.6	+1.4	+4.0	.90	6.14	+.90	+6.14
1.8	1.0	1.8	1.6	2.8	.6	3.0	1.0	2.4	1.2	2.4	1.0	1.6	.98	1.98		
3.6	+1.4	+3.8	+1.4	+2.8	+1.4	+3.4	+1.8	+3.2	+1.4	+3.8	+1.8	+3.0	1.48	2.74	+1.48	+2.74
.4	.6	1.8	1.4	1.8	.8	1.2	.2	1.2	1.0	1.2	1.0	.6	.78	1.28		
5.2	—0.6	+6.6	—1.6	+6.4	—0.2	+7.0	0.0	+5.8	—0.6	+8.8	—0.6	+1.8	.60	5.00	—.24	+5.00
1.6	1.0	1.0	1.2	2.6	1.0	2.0	1.2	2.2	1.4	2.2	1.4	1.8	1.12	1.86		
4.6	+0.2	+6.4	0.0	+8.0	—0.8	+9.4	+0.8	+6.0	+0.8	+4.2	+0.8	+4.6	.60	4.90	+.44	+4.78
2.6	1.0	2.4	1.2	3.0	1.6	1.6	1.6	2.4	.4	2.4	1.2	2.6	1.16	2.24		
6.4	+1.6	+7.2	+2.0	+8.6	+2.2	+9.6	+2.0	+8.8	+1.6	+8.0	+2.4	+6.2	1.86	6.52	+1.86	+6.52
1.6	.4	1.2	.4	2.4	.2	2.6	.4	1.6	.8	1.8	.4	1.0	.54	1.54		
7.2	+0.6	+7.4	+0.6	+6.4	+1.2	+6.6	+0.8	+5.2	+0.4	+5.6	+0.6	+4.8	.74	5.82	+.74	+5.82
1.6	.6	1.4	1.0	1.4	2.0	.8	.8	1.6	.8	1.4	.6	.8	.98	1.36		
1.2	+1.6	+11.8	+1.4	+11.6	+1.0	+10.6	+0.8	+10.8	+1.4	+9.6	+0.6	+8.2	1.60	9.10	+1.60	+9.10
2.0	1.2	2.2	1.0	4.1	1.4	3.6	1.6	4.4	1.0	3.0	.6	3.4	1.12	2.91		
3.6	+1.0	+4.8	+1.2	+5.8	+0.6	+5.8	+1.0	+6.0	+0.8	+5.8	+0.8	+4.6	1.02	4.34	+1.02	+4.34
1.6	1.4	1.2	1.2	1.2	1.4	1.6	1.4	1.2	1.6	.8	1.6	1.4	1.18	1.28		

TABLE VIII. Summary from Table VII

	0-8		0-5		[illegible]		0-2		0-1		0-.5		Average Arithmetic O	Average Arithmetic V	Average Algebraic O	Average Algebraic V
A. Men																
40	128	136	128	133	128	131	128	130	128	129	128	128.5	128	138	128	138
.43	1.53	3.28	1.54	3.21	1.61	3.25	1.53	3.10	1.62	2.89	1.54	2.72	1.54	3.05		
96	31	100	32	99	31	100	34	96	35	93	34	89	33	91		
3	64	0	67	0	66	0	63	4	64	5	64	11	65	8		
1	5	0	1	1	3	0	3	0	1	2	2	0	2	1		
.34	1.59	6.81	1.47	7.15	1.55	6.97	1.62	6.35	1.66	5.64	1.61	4.85	1.54	5.32		
.31	—.63	+6.81	—.53	+7.15	—.51	+6.97	—.57	+6.27	—.57	+5.55	—.64	+4.71			—.53	+5.03
B. Women																
68	256	264	256	261	256	259	256	258	256	257	256	256.5	256	266	256	266
.32	1.28	2.50	1.35	2.59	1.28	2.45	1.27	2.29	1.38	2.09	1.39	1.92	1.29	2.21		
98	65	98	65	98	63	98	63	97	60	97	68	94	67	94		
1	34	2	30	2	32	2	36	3	37	3	30	5	30	5		
1	1	0	5	0	5	0	1	0	3	0	2	1	3	1		
.48	1.52	6.36	1.59	6.71	1.73	6.26	1.81	5.74	1.79	4.98	1.77	4.06	1.65	4.86		
.46	+.71	+6.29	+.80	+6.56	+.70	+6.10	+.66	+5.59	+.64	+4.95	+.69	+3.85			+.77	+4.68

tone. Hence the presentation of the increment fork followed immediately upon the close of the observer's reproduction of o, the subject being encouraged to make the reproduction about 1 second in length. The increment forks were struck while the o was being reproduced. This was, however, no distraction as only a slight blow on the practically noiseless sounder was necessary, and the forks could not be heard until presented before the resonators. Following the reproduction of each increment fork there was a period of about 2.5 seconds before the next sounding of o.

Other factors. In the matter of intensity of standard and intensity and vowel quality of the voice we took direct advantage of our previous work and adopted such conditions as would give the most normal results according to those findings. By the use of resonators at a considerable distance from the observer's ear we found a satisfactory means of controlling the intensity of the standards,[13] while the intensity of the voice had to be judged subjectively and watched by the experimenter. And in the selection of *"ä"* we are using that vowel quality which according to Berlage and our own results affects least the constant error of the reproductions.

Tables of data

The constant error (C. E.) and mean variation (m.v.) were found for the ten trials on each fork of the ten pairs given in the test. These twenty C. E.'s and twenty m.v.'s for each individual tested are embodied in Table VII, which has been divided into two parts, A. and B., for the men and women respectively. In the first column of the table, at the left, are given the numbers which stand for the individual observers. This numbering is in no sense a ranking, but simply for convenience in handling the data and aid in identification. Odd numbers are used throughout to refer to women and even numbers to men. The second column from the left shows the C. E. and m.v. (the latter is under the former) for the ten trials on o. when used in the pair o-30. The same measures for the ten trials on variant (or interval) tone 30 are given in column three, and each of the successive smaller increments are represented in the same manner. The arithmetic averages for the constant error

[13] It is possible that the pitch of a standard is not only varied by its intensity but also by its position when held near the ear.

Obs.	0-30		0-23		0-17		0-12		0-8		0-5		0-3	
	128	158	128	151	128	145	128	140	128	136	128	133	128	
2	−0.2	+4.0	−0.3	+2.7	+1.0	+4.6	−0.1	+8.9	+0.1	+7.6	−0.2	+11.4	−0.1	+
	1.8	2.2	2.1	2.5	1.0	1.8	1.5	2.3	1.7	4.0	1.6	2.8	2.1	
4	−1.9	−6.6	−2.9	−2.3	−1.3	−0.3	−3.0	−1.5	−4.8	+4.6	−2.4	+6.0	−3.5	+
	2.7	4.8	1.9	5.5	1.5	7.7	2.8	6.9	1.4	6.8	3.0	6.8	1.7	
6	−0.1	+1.7	0	+4.5	−0.2	+4.7	+0.1	+5.0	0	+6.0	+0.1	+4.5	−0.2	+
	1.1	.7	.8	1.9	1.0	3.1	1.1	4.0	.4	3.4	.9	2.5	.6	
8	+1.0	+3.6	+1.1	+2.9	+1.3	+5.2	+1.2	+5.5	+0.9	+3.9	+0.8	+4.7	+1.1	+
	.4	1.2	.7	1.3	.9	3.	.4	1.9	.3	1.7	.4	1.7	.5	
10	+0.3	−0.1	+0.3	+0.7	+0.1	−0.1	0.0	+2.8	0.0	+4.5	+0.1	+4.8	−0.2	+
	.7	.5	.7	1.1	.5	2.3	.4	1.0	.4	.9	.1	1.2	.6	
12	−0.1	+1.2	+0.4	+0.8	+0.3	+3.0	+0.3	+2.3	+0.4	+2.3	+0.2	+3.0	−0.2	+
	.7	3.0	.6	2.2	.7	3.8	.7	2.7	.6	1.9	.8	1.2	.2	
14	+4.0	+17.3	+2.0	+21.8	+1.3	+25.0	+4.0	+26.5	+3.5	+28.8	+0.3	+36.0	−0.2	+3
	6.0	9.0	2.0	4.1	2.0	6.8	5.0	6.5	5.1	5.1	4.0	5.3	3.1	
16	+0.4	−0.3	0.0	+0.3	−0.2	+2.0	0.0	+3.8	−0.5	+4.1	−0.5	+5.0	−1.1	+
	.8	1.4	.9	2.1	1.0	1.0	.8	1.6	1.0	1.7	.6	1.9	.5	
18	−0.7	+2.8	−1.1	+2.4	−0.9	+5.4	−0.5	+5.9	−0.9	+7.9	−0.4	+8.9	0.0	+1
	1.3	2.2	1.7	4.5	1.7	2.5	2.0	4.3	1.3	4.9	1.4	3.7	1.3	
20	−1.0	+4.4	−1.1	+8.1	−2.9	+10.9	−2.3	+7.0	−1.5	+9.4	−1.6	+9.5	−3.2	+
	1.6	2.5	1.9	2.1	2.5	3.4	2.1	3.6	2.1	4.7	2.2	4.3	2.2	
22	−2.2	−3.4	−0.5	−0.9	−0.7	+0.8	−0.7	+6.1	−3.3	+7.2	−3.7	+5.5	−2.8	+
	2.8	2.7	2.4	5.5	4.1	6.8	2.8	7.1	2.6	9.5	1.2	8.0	2.8	
24	−0.8	−1.1	−1.6	+1.3	−1.9	+5.1	−1.0	+7.5	−1.6	+9.3	−1.8	+11.1	−2.4	+1
	2.4	2.3	2.3	3.6	1.1	4.1	1.6	5.0	2.2	4.9	1.8	4.9	2.3	
26	−2.4	−1.9	−2.2	−4.2	−2.4	−2.6	−2.6	−0.1	−2.8	+0.1	−2.9	+1.4	−3.2	+
	1.6	1.9	1.4	4.2	1.5	2.4	1.9	2.1	1.4	1.5	.9	2.1	.8	
28	+0.1	2.2	+0.4	+0.6	−0.2	+2.4	−0.1	+4.8	0.0	+5.2	−0.4	+5.0	+0.2	+
	.7	1.2	.7	1.3	.4	2.0	.9	1.4	.6	2.8	.7	2.6	.8	
30	+0.7	+0.6	+0.3	+1.6	+1.0	+4.5	+0.8	+6.4	−0.2	+8.5	+0.9	+9.2	+1.1	+
	1.2	1.9	1.0	3.9	1.6	4.2	1.8	3.7	1.5	5.0	.9	3.1	1.3	
32	−2.3	−0.5	−2.9	−1.6	−3.0	+0.4	−2.9	+4.4	−3.6	+4.3	−3.6	+4.0	−3.3	+
	2.2	4.1	1.2	6.0	1.4	4.7	1.3	4.9	1.4	5.1	1.4	5.8	1.9	4
34	−0.8	+3.6	0	+6.8	+0.4	+8.5	−1.1	+9.9	0	+10.1	0.7	+10.4	+0.6	+
	1.0	3.2	1.0	1.6	1.1	3.6	2.1	3.5	1.6	4.3	.7	4.2	.9	3
36	−1.8	+1.4	−1.9	+4.0	−2.1	+6.7	−2.8	+7.6	−2.7	+9.5	−2.0	+8.0	−2.2	+7
	.8	1.9	1.1	4.2	.7	6.0	.8	6.3	1.4	3.4	.6	4.6	1.2	6
38	+1.1	+5.9	+1.0	+7.3	+0.9	+9.3	+0.7	+9.9	+0.4	+8.6	−0.2	+8.1	−0.1	+5
	.9	2.3	.8	2.0	1.1	1.9	1.1	3.9	.8	2.4	.7	2.1	6.0	1
40	+3.2	+2.2	+3.1	+7.7	+3.0	+7.6	+2.9	+9.0	+3.3	+2.4	+3.0	+5.6	+3.3	+2
	1.6	.6	1.9	1.6	1.8	8.4	1.9	11.8	1.9	8.5	1.8	8.5	2.2	1
42	−2.2	−2.0	−2.5	+1.8	−2.6	+6.2	−4.0	+4.4	−3.9	+5.7	−2.8	+4.5	−3.5	+3
	3.0	9.4	3.0	10.5	3.2	11.4	2.5	12.0	2.7	12.3	3.7	13.7	3.5	12
44	−2.9	−3.8	−3.4	−1.2	−3.0	+5.5	−1.7	+11.8	−4.7	+17.1	−3.9	+16.0	−3.7	+17
	2.1	4.7	2.6	5.3	2.2	4.3	2.7	4.2	2.3	4.8	4.3	6.0	4.1	6
46	−4.3	−1.0	−3.7	+0.6	−3.3	+5.9	−4.7	+7.0	−3.7	+4.8	−1.5	+6.4	−1.3	+6
	3.9	7.8	4.7	3.7	4.7	3.7	4.4	2.2	4.0	6.2	3.7	6.5	4.2	6
48	−1.3	+8.5	−2.0	+3.5	−1.3	−0.5	−1.4	+1.3	−2.1	+1.2	−1.1	+3.2	−0.2	+3
	1.7	3.9	1.6	5.0	2.3	3.9	2.6	2.6	1.7	3.2	1.9	3.4	1.9	3
50	+1.3	+2.6	+1.1	+1.4	+1.3	+4.4	+1.1	+5.4	+1.0	+5.6	+1.1	+6.0	+1.0	+5
	.5	1.6	.7	1.4	.3	1.6	.5	2.4	.4	2.8	.9	2.0	.8	2
52	+2.2	+3.6	+2.6	+0.6	+2.1	+2.8	+2.3	+5.9	+2.6	+7.2	+2.6	+5.8	+2.4	+6.
	.6	.8	.8	2.4	.7	2.0	.7	1.1	1.0	1.6	.4	2.4	.6	1.
54	+0.1	+0.3	−0.2	+3.8	−1.0	+7.6	−0.1	+0.4	−0.6	+16.7	−0.3	+16.3	+0.6	+16.
	2.1	3.5	1.8	2.0	2.2	2.0	1.9	3.6	2.0	2.1	1.5	1.9	2.0	3.
56	−1.1	+0.8	−1.7	+4.4	−2.3	+6.7	−1.7	+5.4	−0.5	+5.1	−1.1	+6.2	−0.5	+6.
	1.7	2.0	1.7	2.2	2.1	3.9	1.3	4.2	1.7	2.3	2.1	2.6	2.3	3.
58	−0.4	+1.0	−0.3	+2.4	−0.6	+7.9	−0.3	+8.9	+0.2	+11.7	+0.2	+11.1	+0.9	+10.
	1.8	4.6	1.9	3.8	1.8	3.9	2.1	2.9	1.4	3.3	2.2	8.9	1.7	7.
60	−1.6	+1.1	−1.4	+1.5	−1.8	+3.7	−1.6	+4.6	−1.5	+6.8	−2.1	+6.9	−2.2	+8.
	.8	2.1	1.4	4.5	1.2	5.5	1.4	5.2	.7	3.6	.7	4.3	1.2	3.
62	+0.1	+3.7	+0.5	+1.1	+0.1	+3.0	+0.1	+1.7	−0.8	+1.1	−1.0	0.0	−1.0	−0.
	1.1	1.5	1.1	2.9	1.1	2.4	1.1	3.1	1.4	1.7	.8	1.6	1.0	2.
64	−0.9	−3.6	−0.8	−1.1	−0.5	+0.6	−0.4	+2.6	−1.1	+4.7	−0.8	+6.6	−0.7	−8.

y in singing: men

	0-3		0-2		0-1		0-.5		Average Arithmetic O	Average Arithmetic V	Average Algebraic O	Average Algebraic V
3	128	131	128	130	128	129	128	128.5	128	138	128	138

	0-3		0-2		0-1		0-.5		Average Arithmetic O	Average Arithmetic V	Average Algebraic O	Average Algebraic V
3	128	131	128	130	128	129	128	128.5	128	138	128	138

4	—0.1	+9.5	+0.5	+8.2	+0.8	+5.7	+1.3	+4.3	.46	6.69	+.29	+6.69
8	2.1	2.1	1.9	1.8	2.2	2.5	1.5	1.4	1.74	2.34		
0	—3.5	+7.7	—3.8	+5.9	—2.3	+10.5	—3.3	+5.5	2.92	5.09	—2.92	+2.90
3	1.7	5.7	1.6	6.3	1.5	8.3	2.1	4.8	2.02	6.36		
5	—0.2	+4.2	—0.2	+3.4	—0.3	+3.5	+0.1	+3.0	.12	4.05	—.06	+4.05
5	.6	1.2	.6	1.4	.8	1.3	.5	1.4	.78	2.06		
7	+1.1	+5.4	+0.9	+5.6	+0.7	+5.1	+0.9	+5.1	.99	4.70	+.99	+4.70
7	.5	2.6	.7	1.8	.5	1.7	.3	1.4	.51	1.83		
8	—0.2	+4.6	—0.2	+3.9	+0.1	+2.6	0.0	+1.7	.13	2.58	+.05	+2.54
2	.6	1.4	.8	1.5	.5	1.2	.2	1.2	.49	1.23		
0	—0.2	+3.3	+0.1	+3.2	+0.1	+3.2	+0.3	+2.3	.24	2.46	+.18	+2.46
2	.2	1.1	.5	1.4	.7	1.8	.5	1.0	.60	2.02		
0	—0.2	+34.5	+0.5	+27.3	+1.1	+33.0	+0.5	+28.0	1.74	27.0	+1.7	+27.8
3	3.1	4.5	2.5	11.6	5.1	8.0	3.1	3.8	3.25	6.47		
0	—1.1	+5.3	—1.0	+5.8	—1.3	+4.5	—1.7	+3.7	.67	3.48	—.57	+3.48
9	.5	1.2	.8	1.8	1.4	1.4	1.3	1.6	.91	1.55		
9	0.0	+10.6	+0.7	+8.9	+0.3	+7.6	0.0	+5.8	.55	6.62	—.35	+6.62
7	1.3	5.3	1.1	4.9	1.4	3.6	1.2	2.8	1.43	3.97		
5	—3.2	+3.5	—2.5	+2.1	—2.8	+2.1	—2.0	—0.1	2.09	5.71	—2.09	+5.71
3	2.2	4.7	3.4	6.1	2.6	3.9	3.4	5.0	2.40	4.03		
.5	—2.8	+2.3	—4.9	+6.3	—3.9	+1.2	—3.9	+0.2	2.66	3.39	—2.66	+2.53
.0	2.8	2.8	1.4	4.3	2.1	2.7	3.1	4.2	2.52	5.36		
.1	—2.4	+11.3	—2.6	+7.4	—2.2	+7.5	—4.2	+6.8	2.01	6.84	—2.01	+6.62
.9	2.3	5.4	2.2	6.0	2.2	4.9	2.2	4.7	2.03	4.58		
.4	—3.2	+1.0	—3.8	—0.4	—3.3	—2.1	—3.4	—2.5	2.90	1.63	—2.90	—1.23
.1	.8	3.2	1.0	3.2	1.5	3.3	1.3	1.9	1.33	2.58		
.0	+0.2	+5.2	—0.1	+3.1	—0.8	+2.8	—0.1	+1.4	.24	3.27	—.10	+3.27
.6	.8	1.8	.9	1.5	.6	1.4	.7	1.3	.76	1.73		
.2	+1.1	+9.6	+1.0	+7.7	+2.0	+8.8	+1.5	+6.5	.95	6.34	+.91	+6.34
.1	1.3	3.3	.6	2.9	1.2	3.6	.9	1.8	1.20	3.34		
.0	—3.3	+5.5	—2.8	+4.7	—3.1	+4.2	—3.7	+4.2	3.12	3.38	—3.12	+2.96
.8	1.9	4.7	2.2	1.8	1.9	1.4	2.3	3.4	1.73	4.19		
.4	+0.6	+9.7	+1.1	+8.5	+1.0	+5.1	+0.5	+5.7	.62	7.83	+.24	+7.83
.2	.9	3.9	1.3	4.6	.8	2.1	1.3	3.0	1.18	3.40		
.0	—2.2	+7.7	—2.7	+1.8	—2.7	+0.6	—2.3	+0.9	2.32	4.82	—2.32	+4.82
.6	1.2	6.8	.9	4.6	.7	4.2	1.0	3.6	.92	4.56		
.1	—0.1	+5.5	0	+5.8	—0.8	+4.8	—0.2	+4.1	.54	6.93	+.28	+6.93
.1	6.0	1.9	.9	1.2	.6	1.8	.5	1.7	.80	2.12		
.6	+3.3	+2.0	+4.3	+4.7	+4.2	+4.1	+4.4	+4.2	3.47	4.95	+3.47	+4.95
.5	2.2	1.2	1.6	4.5	1.4	1.5	1.3	1.6	1.74	4.82		
.5	—3.5	+3.2	—3.8	+5.0	—2.8	+4.1	—2.3	—0.5	3.04	3.74	—3.04	+3.24
.7	3.5	12.5	3.4	12.0	3.4	13.2	2.7	7.7	3.11	11.2		
.0	—3.7	+17.0	+0.5	+16.6	—0.5	+17.5	—2.5	+20.1	2.68	12.6	—2.58	+11.7
.0	4.1	6.6	5.6	4.8	6.5	3.7	5.6	3.7	3.90	4.82		
.4	—1.3	+6.9	—2.8	+1.7	—1.7	—0.9	—1.1	—0.3	2.81	3.55	—2.81	+3.11
.5	4.2	6.3	4.2	6.4	4.7	5.7	4.7	5.0	4.32	5.35		
.2	—0.2	+3.1	—0.5	+1.9	+0.7	—0.1	—1.2	—0.3	1.18	2.36	—1.18	+2.16
.4	1.9	3.9	1.6	4.3	1.1	1.7	1.6	2.7	1.80	3.47		
.0	+1.0	+5.8	+1.2	+4.7	+0.6	+3.7	+0.9	+1.4	1.06	4.10	+1.06	+4.10
.0	.8	2.2	.8	2.1	.8	2.5	1.1	1.7	.68	2.03		
.8	+2.4	+6.7	+2.3	+5.6	+2.6	+5.1	+2.0	+4.5	2.37	4.78	+2.37	+4.78
.4	.6	1.9	1.1	1.6	.8	1.1	.8	1.4	.75	1.63		
.3	+0.6	+16.6	+0.4	+16.8	+0.7	+17.0	+0.7	+15.7	.38	11.1	+.12	+11.1
.9	2.0	3.6	1.2	3.0	1.5	3.0	1.3	3.2	1.75	2.89		
.2	—0.5	+6.0	—0.6	+8.8	0	+9.1	—1.3	+9.1	1.08	6.16	—1.08	+6.16
.6	2.3	3.8	1.4	4.2	2.4	4.3	.7	4.4	1.74	3.39		
.1	+0.9	+10.0	+0.9	+9.2	+0.8	+4.4	+2.0	+3.7	.66	7.03	+.34	+7.03
.9	1.7	7.4	1.9	5.8	1.6	2.8	1.8	2.8	.82	4.60		
.9	—2.2	+8.6	—2.2	+8.1	—2.1	+6.4	—1.1	+4.9	1.72	5.26	—1.76	+5.26
.3	1.2	3.2	1.0	2.7	1.1	1.6	1.3	2.4	1.08	3.51		
.0	—1.0	—0.6	—1.4	—0.6	—1.8	—1.0	—0.9	—0.2	.77	1.30	—.61	+.94
.6	1.0	2.2	1.2	1.2	.8	1.2	.9	1.5	1.05	1.93		
.6	—0.[illegible]	-8.2	—0.7	+8.2	—1.0	+7.2	—0.9	+5.9	.78	4.87	—.78	+3.93
.6	1.1	3.2	1.1	4.7	.6	2.2	.9	1.6	.94	2.63		

TABLE VII, B. *Accuracy in singing: women*

Obs. 0-30 0-23 0-17 0-12 0-8 0-5 0-3 0-2
256 286 256 279 256 273 256 268 256 264 256 261 256 259 256 258

Obs. 0-30 0-23 0-17 0-12 0-8 0-5 0-3 0-2
256 286 256 279 256 273 256 268 256 264 256 261 256 259 256 258

Obs. 0-30 0-23 0-17 0-12 0-8 0-5 0-3 0-2
256 286 256 279 256 273 256 268 256 264 256 261 256 259 256 258

Obs.	0-30		0-23		0-17		0-12		0-8		0-5		0-3		0-2	
	256	286	256	279	256	273	256	268	256	264	256	261	256	259	256	258
1	+1.0	+1.0	+0.2	+2.6	+0.2	+2.0	+0.6	+4.2	+0.6	+6.0	+0.6	+6.0	+1.2	+4.6	+1.4	+5.2
	1.0	1.4	.6	2.0	.6	1.8	1.4	1.8	1.0	2.4	1.0	2.2	1.2	1.2	1.0	2.4
3	+2.8	+3.6	+2.2	+6.2	+2.2	+5.8	+2.0	+4.4	+1.8	+3.6	+2.4	+3.0	+1.8	+3.2	+1.6	+2.0
	.8	2.8	.6	2.4	1.0	4.0	1.2	2.8	1.4	2.4	1.2	1.2	1.4	1.0	1.6	1.2
5	+0.6	+6.0	+1.6	+5.0	+1.0	+5.0	+1.0	+8.0	+0.8	+7.2	+1.0	+8.2	+1.0	+8.2	+0.6	+8.2
	1.4	1.6	1.6	2.0	2.2	2.0	1.8	2.0	1.2	1.6	1.8	2.0	1.4	1.6	1.0	1.8
7	+1.4	+0.8	+1.8	+2.8	+0.8	+4.8	+1.4	+6.4	+1.2	+6.4	+1.0	+6.2	+1.8	+5.8	+1.2	+5.0
	1.0	2.0	1.0	1.4	1.2	2.2	1.0	2.8	.8	2.4	1.0	2.4	.6	2.4	.8	1.4
9	+0.6	+1.6	+0.2	−0.6	+0.5	+0.4	−0.6	0.0	−1.0	+1.2	+0.2	+3.0	0.0	+2.2	+0.2	+2.6
	1.0	1.2	1.4	2.0	.9	2.4	1.4	1.2	1.8	1.6	1.6	2.2	1.2	2.3	1.0	1.6
11	+4.4	+4.4	+4.6	+6.4	+4.8	+8.0	+5.4	+8.8	+5.4	+9.8	+5.6	+9.6	+5.4	+8.6	+5.0	+9.2
	1.6	2.4	1.6	4.0	1.2	3.0	1.6	1.6	1.4	3.4	1.2	2.8	2.0	2.0	1.0	1.6
13	+1.6	+1.0	+2.2	+2.8	+2.2	+4.6	+1.4	+5.2	+2.0	+6.6	+3.0	+4.8	+2.6	+3.6	+2.2	+3.4
	1.1	1.0	.9	1.0	1.0	1.2	1.2	1.4	.8	3.0	.9	2.2	1.1	1.4	1.0	1.6
15	−0.8	−0.6	−0.8	−0.2	−0.8	+1.2	−1.0	+1.8	−1.2	+3.2	−0.4	+4.2	−1.4	+1.6	−0.8	+1.6
	1.0	1.0	1.0	2.4	1.4	3.0	.9	1.8	1.2	1.8	1.2	1.6	1.6	2.8	.9	2.0
17	+1.2	+8.0	+0.2	+10.0	0.0	+10.6	−0.4	+8.6	−0.4	+6.2	+0.2	+4.2	−1.2	+4.6	−0.4	+5.4
	1.4	2.8	1.8	4.4	.8	6.0	1.2	4.2	1.2	7.0	1.4	5.6	1.8	6.0	1.6	4.6
19	+2.2	+0.8	+2.2	+2.0	+1.2	+2.2	+0.4	+2.8	0.0	+5.6	−0.2	+4.6	+0.8	+3.8	−0.4	+2.6
	.6	2.2	1.0	2.4	1.2	2.2	1.2	1.6	.8	2.8	1.0	3.2	1.4	2.4	1.2	1.8
21	+1.0	+0.6	+0.8	+0.8	+1.4	+2.6	+0.8	+4.8	+1.5	+5.8	−1.6	+7.0	+1.4	+6.2	+2.0	+6.6
	1.1	2.0	1.2	1.4	1.0	1.6	1.2	2.0	1.5	2.3	1.2	1.6	1.4	2.3	.8	2.2
23	+2.0	+0.8	+2.8	+2.0	+2.6	+2.4	+2.6	+4.8	+2.8	+6.8	+2.6	+7.0	+2.4	+8.4	+2.0	+7.4
	1.0	1.0	1.4	1.4	1.6	1.4	.8	1.0	1.0	1.6	1.2	2.4	.4	1.4	0	1.6
25	+1.6	+2.0	+1.4	+3.8	+1.4	+3.4	+1.0	+4.4	+1.2	+6.6	+1.8	+8.8	+2.4	+6.6	+2.0	+7.6
	1.2	.4	2.2	2.4	1.6	1.2	1.6	1.2	2.1	1.4	1.4	1.8	1.6	3.6	1.9	1.6
27	+2.2	−3.6	+2.0	−1.4	+2.0	+1.6	+1.4	+4.0	+1.0	−7.2	−0.5	+7.8	+1.0	+9.4	+1.2	+6.2
	2.2	2.0	2.4	2.8	2.0	4.4	2.0	4.8	2.2	5.0	1.7	6.0	2.0	4.4	1.4	3.8
29	+0.7	0.0	+1.6	+2.8	+1.0	+5.6	+0.6	+6.0	+0.6	+8.4	+0.6	+10.8	+1.6	+10.6	+0.9	+10.2
	1.4	2.0	1.2	2.2	1.0	1.8	1.6	2.0	1.6	1.6	1.8	1.4	1.2	2.0	1.0	1.0
31	+1.8	−0.6	+2.2	−0.2	+1.9	+1.8	+2.7	+4.0	+2.8	+4.8	+2.0	+3.6	+1.2	+2.8	+1.6	+4.0
	1.1	1.6	1.0	2.0	.9	3.0	1.1	2.4	1.4	2.0	1.2	2.0	1.4	1.0	2.4	1.2
33	−0.4	+0.4	−1.0	+0.2	−1.4	+0.6	−0.8	+2.6	−0.8	+4.0	−0.8	+3.8	−0.5	+3.6	−1.8	+3.8
	1.0	2.0	1.8	1.8	.9	2.4	1.2	2.6	1.5	2.5	1.2	2.2	1.7	2.0	1.0	2.2
35	+3.0	−0.4	+3.9	+0.4	+3.6	+2.8	+3.0	+6.4	+2.8	+7.2	+3.0	+8.0	+3.2	+7.8	+2.6	+8.4
	1.8	2.0	1.3	2.4	.8	3.0	1.4	4.0	1.6	3.8	1.2	3.2	1.0	3.0	1.4	3.2
37	+0.8	−1.0	+0.1	+0.8	+0.4	+1.0	−0.3	−0.6	−0.8	−0.8	−1.4	−1.2	−2.0	−1.6	−1.4	−1.4
	.9	1.4	.7	2.6	.8	2.0	1.5	3.2	1.4	2.8	1.4	3.3	1.2	2.2	1.4	1.4
39	+1.6	+2.6	+0.2	+4.0	+1.2	+2.6	+1.2	+4.0	+0.8	+6.2	0.0	+6.8	0.0	+6.0	−0.2	+6.2
	.8	1.4	1.8	2.2	1.6	2.0	1.6	1.6	1.6	1.8	2.0	2.2	2.0	1.8	1.4	2.6
41	+2.0	+3.6	+1.8	+4.4	+2.6	+5.4	+2.8	+7.4	+3.4	+9.2	+3.0	+10.2	+2.0	+10.0	+2.8	+8.6
	.8	.8	1.0	1.4	1.0	2.0	.8	2.2	1.0	2.4	1.0	2.8	.8	3.0	1.2	2.6
43	+3.2	+3.6	+2.0	+5.0	+3.0	+9.0	+2.4	+5.4	+2.4	+3.8	+2.2	+0.8	+2.4	+1.6	+2.0	+0.6
	2.8	1.6	1.6	6.0	1.8	5.2	2.0	7.4	1.2	5.0	1.8	3.0	1.2	2.6	1.2	1.4
45	−0.6	−1.0	−1.4	+0.8	−1.8	+1.0	−0.6	+3.6	−1.0	+4.2	−1.0	+4.2	−0.8	+4.2	−1.3	+3.8
	.6	1.4	.6	1.8	.6	3.2	1.8	4.0	1.0	1.8	1.4	2.0	1.6	3.6	1.9	3.4
47	−0.6	+0.6	−1.2	+4.6	+0.4	+5.0	−0.6	+5.4	−0.4	+6.2	0.0	+4.8	+0.2	+3.2	−0.4	+2.8
	1.4	2.6	.8	1.6	1.6	1.6	2.2	2.2	1.2	2.2	.8	3.4	.6	3.0	.8	2.4
49	+2.0	+1.8	+2.4	+6.0	+2.2	+8.8	+1.4	+9.2	+1.6	+13.4	+1.6	+9.0	+1.2	+14.2	+2.8	+13.0
	3.6	1.4	2.4	2.6	2.6	2.2	2.2	5.2	2.4	5.0	2.0	7.0	3.6	4.4	3.2	4.6
51	+2.0	+1.2	+2.8	+2.4	+2.8	+1.8	+2.6	+1.0	+2.0	+3.8	+1.2	+3.8	+2.8	+4.0	+3.[illegible]	+3.[illegible]
	1.6	1.6	1.6	3.4	1.6	2.0	1.0	1.4	.8	1.8	1.2	1.6	.8	1.4	1.[illegible]	1.0
53	+1.2	−0.4	−0.4	+2.8	0.0	+7.6	+0.2	+10.8	+0.4	+9.0	+0.8	+6.0	+0.8	+4.0	−0.[illegible]	+6.0
	1.6	1.6	2.0	2.2	2.0	1.8	1.8	3.2	1.6	5.4	1.2	5.4	1.6	2.6	1.[illegible]	6.4

TABLE VII, B. Accuracy in singing

Obs. 0-30 0-23 0-17 0-12 0-8 0-5 0-3
256 286 256 279 256 273 256 268 256 264 256 261 256
Obs. 0-30 0-23 0-17 0-12 0-8 0-5 0-3
256 286 256 279 256 273 256 268 256 264 256 261 256
Obs. 0-30 0-23 0-17 0-12 0-8 0-5 0-3
256 286 256 279 256 273 256 268 256 264 256 261 256

Obs.	0-30		0-23		0-17		0-12		0-8		0-5		0-3
	256	286	256	279	256	273	256	268	256	264	256	261	256
1	+1.0	+1.0	+0.2	+2.6	+0.2	+2.0	+0.6	+4.2	+0.6	+6.0	+0.6	+6.0	+1.2
	1.0	1.4	.6	2.0	.6	1.8	1.4	1.8	1.0	2.4	1.0	2.2	1.2
3	+2.8	+3.6	+2.2	+6.2	+2.2	+5.8	+2.0	+4.4	+1.8	+3.6	+2.4	+3.0	+1.8
	.8	2.8	.6	2.4	1.0	4.0	1.2	2.8	1.4	2.4	1.2	1.2	1.4
5	+0.6	+6.0	+1.6	+5.0	+1.0	+5.0	+1.0	+8.0	+0.8	+7.2	+1.0	+8.2	+1.0
	1.4	1.6	1.6	2.0	2.2	2.0	1.8	2.0	1.2	1.6	1.8	2.0	1.4
7	+1.4	+0.8	+1.8	+2.8	+0.8	+4.8	+1.4	+6.4	+1.2	+6.4	+1.0	+6.2	+1.8
	1.0	2.0	1.0	1.4	1.2	2.2	1.0	2.8	.8	2.4	1.0	2.4	.6
9	+0.6	+1.6	+0.2	—0.6	+0.5	+0.4	—0.6	0.0	—1.0	+1.2	+0.2	+3.0	0.0
	1.0	1.2	1.4	2.0	.9	2.4	1.4	1.2	1.8	1.6	1.6	2.2	1.2
11	+4.4	+4.4	+4.6	+6.4	+4.8	+8.0	+5.4	+8.8	+5.4	+9.8	+5.6	+9.6	+5.4
	1.6	2.4	1.6	4.0	1.2	3.0	1.6	1.6	1.4	3.4	1.2	2.8	2.0
13	+1.6	+1.0	+2.2	+2.8	+2.2	+4.6	+1.4	+5.2	+2.0	+6.6	+3.0	+4.8	+2.6
	1.1	1.0	.9	1.0	1.0	1.2	1.2	1.4	.8	3.0	.9	2.2	1.1
15	—0.8	—0.6	—0.8	—0.2	—0.8	+1.2	—1.0	+1.8	—1.2	+3.2	—0.4	+4.2	—1.4
	1.0	1.0	1.0	2.4	1.4	3.0	.9	1.8	1.2	1.8	1.2	1.6	1.6
17	+1.2	+8.0	+0.2	+10.0	0.0	+10.6	—0.4	+8.6	—0.4	+6.2	+0.2	+4.2	—1.2
	1.4	2.8	1.8	4.4	.8	6.0	1.2	4.2	1.2	7.0	1.4	5.6	1.8
19	+2.2	+0.8	+2.2	+2.0	+1.2	+2.2	+0.4	+2.8	0.0	+5.6	—0.2	+4.6	+0.8
	.6	2.2	1.0	2.4	1.2	2.2	1.2	1.6	.8	2.8	1.0	3.2	1.4
21	+1.0	+0.6	+0.8	+0.8	+1.4	+2.6	+0.8	+4.8	+1.5	+5.8	—1.6	+7.0	+1.4
	1.1	2.0	1.2	1.4	1.0	1.6	1.2	2.0	1.5	2.3	1.2	1.6	1.4
23	+2.0	+0.8	+2.8	+2.0	+2.6	+2.4	+2.6	+4.8	+2.8	+6.8	+2.6	+7.0	+2.4
	1.0	1.0	1.4	1.4	1.6	1.4	.8	1.0	1.0	1.6	1.2	2.4	.4
25	+1.6	+2.0	+1.4	+3.8	+1.4	+3.4	+1.0	+4.4	+1.2	+6.6	+1.8	+8.8	+2.4
	1.2	.4	2.2	2.4	1.6	1.2	1.6	1.2	2.1	1.4	1.4	1.8	1.6
27	+2.2	—3.6	+2.0	—1.4	+2.0	+1.6	+1.4	+4.0	+1.0	—7.2	—0.5	+7.8	+1.0
	2.2	2.0	2.4	2.8	2.0	4.4	2.0	4.8	2.2	5.0	1.7	6.0	2.0
29	+0.7	0.0	+1.6	+2.8	+1.0	+5.6	+0.6	+6.0	+0.6	+8.4	+0.6	+10.8	+1.6
	1.4	2.0	1.2	2.2	1.0	1.8	1.6	2.0	1.6	1.6	1.8	1.4	1.2
31	+1.8	—0.6	+2.2	—0.2	+1.9	+1.8	+2.7	+4.0	+2.8	+4.8	+2.0	+3.6	+1.2
	1.1	1.6	1.0	2.0	.9	3.0	1.1	2.4	1.4	2.0	1.2	2.0	1.4
33	—0.4	+0.4	—1.0	+0.2	—1.4	+0.6	—0.8	+2.6	—0.8	+4.0	—0.8	+3.8	—0.5
	1.0	2.0	1.8	1.8	.9	2.4	1.2	2.6	1.5	2.5	1.2	2.2	1.7
35	+3.0	—0.4	+3.9	+0.4	+3.6	+2.8	+3.0	+6.4	+2.8	+7.2	+3.0	+8.0	+3.2
	1.8	2.0	1.3	2.4	.8	3.0	1.4	4.0	1.6	3.8	1.2	3.2	1.0
37	+0.8	—1.0	+0.1	+0.8	+0.4	+1.0	—0.3	—0.6	—0.8	—0.8	—1.4	—1.2	—2.0
	.9	1.4	.7	2.6	.8	2.0	1.5	3.2	1.4	2.8	1.4	3.3	1.2
39	+1.6	+2.6	+0.2	+4.0	+1.2	+2.6	+1.2	+4.0	+0.8	+6.2	0.0	+6.8	0.0
	.8	1.4	1.8	2.2	1.6	2.0	1.6	1.6	1.6	1.8	2.0	2.2	2.0
41	+2.0	+3.6	+1.8	+4.4	+2.6	+5.4	+2.8	+7.4	+3.4	+9.2	+3.0	+10.2	+2.0
	.8	.8	1.0	1.4	1.0	2.0	.8	2.2	1.0	2.4	1.0	2.8	.8
43	+3.2	+3.6	+2.0	+5.0	+3.0	+9.0	+2.4	+5.4	+2.4	+3.8	+2.2	+0.8	+2.4
	2.8	1.6	1.6	6.0	1.8	5.2	2.0	7.4	1.2	5.0	1.8	3.0	1.2
45	—0.6	—1.0	—1.4	+0.8	—1.8	+1.0	—0.6	+3.6	—1.0	+4.2	—1.0	+4.2	—0.8
	.6	1.4	.6	1.8	.6	3.2	1.8	4.0	1.0	1.8	1.4	2.0	1.6
47	—0.6	+0.6	—1.2	+4.6	+0.4	+5.0	—0.6	+5.4	—0.4	+6.2	0.0	+4.8	+0.2
	1.4	2.6	.8	1.6	1.6	1.6	2.2	2.2	1.2	2.2	.8	3.4	.6
49	+2.0	+1.8	+2.4	+6.0	+2.2	+8.8	+1.4	+9.2	+1.6	+13.4	+1.6	+9.0	+1.2
	3.6	1.4	2.4	2.6	2.6	2.2	2.2	5.2	2.4	5.0	2.0	7.0	3.6
51	+2.0	+1.2	+2.8	+2.4	+2.8	+1.8	+2.6	+1.0	+2.0	+3.8	+1.2	+3.8	+2.8
	1.6	1.6	1.6	3.4	1.6	2.0	1.0	1.4	.8	1.8	1.2	1.6	.8
53	+1.2	—0.4	—0.4	+2.8	0.0	+7.6	+0.2	+10.8	+0.4	+9.0	+0.8	+6.0	+0.8
	1.6	1.6	2.0	2.2	2.0	1.8	1.8	3.2	1.6	5.4	1.2	5.4	1.6

singing: women*

0-3		0-2		0-1		0-.5		Average Arithmetic		Average Algebraic	
								O	V	O	V
256	259	256	258	256	257	256	256.5	256	266	256	266

0-3		0-2		0-1		0-.5		Average Arithmetic		Average Algebraic	
								O	V	O	V
256	259	256	258	256	257	256	256.5	256	266	256	266

0-3		0-2		0-1		0-.5		Average Arithmetic		Average Algebraic	
								O	V	O	V
256	259	256	258	256	257	256	256.5	256	266	256	266

0-3		0-2		0-1		0-.5		Average Arithmetic		Average Algebraic	
								O	V	O	V
256	259	256	258	256	257	256	256.5	256	266	256	266
+1.2	+4.6	+1.4	+5.2	+1.6	+3.2	+1.2	+2.4	.86	3.72	+.86	+3.72
1.2	1.2	1.0	2.4	.4	.6	.8	.8	.90	1.66		
+1.8	+3.2	+1.6	+2.0	+2.0	+2.8	+1.4	+3.0	2.02	3.76	+2.02	+3.76
1.4	1.0	1.6	1.2	.4	1.2	1.0	1.0	1.06	2.00		
+1.0	+8.2	+0.6	+8.2	+0.2	+7.4	+0.4	+4.5	.82	6.77	+.82	+6.77
1.4	1.6	1.0	1.8	1.4	.8	1.2	1.4	1.50	1.68		
+1.8	+5.8	+1.2	+5.0	+3.2	+4.2	+1.0	+3.5	1.48	4.59	+1.48	+4.59
.6	2.4	.8	1.4	2.8	1.6	1.8	1.6	1.20	2.02		
0.0	+2.2	+0.2	+2.6	—0.2	+1.6	+0.6	+1.5	.41	1.47	+.05	+1.35
1.2	2.3	1.0	1.6	1.7	2.6	1.4	1.3	1.34	1.84		
+5.4	+8.6	+5.0	+9.2	+5.8	+8.6	+5.0	+8.4	5.14	8.18	+5.14	+8.18
2.0	2.0	1.0	1.6	1.4	1.6	1.0	1.6	1.40	2.40		
+2.6	+3.6	+2.2	+3.4	+2.4	+3.4	+2.2	+2.6	2.18	3.80	+2.18	+3.80
1.1	1.4	1.0	1.6	.8	2.0	.7	.9	.95	1.57		
—1.4	+1.6	—0.8	+1.6	—0.6	+0.4	—1.8	—0.6	.96	1.54	—.96	+1.26
1.6	2.8	.9	2.0	1.2	1.6	.5	.8	1.09	1.98		
—1.2	+4.6	—0.4	+5.4	—0.8	+3.2	—0.4	+5.0	.52	6.58	—.20	+6.58
1.8	6.0	1.6	4.6	2.2	3.2	1.6	3.8	1.50	4.76		
+0.8	+3.8	—0.4	+2.6	—0.8	+1.8	+0.4	+1.0	.86	2.72	+.58	+2.72
1.4	2.4	1.2	1.8	1.4	1.6	1.6	1.5	1.14	2.17		
+1.4	+6.2	+2.0	+6.6	+1.8	+5.2	+1.6	+4.2	1.39	4.38	+1.07	+4.34
1.4	2.3	.8	2.2	.6	.6	.9	1.6	1.09	1.76		
+2.4	+8.4	+2.0	+7.4	+2.8	+6.2	+2.4	+5.2	2.50	5.10	+2.50	+5.10
.4	1.4	0	1.6	1.2	1.9	.5	.8	.91	1.45		
+2.4	+6.6	+2.0	+7.6	+2.2	+6.0	+2.4	+4.5	1.74	5.37	+1.74	+5.37
1.6	3.6	1.9	1.6	2.2	2.0	1.6	1.9	1.74	1.75		
+1.0	+9.4	+1.2	+6.2	+0.2	+4.4	0.0	+1.6	1.15	4.72	+1.05	+3.72
2.0	4.4	1.4	3.8	1.4	3.6	2.0	4.0	1.93	4.08		
+1.6	+10.6	+0.9	+10.2	+0.6	+7.2	+0.8	+6.4	.90	6.80	+.90	+6.80
1.2	2.0	1.0	1.0	1.8	2.2	1.6	1.3	1.42	1.75		
+1.2	+2.8	+1.6	+4.0	+2.0	+3.6	+2.6	+4.1	2.08	2.95	+2.08	+2.79
1.4	1.0	2.4	1.2	.8	1.3	1.8	1.9	1.31	1.84		
—0.5	+3.6	—1.8	+3.8	—1.8	+2.8	—1.2	+1.6	1.05	2.34	—1.05	+2.34
1.7	2.0	1.0	2.2	1.4	2.4	2.0	1.3	1.37	2.14		
+3.2	+7.8	+2.6	+8.4	+3.0	+8.8	+2.4	+7.6	3.05	5.78	+3.05	+5.70
1.0	3.0	1.4	3.2	1.4	2.6	.8	2.8	1.27	3.00		
—2.0	—1.6	—1.4	—1.4	—2.2	—0.8	—2.0	—1.2	1.14	1.04	—.88	—.68
1.2	2.2	1.4	1.4	1.8	1.2	1.3	1.3	1.24	2.14		
0.0	+6.0	—0.2	+6.2	0.0	+5.0	0.0	+4.8	.52	4.82	+.48	+4.80
2.0	1.8	1.4	2.6	2.0	2.8	1.6	2.4	1.64	2.08		
+2.0	+10.0	+2.8	+8.6	+2.6	+7.6	+2.6	+5.6	2.56	7.20	+2.56	+7.20
.8	3.0	1.2	2.6	1.0	1.4	1.0	1.6	.96	2.02		
+2.4	+1.6	+2.0	+0.6	+1.8	+1.6	+1.4	+0.4	2.28	3.18	+2.28	+3.18
1.2	2.6	1.2	1.4	1.4	1.9	1.8	2.8	1.68	3.69		
—0.8	+4.2	—1.3	+3.8	—1.6	+3.2	—2.0	+2.0	1.21	2.80	—1.21	+2.60
1.6	3.6	1.9	3.4	2.0	2.6	1.2	2.0	1.27	2.58		
+0.2	+3.2	—0.4	+2.8	—0.6	+1.6	—0.7	0.0	.51	3.42	—.39	+3.42
.6	3.0	.8	2.4	1.2	1.8	1.1	1.3	1.17	2.21		
+1.2	+14.2	+2.8	+13.0	+1.8	+10.4	+2.4	+5.8	1.94	9.16	+1.94	+9.16
3.6	4.4	3.2	4.6	3.4	5.4	2.4	4.6	2.78	4.24		
+2.8	+4.0	+3.0	+3.8	+2.2	+4.2	+3.6	+4.4	2.50	3.04	+2.50	+3.04
.8	1.4	1.0	1.0	1.0	.8	.8	1.4	1.14	1.64		
+0.8	+4.0	—0.8	+6.0	—0.4	+1.8	+0.6	+1.2	.56	4.96	+.24	+4.88
1.6	2.6	1.6	6.4	2.8	4.8	2.6	2.4	1.88	3.58		

	.5	2.0	.8	2.3	.9	2.0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
66	0.0	+1.9	−0.3	+2.9	+0.2	+0.9	+0.2	+4.8	+0.1	+4.2	−0.1	+4
	1.2	.9	1.3	3.1	.8	3.1	1.4	3.2	1.1	1.8	1.5	1
68	+1.4	+3.8	+1.2	+3.2	+1.3	+2.8	+1.8	+4.5	+1.2	+6.5	+0.8	+8
	1.2	1.0	.4	2.2	.3	1.8	.8	.7	.6	1.1	.4	1
70	+1.4	−4.3	+1.4	−3.8	+1.3	0.0	+2.0	+1.8	+2.2	+3.3	+2.8	+7
	2.4	3.7	2.4	5.4	2.5	4.8	3.0	4.6	2.0	3.7	2.4	2
72	+2.3	−1.0	+1.4	+0.6	+1.5	+1.0	+1.7	+1.8	+1.4	+2.3	+0.9	+1
	1.1	.8	1.2	2.0	.9	.8	.9	1.6	.8	1.7	1.3	1
74	−2.6	−2.2	−2.4	+2.7	−1.6	+6.5	−2.3	+10.3	−1.4	+14.3	−1.5	+15
	4.0	2.4	4.2	2.9	3.4	2.9	3.1	2.1	3.0	3.1	3.5	3
76	−3.6	+4.5	−3.5	+4.9	−3.7	+5.7	−2.9	2.1	−3.9	+4.6	−2.5	+3
	1.0	.7	1.1	1.7	1.7	2.1	3.1	2.9	1.9	4.2	2.9	2
78	−2.0	−0.2	−2.0	+4.4	−2.4	+0.6	−2.5	+3.5	−2.1	+5.5	−1.8	+8
	.8	.8	1.0	1.0	.8	2.8	.9	2.1	.5	2.5	.8	1
80	+0.1	−0.2	0.0	+0.3	−1.1	+3.0	−1.6	+5.7	−1.6	+6.0	−1.6	+7
	2.5	1.4	2.8	2.1	2.5	1.8	3.0	3.1	1.8	2.0	2.2	3
82	−1.1	+1.5	−1.2	+2.9	−1.2	+4.1	−1.3	+4.5	−1.3	+5.5	−1.0	+4
	1.5	1.9	1.2	1.3	1.4	2.1	1.1	2.5	.9	1.3	1.0	2
84	−1.0	+4.1	−1.0	+4.0	−1.4	+3.5	−1.0	+3.0	−1.7	+3.1	−1.9	+3
	1.2	2.1	1.4	2.0	1.2	2.7	2.0	2.0	1.9	2.1	1.7	2
86	−1.2	+0.5	−1.2	+3.9	−0.9	+6.5	−0.9	+6.7	−0.9	+8.9	−1.3	+9
	2.4	1.1	2.0	1.3	1.5	3.9	1.9	4.9	1.9	2.3	2.1	1
88	−0.1	+3.5	−0.5	+2.6	+0.2	+5.4	+0.5	+6.0	−0.6	+6.1	−0.8	+4
	1.3	1.5	.7	1.4	1.0	1.6	.9	1.6	1.0	2.1	1.0	
90	−1.7	+0.7	−2.5	+2.9	−2.3	+4.4	−2.6	+5.4	−2.0	+5.4	−2.3	+7
	1.7	2.5	1.7	3.1	1.5	3.0	1.8	4.4	1.8	2.2	1.3	2
92	+2.5	+2.5	+3.2	+6.3	+3.6	+3.5	+3.2	+6.7	+3.6	+9.5	+4.3	+9
	1.9	1.9	1.0	3.7	2.0	2.3	1.4	2.7	2.6	2.3	4.7	4
94	+0.6	+3.3	+0.9	+5.9	+1.9	+5.8	+1.1	+7.0	+0.9	+8.4	+1.1	+7
	1.0	.7	.9	1.9	1.5	1.0	.7	2.2	1.1	3.8	1.1	2
96	+0.2	+0.4	−0.4	+2.5	−0.5	+3.8	0	+2.6	−0.2	+3.4	+0.9	+4
	1.0	2.0	1.0	2.3	.7	3.4	.6	4.4	1.2	3.0	.9	3
98	+1.5	+3.3	+1.7	+4.0	+2.0	+5.1	+1.9	+7.3	+1.7	+7.8	+1.7	+7
	.5	.7	.7	2.0	.6	2.7	.5	1.7	.5	2.2	.7	2
100	−1.5	+1.7	−1.5	+6.7	−1.7	+6.6	−1.4	+6.2	−1.6	+6.9	−2.4	+5
	1.5	.7	1.7	1.5	1.3	4.0	1.6	3.4	2.0	3.1	2.0	4
102	−0.2	+1.3	+0.1	+3.1	+0.1	+2.4	+0.5	+1.4	0.0	+3.0	+0.1	+4
	.6	.9	1.1	2.1	1.1	3.0	.7	1.1	.8	1.0	.7	1
104	−1.0	+1.0	−1.5	+4.7	−1.1	+5.1	−1.1	+4.6	−1.1	+7.4	−1.4	+8
	1.2	1.4	.9	1.5	1.5	1.9	1.1	1.9	1.3	3.2	1.6	1
106	−0.4	+3.7	−0.4	+2.9	−0.2	+5.8	−0.2	+6.7	+0.2	+6.9	−0.3	+4
	.4	1.1	1.0	3.3	.8	2.0	1.0	2.7	.4	2.9	.5	2
108	−1.3	+1.2	−1.0	+5.3	−1.6	+6.8	−1.7	+6.4	−1.7	+7.3	−0.3	+8
	1.9	2.2	2.2	2.1	2.1	3.0	1.1	2.4	1.3	3.6	1.9	2
110	+1.7	+1.4	+1.4	+3.8	+1.7	+4.7	+1.3	+6.2	+1.5	+6.0	+1.6	+6
	.9	1.0	.6	1.4	.7	1.5	.7	1.6	.9	1.6	1.2	1
112	+0.6	−4.5	+0.6	−2.2	−1.1	+5.1	−0.2	+8.1	+0.2	+11.9	−0.6	+11
	1.6	3.9	3.2	3.8	1.7	4.7	1.4	3.3	2.0	3.1	1.2	1
114	−0.3	+1.6	−0.8	+1.3	−0.2	+3.8	−0.1	+3.5	−0.4	+4.6	−0.8	+2
	.9	1.0	.8	.9	1.0	2.0	1.3	1.9	1.0	2.6	.6	1
116	−0.5	+1.8	−1.5	+2.5	−2.3	+3.6	−2.5	+3.6	−2.9	+7.0	−2.8	+9
	1.1	2.8	1.3	5.5	1.3	4.8	1.3	4.4	1.5	3.8	1.6	3
118	−0.6	−0.1	−0.5	+0.9	−0.9	+4.7	−1.1	+4.3	−0.1	+8.1	−0.5	+7
	1.0	2.3	1.1	4.1	1.3	4.5	1.7	2.1	1.9	2.1	1.9	4
120	+03	+1.9	+0.6	+3.4	+0.3	+4.2	+0.1	+6.8	+0.3	+7.4	+0.3	+6
	.7	.7	1.2	1.4	.9	2.0	.7	2.4	1.3	1.6	1.1	1
122	−2.5	−0.7	−3.4	+2.3	−3.9	+5.6	−2.9	+7.3	−2.7	+10.5	−1.2	+12
	1.7	3.6	1.1	3.8	1.8	3.8	1.3	4.3	.8	2.7	2.3	3
124	−1.2	−1.1	−1.3	+3.6	−1.3	+6.7	−1.1	+9.4	−1.2	+12.3	−1.1	+11
	1.2	3.5	1.1	3.0	1.5	3.1	1.3	4.6	1.4	6.4	1.5	8
126	−0.5	+0.9	−0.5	+2.1	−0.8	+2.0	−0.7	+3.1	−0.9	+3.6	−0.8	+2
	.5	.3	.7	1.1	.6	1.4	.3	.9	.5	1.6	.4	1
128	+0.1	+0.7	0.0	+3.4	−0.6	+3.2	−0.5	+4.6	−0.6	+5.9	−0.4	+5
	.7	1.5	1.0	1.8	.8	2.2	.5	2.0	.6	.7	.6	1
130	−0.3	−1.2	−0.8	+3.1	−1.1	−0.6	−0.8	+2.2	−1.2	+6.3	−0.8	+8
	.5	1.8	1.0	1.5	.7	2.8	.4	2.2	.8	2.5	1.0	2
132	−1.5	−3.2	−2.5	−2.7	−2.7	+1.8	−2.0	+5.0	−2.3	+7.6	−1.7	+10
	2.0	5.1	2.5	3.6	2.8	2.8	2.5	2.0	2.5	2.1	3.1	2
134	−2.6	+0.7	−2.7	+3.1	−3.0	+3.0	−2.7	+3.7	−3.1	+5.9	−3.9	+5
	2.2	6.7	2.5	4.1	2.6	4.4	2.7	2.9	2.7	3.1	2.9	4
136	−1.7	+0.3	−0.8	−1.9	−1.2	0.0	−1.2	+4.8	−1.4	+6.7	−1.7	+4
	.5	2.1	.8	2.9	.6	1.6	.6	2.6	.4	3.1	.3	2
138	−2.5	−16.0	−7.0	−8.1	−1.4	−3.4	−6.2	+4.1	−1.2	+2.8	+1.5	+8
	8.3	6.1	7.3	6.1	3.6	3.8	7.8	12.1	3.1	1.8	4.1	6
140	0.0	+3.2	0.0	+4.2	−0.2	+8.3	−0.2	+8.4	+0.2	+8.8	−0.5	+2
	1.0	3.6	.8	1.8	.6	2.7	.6	5.2	1.0	4.8	1.5	3
142	−0.2	+0.7	+0.8	+1.7	+0.1	+5.8	+1.2	+7.3	+0.5	+4.6	+0.1	5
	.6	[illegible]	1.4	2.1	1.5	2.8	2.2	3.7	1.0	4.8	1.3	4

-0.1	+4.7	-0.6	+5.7	-0.8	+5.0	-0.5	+5.7	-0.3	+5.1	.31	4.09	-.21	+4.09	55
1.5	1.7	1.4	1.1	1.6	2.8	1.3	2.3	1.3	2.6	1.29	2.26			
-0.8	+8.5	+0.6	+8.1	+0.8	+8.3	+0.7	+6.9	+1.1	+5.0	1.09	5.76	+1.09	+5.76	57
.4	1.1	.6	1.3	.8	1.9	.9	1.9	.5	1.1	.65	1.41			
-2.8	+7.5	+2.8	+8.7	+2.2	+8.5	-1.9	+7.2	+1.2	+5.5	1.92	5.06	+1.64	+3.44	59
2.4	2.3	2.4	5.1	2.0	4.7	1.9	4.4	1.0	1.6	2.20	4.03			
-0.9	+1.7	+1.0	+3.4	+0.8	+2.9	+0.7	+3.7	+0.8	+3.3	1.25	2.08	+1.25	+2.08	61
1.3	1.5	.8	1.4	1.2	1.1	1.1	.8	1.0	1.0	1.03	1.27			
-1.5	+15.6	0.0	+14.7	+0.6	+13.8	+1.9	+17.0	+1.8	+15.6	1.61	11.3	-.75	+10.8	63
3.5	3.0	2.6	5.5	2.4	6.2	2.5	6.0	2.6	7.7	3.03	4.18			
-2.5	+3.6	-2.5	+4.0	-2.6	+4.6	-3.2	+2.7	-2.9	+1.5	3.13	3.82	-3.13	+3.82	65
2.9	2.8	2.3	3.2	3.4	3.0	2.8	3.5	2.3	3.4	2.15	2.85			
-1.8	+8.3	-2.1	+7.3	-2.3	+7.4	-2.2	+6.8	-1.8	+5.2	2.12	4.92	-2.12	+4.90	67
.8	1.5	.5	2.5	.5	2.2	.8	3.4	1.0	3.5	.76	2.23			
-1.6	+7.7	-2.1	+7.4	-1.8	+8.9	-2.6	+8.3	-2.2	+8.4	1.47	5.59	-1.45	+5.57	69
2.2	3.5	2.3	3.4	1.6	2.1	1.8	2.3	2.0	1.7	2.25	2.34			
-1.0	+4.3	-1.2	+2.8	-2.3	+0.6	-2.5	+0.8	-2.2	-1.3	1.53	2.83	-1.53	+2.57	71
1.0	2.5	1.0	2.2	.9	1.4	1.1	2.2	.8	.6	1.09	1.80			
-1.9	+3.4	-2.0	+3.9	-2.2	+4.5	-2.7	+2.1	-2.5	+0.2	1.74	3.18	-1.74	+3.18	73
1.7	2.2	1.2	1.7	1.6	1.5	1.5	3.1	1.1	2.3	1.48	2.17			
-1.3	+9.2	-1.3	+9.8	-1.3	+9.5	-1.9	+7.8	-1.0	+8.4	1.19	7.10	-1.19	+7.10	75
2.1	1.0	2.1	2.2	1.7	2.3	1.5	2.6	2.8	2.1	1.99	2.47			
-0.8	+4.4	-0.5	+4.6	-0.8	+4.4	-1.1	+3.1	-0.4	+1.8	.55	4.19	-.41	+4.19	77
1.0	.8	1.3	1.6	1.2	1.6	1.3	2.3	1.0	2.1	1.07	1.66			
-2.3	+7.3	-1.7	+8.9	-1.8	+7.2	-2.2	+7.2	-1.9	+6.7	2.10	5.61	-2.10	+5.61	79
1.3	2.3	1.3	1.9	1.0	1.4	.8	2.8	.9	2.8	1.38	2.64			
+4.3	+9.4	+3.0	+10.0	+4.2	+10.1	+4.9	+10.8	+4.4	+10.8	3.69	7.96	+3.69	+7.96	81
4.7	4.4	1.6	2.6	2.8	3.1	2.9	2.4	3.0	3.1	2.19	2.85			
+1.1	+7.9	+0.6	+6.0	+0.5	+4.8	+0.5	+4.6	+0.7	+4.2	.88	5.79	+.88	+5.79	83
1.1	2.7	.8	2.2	.9	2.4	1.1	2.2	.9	2.3	1.00	2.14			
+0.9	+4.2	+0.1	+4.7	+0.2	+4.9	+0.3	+5.6	+0.8	+5.9	.36	3.80	+.14	+3.80	85
.9	3.0	.9	3.8	.8	3.1	1.1	3.6	.8	3.2	.90	3.18			
+1.7	+7.9	+1.9	+7.5	+1.9	+3.4	+1.6	+3.1	+1.3	+2.3	1.72	5.17	+1.72	+5.17	87
.7	2.1	1.1	2.3	.7	1.6	.6	1.1	.5	.6	.64	1.70			
-2.4	+5.8	-2.8	+7.7	-2.5	+8.2	-2.9	+5.9	-2.3	+4.6	2.06	6.03	-2.06	+6.03	89
2.0	4.0	2.0	3.9	1.9	2.6	2.1	3.1	1.7	3.3	1.78	2.96			
+0.1	+4.2	0.0	+6.0	+0.3	+6.2	+0.4	+7.0	+0.4	+7.2	.21	4.18	+.17	+4.18	91
.7	1.4	1.2	1.2	.7	1.2	1.4	1.4	1.0	1.7	.93	1.50			
-1.4	+8.7	-1.2	+7.4	-0.9	+5.9	+0.1	+3.4	-0.4	+1.1	.96	4.93	-.96	+4.93	93
1.6	1.9	1.6	2.6	1.7	2.1	1.1	1.2	1.0	1.6	1.30	1.83			
-0.3	+4.4	+0.1	+4.2	-0.8	+4.5	-0.2	+3.8	-0.1	+4.1	.29	4.70	-.23	+4.70	95
.5	2.4	1.7	2.4	.8	2.3	.8	1.8	.7	1.8	.71	2.27			
-0.3	+8.6	-0.6	+7.8	-1.3	+6.6	-0.9	+3.5	-1.2	+3.9	1.16	5.74	-1.16	+5.74	97
1.9	2.6	3.0	3.4	1.5	4.2	1.7	3.5	1.4	3.0	1.71	3.00			
+1.6	+6.5	+1.6	+5.4	+1.4	+3.7	+1.4	+3.6	+1.3	+3.0	1.49	4.43	+1.49	+4.43	99
1.2	1.7	.8	1.2	.8	1.5	.8	1.6	.7	.9	.81	1.40			
-0.6	+11.4	+0.6	+12.8	+0.9	+13.4	+0.1	+14.4	-0.5	+12.6	.54	9.64	+.06	+8.30	101
1.2	1.8	1.9	3.4	1.5	1.4	1.5	3.6	1.7	5.3	1.72	3.43			
-0.8	+2.8	-0.7	+4.4	-0.9	+3.8	-1.2	+3.3	-1.3	+2.9	.67	3.20	-.67	+3.20	103
.6	1.0	.9	1.6	1.3	1.4	1.2	1.1	1.3	.8	1.03	1.43			
-2.8	+9.1	-2.0	+9.0	-2.4	+9.7	-2.9	+8.7	-2.7	+9.3	2.25	6.43	-2.25	+6.43	105
1.6	3.1	2.0	2.0	.8	2.9	1.1	2.7	1.3	3.0	1.33	3.50			
-0.5	+7.3	-1.0	+8.2	-1.4	+7.1	-1.4	+6.6	-2.1	+2.0	.96	4.93	-.96	+4.91	107
1.9	4.3	2.2	4.8	2.2	4.1	2.8	4.6	2.5	4.7	1.86	3.76			
+0.3	+6.5	-0.1	+6.1	-0.2	+5.3	-0.3	+4.2	-0.1	+3.4	.26	4.92	+.12	+4.92	109
1.1	1.7	.9	1.7	.8	1.7	.9	1.0	.9	.5	.94	1.47			
-1.2	+12.0	-1.9	+13.2	-3.8	+12.2	-2.8	+12.2	-3.5	+13.5	2.86	8.95	-2.86	+8.81	111
2.3	3.8	1.8	4.2	1.5	3.0	2.0	2.7	1.5	3.7	1.58	3.66			
-1.1	+11.9	-1.2	+12.5	-1.8	+16.6	-1.5	+15.6	-1.1	+17.4	1.28	10.7	-1.28	+10.5	113
1.5	8.3	1.8	8.1	1.4	8.2	1.5	8.0	1.7	8.7	1.44	6.24			
-0.8	+2.8	-0.7	+3.8	-0.9	+3.5	-1.0	+2.3	-0.9	+0.9	.77	2.50	-.77	+2.50	115
.4	1.2	.5	1.0	.3	1.1	.4	.7	.5	.4	.47	.97			
-0.4	+5.5	-0.3	+4.5	0.0	+3.8	-0.1	+3.6	+0.1	+2.7	.27	3.79	-.23	+3.79	117
.6	1.5	.5	2.1	.8	2.4	.9	2.3	.5	1.6	.63	1.80			
-0.8	+8.1	-0.8	+9.2	-0.6	+9.5	-0.6	+7.4	-0.9	+6.6	.79	5.42	-.79	+5.08	119
1.0	2.5	.8	1.8	.8	1.5	.6	2.0	1.1	2.7	.77	2.13			
-1.7	+10.3	-0.4	+9.8	-1.3	+11.6	-0.4	+9.5	-0.3	+10.2	1.51	7.17	-1.51	+5.99	121
3.1	2.1	2.6	3.6	2.2	2.3	3.1	3.7	2.0	2.0	2.53	2.93			
-3.9	+5.9	-3.2	+6.4	-3.7	+7.2	-4.0	+3.0	-3.5	+1.7	3.24	4.06	-3.24	+4.06	123
2.9	4.5	2.8	3.8	2.9	3.6	2.6	5.0	2.5	5.0	2.37	4.31			
-1.7	+4.4	-1.5	+5.4	-1.5	+5.1	-1.6	+3.1	-1.2	+2.1	1.38	3.38	-1.38	+3.00	125
.3	2.6	.5	2.4	.5	1.3	.6	1.9	.6	2.0	.54	2.25			
+1.5	+8.6	-3.7	+4.6	-1.5	+8.6	-2.5	+4.1	-5.2	+1.6	3.27	6.19	-2.97	+.69	127
4.1	6.0	10.1	12.1	5.6	8.1	5.0	6.8	5.5	12.1	6.04	7.50			
-0.5	+2.8	-0.2	+2.1	-0.6	+0.1	-0.6	0.0	-0.9	-0.4	.34	3.83	-.34	+3.75	129
1.5	3.6	1.4	4.9	1.6	2.5	1.6	2.6	1.1	2.1	1.12	3.38			
+0.1	5.7	+1.3	+2.8	+1.6	+2.7	+1.8	+2.9	+0.7	+1.9	.83	3.61	+.79	+3.61	131
1.3	4.7	[illegible]	1.6	.8	2.3	1.4	1.7	.7	1.0	1.27	2.70			

55	+1.8	—0.2	+1.8	+1.0	+2.8	+2.2	+2.6	+9.2	+2.0	+5.4	+3.2	[illegible]
	1.0	1.4	.6	1.6	1.2	2.8	.6	2.0	.8	3.0	.8	3.4
57	+2.0	+2.4	+0.8	+4.6	+1.2	+6.8	+1.2	+8.0	+1.2	+8.8	+1.2	+6.8
	2.4	1.2	2.8	2.2	2.8	2.2	2.4	1.6	2.4	2.0	2.8	2.2
59	+4.4	+2.2	+3.2	+2.8	+2.6	+2.6	+2.4	+5.4	+2.6	+7.4	+2.6	+5.8
	.4	1.0	1.6	1.0	.6	1.6	.8	2.2	1.0	1.4	.6	1.6
61	+2.2	+2.2	+1.6	—0.4	+2.2	+1.6	+1.0	+4.0	+1.2	+6.4	+2.2	+7.8
	1.0	1.0	1.6	3.0	1.4	1.4	1.4	1.2	1.2	2.0	1.4	2.0
63	+0.2	—1.4	—0.4	+0.4	—0.8	—1.6	—0.2	+1.0	—0.6	+4.4	—0.6	+6.4
	.2	1.0	.4	2.2	1.2	3.0	.6	1.4	1.4	1.6	1.8	1.4
65	—0.8	—2.2	—2.6	—1.2	—2.6	—1.2	—2.8	+1.0	—2.8	+3.0	—2.6	+1.8
	1.2	1.0	1.0	1.8	1.4	2.6	1.6	2.2	1.2	2.2	1.4	1.2
67	—1.6	0.0	—1.4	+2.6	—0.8	+5.0	—0.8	+5.4	—1.0	+4.8	—1.4	+5.4
	3.2	3.6	2.2	2.4	1.6	3.2	1.6	2.6	1.4	2.8	1.8	4.0
69	+3.0	—1.0	+2.8	+1.6	+2.0	+5.4	+1.8	+6.8	+3.0	+8.4	+2.2	+9.6
	1.6	1.6	1.4	2.2	2.0	2.4	1.8	2.6	1.6	2.8	.6	2.2
71	+1.4	+0.6	+1.6	+1.0	+2.8	+4.8	+3.0	+5.4	+3.0	+6.8	+2.8	+7.4
	1.8	1.4	1.6	1.4	1.6	2.2	1.0	1.8	1.8	2.8	1.6	1.6
73	—3.2	+1.0	+3.2	+1.0	—3.2	+1.8	—3.6	+2.2	—2.6	+4.6	—1.4	+6.0
	1.2	1.4	2.0	1.8	1.2	1.6	1.2	1.4	3.4	1.4	3.0	2.2
75	—1.6	+3.4	—1.0	+4.4	—1.8	+4.2	—1.2	+4.8	—1.6	+6.8	+0.2	+8.2
	1.2	3.0	1.4	1.8	1.0	2.4	1.2	2.8	.8	2.8	1.4	2.8
77	—1.0	—0.8	—0.8	+0.8	—1.2	+4.6	—1.0	+6.0	—1.2	+7.4	—0.2	+6.0
	1.0	2.0	.8	2.6	.8	2.0	1.4	2.0	1.2	1.8	.6	2.6
79	+1.2	+1.2	+1.6	+1.0	+2.6	+5.0	+3.2	+6.0	+2.4	+7.2	+2.2	+7.6
	1.6	2.4	1.6	1.6	1.0	2.0	1.2	2.0	2.4	2.8	2.2	3.0
81	—1.4	+1.6	—1.4	+4.2	—1.6	+6.2	—0.8	+9.6	—1.2	+8.2	+1.0	+6.4
	1.4	1.2	1.4	1.2	.8	2.4	1.6	2.8	1.2	3.0	1.8	4.6
83	+0.8	+2.4	+0.8	+4.2	+0.6	+2.8	+0.4	+5.6	+0.6	+6.0	+0.6	+8.6
	1.6	2.0	1.2	2.4	1.0	1.8	1.6	1.6	1.4	2.0	1.8	2.8
85	—1.4	+1.6	+2.0	+4.0	+1.2	+3.8	+1.0	+4.8	+0.6	+7.6	+1.2	+9.0
	.6	1.6	.4	1.4	1.2	2.2	1.0	3.2	1.4	2.4	1.2	1.6
87	+1.6	+1.6	+1.4	+3.0	+0.8	+3.8	—0.2	+3.8	+0.6	+5.6	+0.4	+6.2
	1.2	1.6	1.8	2.4	2.0	2.8	1.4	1.8	1.0	1.2	.8	2.1
89	+1.0	+3.0	+0.8	+6.6	+0.6	+7.0	+0.8	+5.6	+0.2	+4.8	—0.6	+6.4
	1.4	1.8	2.0	1.2	1.0	2.0	1.2	2.0	1.4	2.8	1.4	1.
91	—4.0	+1.0	—3.5	+2.4	—3.0	+4.0	—3.5	+7.2	—4.4	—2.4	—4.4	—7.
	2.0	2.4	2.0	.4	2.0	1.6	2.0	1.4	2.2	6.0	2.2	3.
93	+0.4	—0.3	+0.4	+0.4	—0.2	+2.0	+0.4	+3.2	—0.2	+5.4	+0.4	+5.4
	.4	.9	.8	1.1	1.4	3.0	.8	1.6	1.4	1.0	.8	1.
95	—0.4	+2.4	—0.4	+5.6	+3.6	+5.0	—2.0	+5.8	—1.2	+7.6	—2.6	+7.
	1.6	1.2	1.2	3.0	1.2	1.6	.8	2.2	.8	2.0	1.4	3.
97	+1.2	+1.6	+1.0	+2.8	+1.4	+3.2	+1.4	+4.4	+1.6	+3.8	+1.6	+4.
	1.2	.8	1.0	1.4	.6	1.0	.6	1.2	.8	1.4	.8	.
99	+2.2	+1.0	+3.0	+2.6	+2.8	+5.0	+3.0	+7.4	+2.2	+8.2	+3.2	+9.
	1.0	1.4	1.8	2.0	1.6	2.0	1.8	2.2	1.0	2.2	2.4	2.8
101	—0.8	+0.6	—0.6	+2.0	—0.6	+3.2	—0.8	+3.4	—1.0	+4.0	—1.6	+5.
	.8	1.4	1.4	3.4	1.0	4.2	1.2	3.0	1.4	2.0	1.6	3.
103	+1.8	0.0	+1.0	+3.6	+1.4	+6.8	+1.8	+7.6	+1.2	+9.0	+2.2	+11.
	.6	1.2	1.8	1.8	1.0	2.2	.6	2.4	.8	2.2	1.0	1.
105	—0.2	+1.2	—0.2	+2.0	—0.4	+2.8	—0.2	+3.6	—0.4	+5.2	—0.2	+5.
	.2	1.6	.6	2.2	.4	1.0	1.0	1.2	.4	1.2	.6	.
107	+1.6	+1.6	+1.0	+3.2	+1.4	+4.0	+0.8	+4.6	+1.4	+5.6	+1.2	+6.
	.8	1.2	1.0	1.8	.6	1.4	.8	1.4	1.0	1.6	.8	1.
109	+0.6	+1.6	+0.2	+4.6	—0.6	+6.0	—0.6	+9.0	—0.4	+9.0	—1.2	+9.
	1.8	2.8	1.0	2.0	1.8	2.6	1.4	2.2	1.2	2.2	1.6	2.
111	—0.4	+1.2	+0.2	+2.0	—0.8	+2.4	—0.8	+4.0	—1.0	+5.6	—0.6	+7.
	1.6	.8	1.4	1.4	1.2	1.4	1.2	.8	1.0	2.0	1.8	2.
113	+1.6	+1.2	+1.8	+0.6	+1.4	+2.2	+1.2	+4.8	+0.6	+6.8	+1.8	+6.
	.4	.8	1.4	2.0	1.0	1.2	1.2	1.6	1.4	1.6	1.0	1.
115	+2.2	+3.4	+2.4	+6.2	+3.0	+4.6	+2.4	+7.8	+2.6	+11.6	+2.2	+11.
	1.0	1.4	.4	1.6	1.0	1.6	.4	1.8	.6	2.0	.2	2.
117	0.0	—0.6	0.0	+2.2	—0.6	+3.2	—0.8	+4.2	—0.8	+6.6	0.0	+7.
	.4	.6	1.2	1.6	1.0	1.0	.8	1.8	1.2	2.6	1.2	2.
119	+1.8	+2.8	+2.2	+10.2	+1.6	+12.2	+2.2	+14.8	+1.8	+14.4	+1.6	+16.
	1.4	1.6	1.4	2.4	2.0	3.2	2.6	3.2	2.6	4.0	2.8	1.
121	—0.4	+1.4	—0.6	+3.2	—0.6	+4.2	—1.0	+5.2	—1.0	+5.6	—0.8	+7.
	1.6	1.4	1.4	2.2	1.4	3.2	1.4	4.4	1.8	3.6	.8	3.
123	0.0	—0.6	+1.0	+1.0	+1.8	+2.8	+2.2	+5.0	+2.4	+5.6	+2.6	+5.
	.8	1.8	1.0	2.8	.6	1.4	.2	2.2	1.2	2.4	1.0	1.
125	—1.0	—1.0	—0.4	+1.6	+0.4	+2.0	—1.2	+2.6	—1.0	+3.6	—1.2	+0.
	1.4	1.4	.4	1.8	.8	3.0	1.2	3.0	1.0	2.4	.8	2.
127	—0.2	+1.6	—0.4	+2.0	+0.2	+3.6	—0.6	+4.4	—1.0	+5.6	—1.2	+5.
	1.0	.8	1.6	2.2	1.0	1.8	.6	1.6	1.4	1.6	1.2	.
129	—5.4	+4.0	—6.8	+5.0	—7.0	+7.6	—7.6	+10.1	—5.0	+12.2	—8.0	+14.
	4.2	3.6	2.6	3.2	2.0	2.6	1.2	3.6	2.0	4.6	2.0	3.
131	—1.6	+1.0	—1.6	+2.8	—1.6	+3.6	—2.2	+2.0	—1.0	+3.2	—1.6	+4.
	.8	2.6	2.4	3.0	2.0	2.2	1.8	2.4	2.2	2.0	1.6	2.

3.2	[illegible]	+2.2	+7.6	+2.2	+7.8	+2.6	+6.2	+2.2	+5.0	2.34	5.26	+2.34	+5.22
.8	3.4	1.0	2.6	1.0	2.6	1.4	1.2	1.8	3.0	1.02	2.36		
1.2	+6.8	+1.0	+3.6	+1.6	+4.0	0.0	+2.0	+0.8	+1.8	1.10	4.88	+1.10	+4.88
2.8	2.2	1.8	3.0	1.6	3.2	2.4	2.2	2.0	2.5	2.34	2.23		
2.6	+5.8	+2.8	+5.4	+1.8	+5.6	+3.4	+5.2	+3.0	+4.6	2.88	4.70	+2.88	+4.70
.6	1.6	.8	1.2	.6	2.4	1.0	1.9	1.4	2.2	.88	1.65		
2.2	+7.8	+2.4	+6.8	+2.0	+6.2	+2.0	+6.0	+2.0	+6.0	1.88	4.74	+1.88	+4.66
1.4	2.0	.4	2.6	.8	2.6	.4	1.8	.4	1.6	1.00	1.92		
0.6	+6.4	−1.4	+4.8	−0.8	+2.6	−1.0	+0.6	−1.4	+0.5	.74	2.37	−.70	+1.77
1.8	1.4	1.0	3.8	1.2	1.8	1.4	2.7	1.0	1.7	1.02	2.06		
2.6	+1.8	−2.8	+2.6	−3.2	+2.6	−2.8	+3.2	−2.8	+1.8	2.58	2.06	−2.58	+1.14
1.4	1.2	.8	2.0	1.2	3.0	1.2	2.2	1.2	1.8	1.22	2.00		
1.4	+5.4	−1.6	+5.8	−1.0	+6.6	−2.2	+4.6	−1.6	+4.0	1.34	4.42	−1.34	+4.42
1.8	4.0	1.6	3.2	1.8	4.2	1.8	2.8	2.8	3.6	1.98	3.24		
2.2	+9.6	+1.2	+8.8	+1.8	+8.8	+1.8	+8.6	+1.4	+3.8	2.10	6.19	+2.10	+6.19
.6	2.2	1.2	1.4	1.0	2.8	.6	2.0	.6	2.2	1.24	2.22		
2.8	+7.4	+2.8	+7.8	+2.4	+6.0	+1.8	+6.0	+2.0	+6.2	2.36	5.20	+2.36	+5.20
1.6	1.6	1.2	2.4	1.2	.8	1.4	2.2	.8	1.0	1.40	1.76		
1.4	+6.0	−3.6	+6.2	−4.0	+5.2	−3.8	+5.8	−5.0	+4.8	3.36	3.86	−3.36	+3.86
3.0	2.2	1.6	1.6	1.6	1.6	1.4	1.2	2.2	1.6	1.88	1.58		
0.2	+8.2	−2.0	+7.4	−2.2	+5.4	−2.8	+5.6	−2.4	+5.2	1.46	5.54	−1.46	+5.54
1.4	2.8	1.6	1.6	1.4	1.8	1.6	2.2	1.6	2.4	1.32	2.36		
0.2	+6.0	−0.6	+6.4	−1.4	+4.0	−1.2	+2.6	−1.0	+2.2	.96	4.08	−.96	+3.92
.6	2.6	.6	3.0	1.0	1.6	.8	1.2	1.0	1.3	.92	2.01		
2.2	+7.6	+2.4	+8.0	+1.8	+7.0	+1.6	+5.2	+1.4	+5.0	2.04	5.32	+2.04	+5.32
2.2	3.0	3.2	3.8	2.6	2.6	4.0	3.0	2.6	1.1	2.24	2.43		
1.0	+6.4	+0.4	+4.6	+0.2	+2.4	−0.2	+0.8	+1.0	+1.8	.92	4.58	−.40	+4.58
1.8	4.6	1.2	4.4	.6	3.6	1.0	1.4	1.0	1.4	1.20	2.60		
0.6	+8.6	+0.4	+5.2	+0.8	+5.0	+1.0	+5.0	+0.6	+4.4	.66	4.72	+.66	+4.72
1.8	2.8	1.2	2.2	1.2	2.2	1.8	1.2	1.4	1.8	1.42	2.00		
1.2	+9.0	+1.4	+8.8	+1.6	+7.2	+1.2	+8.4	+1.2	+6.6	1.28	6.18	+1.28	+6.18
1.2	1.6	.6	1.8	1.6	2.4	1.2	2.2	1.2	2.2	1.04	2.10		
0.4	+6.2	0.0	+6.8	−1.0	+6.2	0.0	+4.2	+0.6	+4.0	.66	4.52	+.42	+4.52
.8	2.1	1.2	1.4	1.8	1.8	1.2	2.0	1.8	2.0	1.42	1.90		
0.6	+6.4	−0.4	+4.6	−0.4	+4.6	−0.4	+4.6	−0.4	+2.2	.56	4.94	+.12	+4.94
1.4	1.4	2.0	3.6	1.6	2.2	1.2	2.4	2.4	2.2	1.56	2.16		
4.4	−7.0	−4.0	−6.4	−4.6	−6.0	−5.0	−1.5	−4.4	−5.2	4.08	4.31	−4.08	−1.40
2.2	3.6	1.6	2.0	2.0	1.5	2.0	2.0	1.8	1.8	1.98	2.27		
0.4	+5.4	−0.4	+2.6	−0.6	+0.9	−0.3	+2.2	+0.2	+1.2	.35	2.36	+.01	+2.30
.8	1.2	.8	1.6	1.0	1.9	.9	1.4	1.0	1.5	.93	1.52		
2.6	+7.4	−2.0	+7.8	−3.4	+7.4	−2.8	+7.2	−1.6	+5.2	2.00	6.14	−1.28	+6.14
1.4	3.2	1.2	3.2	1.8	3.0	2.0	2.2	.8	2.4	1.28	2.46		
1.6	+4.6	+2.0	+4.0	+1.8	+4.4	+2.2	+4.2	+1.6	+3.5	1.58	3.65	+1.58	+3.65
.8	.8	.4	1.0	.2	.8	.6	.8	.8	.4	.70	.96		
3.2	+9.8	+3.2	+9.2	+2.8	+8.6	+2.8	+8.2	+2.6	+6.0	2.78	6.60	+2.78	+6.60
2.4	2.8	1.6	3.0	1.6	3.0	1.6	2.2	1.4	3.2	1.58	2.18		
1.6	+5.0	−1.0	+4.6	−1.2	+4.8	−1.6	+4.0	−2.2	+2.6	1.14	3.42	−1.14	+3.42
1.6	3.2	1.8	2.4	1.2	1.6	1.6	1.8	1.4	1.8	1.34	2.48		
2.2	+11.6	+1.6	+9.0	+1.8	+10.6	+1.8	+7.6	+1.8	+5.4	1.64	7.12	+1.64	+7.12
1.0	1.4	.8	2.0	1.4	4.2	1.4	2.6	1.0	2.2	1.04	2.02		
0.2	+5.8	−0.6	+5.8	−0.6	+5.4	−1.2	+4.4	−0.8	+3.2	.48	3.94	−.48	+3.94
.6	.8	.6	1.6	1.0	1.4	.8	.6	1.2	1.5	.68	1.31		
1.2	+6.8	+1.4	+6.4	+1.2	+5.4	+0.6	+5.0	+1.0	+4.8	1.16	4.74	+1.16	+4.74
.8	1.4	.6	1.4	1.2	2.2	1.4	1.6	1.0	1.2	.92	1.52		
1.2	+9.4	−1.4	+7.0	−1.6	+8.6	+2.2	+8.2	−2.4	+5.4	1.12	6.88	−.96	+6.88
1.6	2.0	1.8	3.6	2.0	3.8	2.2	4.0	2.4	3.0	1.72	2.60		
0.6	+7.4	−1.0	+7.0	−1.4	+5.2	−1.6	+5.8	−1.4	+4.0	.92	4.46	−.88	+4.46
1.8	2.4	1.8	2.0	2.2	2.0	1.6	2.4	1.8	1.6	1.56	1.68		
1.8	+6.2	0.0	+2.8	−0.6	+2.6	−0.6	+4.4	+0.2	+2.0	.98	3.36	+.74	+3.36
1.0	1.6	.4	2.6	.6	1.4	1.4	1.6	.6	1.1	.94	1.55		
2.2	+11.6	+2.2	+8.4	+3.4	+6.2	+3.0	+6.4	+2.6	+5.4	2.60	7.16	+2.60	+7.16
.2	2.2	.6	2.2	.6	3.0	1.0	2.2	1.0	1.4	.68	1.94		
0.0	+7.0	−0.4	+5.4	−0.4	+4.2	−0.6	+3.4	−0.6	+2.0	.42	3.88	−.42	+3.76
1.2	2.4	.8	2.8	1.2	2.1	1.0	.8	1.0	1.0	.98	1.67		
1.6	+16.8	+1.6	+16.6	+2.0	+14.0	+1.2	+11.6	+2.0	+10.4	1.80	10.90	+1.80	+10.90
2.8	1.6	2.4	2.8	2.4	4.4	3.2	4.4	3.6	4.0	2.18	3.46		
0.8	+7.0	−0.8	+6.8	−1.0	+5.4	−1.2	+2.2	−1.2	+1.0	.86	4.20	−.86	+4.20
.8	3.2	1.6	3.8	1.4	3.0	1.2	2.0	2.0	1.8	1.46	2.86		
2.6	+5.2	+3.0	+5.2	+2.2	+5.0	+1.2	+4.6	+0.6	+1.8	1.70	3.68	+1.70	+3.56
1.0	1.8	1.0	2.6	1.4	2.2	1.2	1.6	1.0	2.6	.94	2.14		
1.2	+0.8	−0.8	+2.8	−1.0	+1.6	−1.0	+1.8	−1.4	+0.8	.94	1.86	−.94	+1.66
.8	2.4	.8	2.2	1.0	2.0	1.0	2.0	1.0	1.7	.94	2.19		
1.2	+5.0	−1.0	+2.2	−1.8	+3.2	−1.0	+5.0	−1.4	+2.2	.88	3.48	−.86	+3.48
1.2	.8	1.0	1.6	.6	2.0	1.4	2.0	1.0	1.2	1.08	1.56		
8.0	+14.0	−8.8	+12.2	−9.6	+10.6	−9.4	+10.4	−7.0	+12.6	7.46	9.86	−7.46	+9.86
2.0	3.6	1.8	2.6	1.6	2.6	2.6	2.0	2.4	4.8	2.24	3.32		
1.6	+4.2	−1.8	+5.6	−1.8	+2.8	−1.2	+1.4	−2.4	−0.8	1.68	2.74	−1.68	+2.55
1.6	2.4	1.4	1.8	1.4	2.0	.8	2.4	2.4	2.8	1.68	2.36		

	.5	2.3	1.4	2.1	1.3	2.0	2.2	3.7	1.9	[illegible]	[illegible]	[illegible]
144	+0.5	−0.6	+0.4	+1.2	+1.0	+1.1	+0.6	+2.0	+0.5	+3.2	+0.3	+3.8
	.7	1.4	.8	1.8	.4	1.3	.6	2.2	.5	1.8	.5	1.6
146	−0.8	−1.9	−0.3	+1.[illegible]	−0.2	+3.1	−1.0	+2.9	−1.3	+2.6	−1.1	+3.7
	1.0	1.3	.7	2.6	1.0	2.5	1.2	2.5	1.3	2.2	1.1	1.5
148	−1.3	−0.5	−1.3	+3.3	−1.1	+2.9	−1.3	+3.1	−1.6	+4.3	−1.1	+5.1
	.7	1.3	.9	3.3	.9	2.3	.7	1.5	2.0	2.1	.9	3.3
150	+1.4	+9.2	+2.4	0.0	+1.8	−0.8	+1.6	+5.0	+2.8	+3.4	+1.8	+2.4
	.8	7.0	1.6	4.0	1.4	3.0	1.2	5.2	2.6	3.6	1.4	2.4
152	−0.7	−0.1	−1.3	+3.0	−0.7	+2.6	−0.9	+4.0	−1.4	+7.3	−1.0	+6.5
	1.9	2.5	1.7	3.2	1.3	2.4	1.1	3.8	1.2	4.1	1.4	4.0
154	−0.1	+1.3	0.0	−0.3	−0.4	+0.2	−0.3	+3.2	−0.1	+4.7	−0.1	+5.5
	.5	1.3	.4	4.3	1.0	1.4	.7	1.6	.9	2.5	.9	2.7
156	+1.1	+1.1	+1.0	+2.8	+0.8	+3.2	+0.8	+4.3	+0.5	+5.7	+0.7	+6.3
	.7	.7	.4	1.6	.3	2.6	.6	1.9	.5	1.3	.3	1.9
158	−0.1	+2.6	−0.2	+4.1	−0.2	+6.8	−0.6	+9.1	−0.5	+11.3	−0.3	+9.6
	.5	1.8	.8	2.3	1.0	2.0	.4	1.9	.7	1.7	.5	3.0
160	−0.6	+1.1	−0.4	+4.8	−0.6	0.0	−1.2	+5.0	−1.5	+6.4	−1.0	+7.8
	1.0	1.7	1.0	1.4	1.0	1.4	1.0	2.8	1.3	1.8	.6	1.2
162	−1.0	+2.3	−1.0	+0.4	−0.8	+0.8	−1.3	+1.7	−1.2	+5.7	−1.1	+6.
	1.0	.9	.6	3.0	.6	2.4	.7	1.1	1.4	1.5	1.3	1.
164	−0.5	+1.5	+0.5	+2.4	−1.0	+2.4	−0.7	+4.6	−1.0	+5.1	−0.9	+5.
	.9	.9	.9	1.8	.8	1.6	.7	.8	1.0	.7	.9	.
166	−1.5	+1.5	−1.8	+0.7	−1.6	+2.2	−1.4	+1.6	−2.3	+2.5	−2.0	+3.
	1.3	2.1	1.0	2.1	.8	2.0	1.0	3.4	1.3	3.1	1.4	2.
168	−0.8	+5.4	−1.6	+8.7	−0.9	+5.0	−1.4	+6.3	−1.3	+6.4	−1.2	+5.
	1.0	2.4	1.0	2.5	1.1	4.4	1.2	5.1	1.1	3.8	1.6	2.
170	−2.2	+4.8	−3.0	+8.0	−2.7	+6.7	−2.7	+13.4	−1.9	+16.8	−1.3	+13.
	3.2	1.6	3.4	3.2	2.4	5.0	2.9	3.8	2.7	7.4	3.3	3.
172	−1.0	1.0	−1.2	−0.9	−1.8	+1.0	−1.5	+0.3	−0.5	+2.3	−1.7	+2.
	1.6	2.4	1.4	3.9	.8	2.6	1.2	2.1	1.2	1.7	1.4	1.
174	−1.0	+3.6	−0.5	+7.9	0.0	+7.3	+0.4	+4.2	−0.4	+7.7	0.0	+7.
	2.6	2.4	2.5	3.3	2.6	2.1	2.2	4.6	2.1	3.7	2.2	1.
176	+1.1	0.0	+1.7	+3.3	+0.9	+2.7	+1.6	+4.5	+1.7	+6.6	+1.3	+7.
	1.3	.8	.9	2.1	.5	3.1	1.1	2.7	.7	3.0	.9	2.
178	−2.6	−3.9	−3.1	−1.6	−3.3	−1.1	−4.1	0.0	−4.0	+3.9	−4.5	+4.
	2.3	2.9	2.7	5.1	2.7	3.7	2.7	5.4	3.2	4.7	2.7	3
180	+7.7	−2.6	8.0	+4.8	+6.8	+9.1	+7.2	+11.9	+6.2	+13.6	+5.9	+12
	2.4	2.1	2.4	3.6	2.2	3.9	2.0	4.3	2.2	4.1	2.1	4
182	−6.2	−6.3	−5.8	−2.1	−5.2	+0.5	−5.3	+1.4	−6.0	+5.4	−5.4	+5
	3.0	6.2	2.6	7.1	2.4	7.6	2.5	8.8	3.0	8.5	2.6	6
184	+5.3	+0.7	+7.9	+4.4	+5.4	+5.7	+4.5	+7.7	+5.4	+10.5	+6.0	+11
	4.5	2.4	5.5	2.9	1.7	4.1	5.7	4.5	6.4	3.8	5.0	4
186	+1.8	+2.0	+1.9	+4.2	+1.8	+0.7	+2.4	+3.9	+1.9	+5.8	+2.3	+5
	1.0	1.4	.5	3.2	.6	2.5	.6	1.3	.5	1.8	.9	2
188	−0.7	+6.2	−1.0	+3.8	−0.5	+7.8	−0.1	+6.6	−0.8	+10.5	−1.3	+11
	2.3	4.4	1.6	6.8	1.9	5.2	2.5	5.6	1.6	4.9	1.1	5

0.3	+3.8	+0.1	+4.3	−0.2	+4.1	−0.2	+3.8	+0.2	+3.3	.40	2.74	+.32	+2.62
.5	1.6	.9	.9	.8	1.9	.8	1.4	.8	.8	.68	1.51		
-1.1	+3.7	−0.5	+4.6	−1.1	+4.4	−1.3	+2.9	−1.3	+2.5	.89	2.98	−.89	+2.60
1.1	1.5	1.9	1.2	1.1	1.6	1.1	.7	.9	2.0	1.13	1.81		
-1.1	+5.1	+1.8	+3.0	−0.5	+0.4	−0.6	0.0	−0.5	−0.1	1.11	2.27	−.75	+2.15
.9	3.3	.8	3.4	.9	1.4	1.0	1.8	.9	1.0	.97	2.14		
1.8	+2.4	+4.8	+5.4	+2.4	+3.8	+1.4	+4.0	+1.0	+5.7	2.14	3.97	+2.14	+3.81
1.4	2.4	1.4	3.6	1.2	3.0	1.2	1.2	.4	2.6	1.32	3.56		
-1.0	+6.5	−1.0	+3.4	+0.4	+2.2	+0.9	+1.6	−0.7	+0.9	.90	3.16	−.64	+3.14
1.4	4.0	1.0	4.0	1.4	3.4	1.1	2.2	.7	2.0	1.28	3.17		
-0.1	+5.5	+0.7	+5.7	0.0	+4.5	+0.1	+4.3	−0.5	+3.7	.23	3.34	−.09	+3.28
.9	2.7	1.1	.9	1.2	1.5	1.3	1.9	1.7	1.4	.97	1.95		
-0.7	+6.3	+0.7	+5.5	+0.5	+4.5	+1.1	+3.2	+1.1	+3.1	.83	3.97	+.83	+3.97
.3	1.9	.3	2.1	.7	2.1	1.0	1.6	.5	1.2	.43	1.70		
-0.3	+9.6	−0.6	+8.4	−0.1	+7.0	−0.1	+6.1	−0.3	+3.7	.30	6.92	−.30	+6.92
.5	3.0	.4	2.2	.7	2.9	.7	2.7	.5	3.0	.62	2.35		
-1.0	+7.8	−1.3	+4.4	−0.9	+3.7	−1.5	+5.0	−0.8	+1.4	.98	3.95	−.98	+3.96
.6	1.2	.9	4.6	1.1	4.3	1.5	4.6	1.0	2.1	1.04	2.59		
-1.1	+6.7	−1.1	+5.5	−1.4	+2.2	−1.1	+2.0	−1.4	+0.9	1.14	2.82	−1.14	+2.82
1.3	1.3	1.3	2.9	1.8	3.8	.7	4.6	.8	2.4	1.02	2.37		
-0.9	+5.5	−0.9	+6.3	−1.2	+3.5	−1.2	+2.5	−1.3	+1.1	.92	3.49	−.82	+3.49
.9	.7	.7	1.1	.4	1.1	.6	•.9	.9	1.0	.78	1.06		
-2.0	+3.6	−2.5	+4.0	−2.4	+3.6	−2.3	+2.4	−2.0	+0.4	1.98	2.25	−1.98	+2.25
1.4	2.0	.9	3.2	1.0	3.0	1.5	3.2	1.6	1.3	1.18	2.60		
-1.2	+5.1	−1.9	+3.7	−2.3	+2.6	−2.6	+2.2	−2.6	+0.5	1.66	4.59	−1.66	+4.59
1.6	2.7	1.3	2.1	1.5	2.2	1.6	2.2	1.4	1.6	1.28	2.90		
-1.3	+13.6	−3.4	+16.2	−1.8	+15.2	−1.9	+15.1	−2.5	+15.5	2.34	12.5	−2.34	+12.5
3.3	3.1	3.0	5.0	2.2	5.4	2.5	5.1	2.6	8.8	2.84	4.84		
-1.7	+2.9	−0.6	+3.3	−1.3	+3.0	−0.4	+3.7	−0.2	+2.7	1.02	2.11	−1.02	+1.93
1.4	1.9	1.7	2.0	1.6	2.2	2.1	1.4	1.4	2.0	1.44	2.22		
0.0	+7.5	−0.2	+6.1	−0.1	+6.6	+1.5	+4.9	+1.5	+1.9	.56	5.77	+.12	+5.77
2.2	1.8	2.6	2.5	2.3	3.4	2.0	2.9	2.4	3.0	2.35	2.97		
-1.3	+7.5	+1.2	+7.4	+1.3	+6.8	+1.2	+6.2	+0.9	+4.3	1.29	4.93	+1.29	+4.93
.9	2.2	.6	1.7	1.1	.6	1.2	2.4	.7	1.6	.90	2.12		
-4.5	+4.1	−4.4	+5.0	−4.0	+4.3	−4.9	+5.9	−5.0	+5.5	3.99	3.53	−3.99	+2.21
2.7	3.3	3.3	3.0	3.2	3.0	3.3	3.1	3.0	3.0	2.91	3.72		
+5.9	+12.7	+6.9	+13.6	+6.3	+12.7	+6.9	+12.9	+3.4	+14.1	6.53	10.8	+6.53	+10.3
2.1	4.9	2.3	3.6	2.3	3.1	2.1	3.3	4.8	3.6	2.48	3.65		
-5.4	+5.0	−5.2	+1.1	−5.5	−2.7	−5.0	+4.1	−5.9	−0.7	5.55	2.93	−5.55	+.57
2.6	6.4	2.6	4.7	2.0	3.1	2.6	2.9	1.9	4.0	2.52	5.93		
+6.0	+11.5	+6.1	+13.3	+7.0	+11.6	+6.4	+10.4	+6.2	+9.8	6.02	8.56	+6.02	+8.56
5.0	4.4	5.5	4.0	5.2	2.9	5.2	3.3	5.4	3.2	5.31	3.58		
+2.3	+5.0	+1.9	+4.5	+1.8	+3.8	+2.0	+2.9	+1.5	+2.7	1.93	3.55	+1.93	+3.55
.9	2.2	.3	1.5	.4	1.6	1.0	.9	.5	.6	.63	1.70		
-1.3	+11.6	−1.5	+9.4	−2.2	+7.6	−2.2	+5.3	−1.9	+4.5	1.22	7.33	−1.22	+7.33
1.1	5.4	1.9	6.4	1.6	3.4	1.8	5.3	1.5	5.0	1.78	5.24		

2.62	133	+3.4	+3.2	+4.0	+7.8	+4.2	+10.6	+3.8	+10.0	+3.8	+8.0
		1.4	2.8	1.6	2.4	1.8	3.6	1.0	3.2	1.0	4.4
2.60	135	−3.4	0.0	+3.6	+4.0	+4.0	+7.2	+4.0	+8.8	+4.4	+9.0
		1.0	1.6	1.2	1.4	1.2	1.4	1.2	2.0	.8	3.8
2.15	137	+4.6	+3.2	+6.2	+3.6	+5.6	+6.4	+6.0	+9.6	+6.2	+7.4
		1.8	2.0	1.8	2.2	1.4	2.0	1.6	1.2	1.0	1.8
-3.81	139	+1.0	+2.4	+1.4	+3.0	+1.2	+5.6	+1.0	+6.4	+0.4	+7.8
		1.0	.4	1.0	2.0	.8	2.0	1.0	1.6	.4	1.8
-3.14	141	+2.2	+2.0	+2.4	+6.2	+1.8	+6.6	+1.8	+7.4	+2.2	+8.2
		.6	1.2	.8	1.6	1.0	2.4	1.0	1.8	1.4	1.4
-3.28	143	+1.8	+2.6	+2.4	+4.0	+2.4	+6.6	+2.4	+8.2	+2.6	+8.4
		1.0	1.4	1.2	1.0	1.2	1.6	.8	1.8	1.0	1.6
-3.97	145	+1.0	+2.0	+1.4	+4.0	+1.2	+5.0	+0.8	+7.0	+1.0	+8.6
		1.0	1.2	.6	1.0	.8	1.2	.8	1.0	1.0	1.8
6.92	147	+1.2	+1.4	−0.4	+2.2	+1.2	+4.0	+0.4	+7.2	+0.6	+5.6
		1.6	1.8	1.6	2.8	2.0	3.0	1.6	3.6	2.2	2.0
-3.96	149	+2.4	−0.4	+1.8	+1.2	+1.4	+4.0	+2.0	+5.2	+1.2	+7.4
		1.2	3.2	1.4	2.6	1.4	3.0	1.2	2.0	1.2	1.8
-2.82	151	−2.4	−4.4	−2.8	−3.8	−4.4	+1.8	−2.8	+1.2	−2.6	+0.8
		3.6	5.6	2.8	8.4	3.6	6.0	4.4	9.2	2.6	8.0
-3.49	153	+0.6	+1.6	+0.2	+3.6	+0.4	+5.8	0.0	+4.8	−0.8	+5.4
		1.0	1.3	1.0	1.6	1.2	1.2	.8	2.8	1.2	2.2
-2.25	155	−0.6	−1.4	−0.4	+1.8	−1.2	+2.6	−1.0	+3.6	−1.4	+3.6
		1.8	2.2	1.2	1.2	.8	1.6	1.0	1.6	1.4	2.8
-4.59	157	+1.8	−0.4	+1.4	+4.0	+2.0	+5.0	+1.8	+6.6	+2.0	+4.2
		.6	2.0	.6	2.6	.8	3.2	.6	3.8	.8	4.2
2.5	159	+1.2	+1.2	+1.2	+2.4	+0.6	+3.6	+0.1	+5.2	+1.4	+2.6
		1.2	2.4	.8	1.0	1.0	2.2	1.0	1.6	1.8	3.4
-1.93	161	+0.8	+2.4	+0.8	+5.4	+1.0	+7.8	+1.0	+5.4	+1.0	+5.4
		1.6	.8	1.0	1.2	1.7	4.2	2.0	2.6	1.6	3.8
-5.77	163	+3.8	+3.0	+4.0	+5.2	+4.0	+7.0	+4.2	+10.0	+4.4	+10.4
		.6	1.2	1.6	1.8	1.2	1.2	1.4	1.6	1.2	2.8
-4.93	165	−1.2	−1.4	−1.6	−0.2	−1.7	+0.8	−1.8	+3.2	−1.2	+3.4
		2.0	2.0	1.2	2.4	1.5	2.6	.8	2.4	1.4	3.4
-2.21	167	+0.8	−3.4	+0.6	−0.6	0.0	+0.8	−0.4	+2.0	−0.8	+3.8
		1.2	1.8	1.0	2.0	.8	3.0	1.6	3.6	1.2	2.6
10.3	169	+2.0	+1.0	+1.8	+2.8	+2.0	+1.6	+2.4	+4.4	+1.6	+6.8
		.8	1.8	1.0	3.4	0.0	2.2	.8	1.6	.8	1.2
+.57	171	+1.0	+0.8	0.0	+3.6	+1.2	+5.0	0.0	+5.8	+0.4	+4.4
		1.8	2.0	1.2	1.4	1.2	1.2	.8	2.2	1.2	3.2
-8.56	173	+3.6	−2.4	+3.8	−0.8	+4.6	+0.8	+4.6	+1.6	+4.2	+4.2
		1.6	2.0	1.4	1.4	2.2	3.0	1.4	2.0	1.0	1.4
-3.55	175	+0.8	+3.8	+0.6	+4.6	+0.4	+6.0	+0.4	+7.0	−0.4	+6.2
		.8	3.0	.6	1.2	1.6	3.0	1.2	3.4	.8	2.6
-7.33	177	+1.4	+1.4	+2.0	+5.0	+2.0	+1.8	+1.0	+5.4	+1.0	+6.4
		1.0	1.0	.8	1.2	1.2	2.8	1.0	1.4	1.4	1.2
	179	+0.6	+0.8	+0.8	+5.2	+0.6	+4.2	0.0	+5.4	−0.4	+8.0
		1.8	1.6	2.0	2.4	1.8	2.8	1.2	3.0	1.6	2.0
	181	+1.8	+1.2	+1.6	+1.4	+1.6	+2.2	+1.6	+3.8	+0.8	+5.4
		.2	.8	.4	1.6	.4	1.6	.4	.6	.8	1.4
	183	+1.4	+0.4	+1.6	+2.4	+1.6	+4.0	+1.0	+6.0	+1.0	+7.2
		1.0	1.2	1.2	3.0	1.2	3.8	1.4	3.2	1.4	3.6
	185	+2.2	−3.2	+1.8	−4.6	+0.4	−3.0	+0.4	+0.2	+1.2	+1.2
		1.0	2.6	.6	2.8	1.6	2.4	1.6	2.2	.8	2.0
	187	+1.2	+0.2	+0.4	+4.2	+0.2	+5.6	+0.8	+5.0	+0.4	+6.0
		2.4	1.8	2.4	2.0	2.6	3.4	2.4	3.4	2.4	3.2
	189	+1.8	+3.2	+1.6	+5.0	+1.6	+5.8	+2.0	+7.0	+1.4	+10.0
		.2	1.2	.4	2.0	.8	1.6	0.0	2.6	.6	1.6
	191	+1.2	+1.4	+0.8	+2.8	+1.6	+1.0	+1.0	+3.6	+1.8	+6.4
		.8	1.0	1.2	2.6	1.6	2.0	1.0	2.0	1.0	1.6
	193	+0.1	+0.4	+0.8	+2.2	+0.6	+6.2	+0.8	+7.0	+0.4	+8.0
		1.0	1.6	1.2	2.2	.6	1.6	1.2	2.6	.8	2.8
	195	−0.2	−0.2	+0.2	+3.6	0.0	+5.4	−0.4	+7.6	−0.2	+9.6
		1.0	1.4	.6	1.4	.8	1.6	.4	2.0	1.0	4.4
	197	+1.2	+0.4	+1.6	+3.2	+2.0	+3.4	+1.0	+5.8	+0.6	+6.6
		.8	1.2	1.2	1.8	.4	1.6	1.8	1.8	.6	1.4
	199	+1.2	+2.2	+1.0	+4.0	+1.2	+5.2	+0.8	+6.6	+0.6	+8.2
		.8	1.0	1.0	2.6	.8	1.4	.8	1.8	1.0	1.8
	201	+1.6	+1.0	+1.4	+0.8	+1.2	+2.0	+1.4	+3.6	+1.4	+3.8
		.8	1.8	.6	1.4	.8	1.4	.6	.4	.6	1.8
	203	+0.8	+0.2	+0.8	+4.0	+0.2	+4.2	−0.6	+5.2	−0.6	+6.6
		.8	1.4	.8	1.4	1.4	2.4	1.0	1.6	1.0	1.0
	205	+0.6	−0.6	0.0	+2.6	+1.2	+2.6	+0.8	+4.6	+0.2	+6.4
		1.0	1.0	.8	1.6	1.2	2.8	1.6	2.6	1.0	2.4
	207	+1.8	+1.0	+1.6	+3.8	+1.8	+5.6	+1.6	+6.4	+1.6	+7.2
		.2	1.0	1.2	1.2	.2	1.0	1.2	1.6	.4	1.2
	209	+0.8	+3.0	+0.8	+5.2	+0.8	+6.8	+0.8	+7.2	+0.6	+7.4
		.8	1.4	1.2	1.8	.8	1.4	1.2	1.6	.6	1.4

+8.0	+7.0	+9.8	+4.6	+8.4	+4.4	+7.0	+4.8	+7.2	+9.0	+8.4	4.90	8.04	+4.90	+8.04
4.4	3.8	3.6	1.4	1.8	1.2	1.4	1.6	1.8	1.0	1.6	1.58	2.66		
+9.0	+3.6	+9.6	+3.4	+9.2	+4.0	+7.6	+4.2	+8.0	+4.0	+6.4	3.86	6.98	+3.86	+6.98
3.8	1.2	3.4	.6	3.4	1.2	2.0	.6	2.6	1.2	1.2	1.02	1.94		
+7.4	+6.4	+10.6	+7.0	+10.4	+6.6	+10.2	+6.0	+10.0	+5.6	+8.6	6.02	8.00	+6.02	+8.00
1.8	.8	2.0	1.0	1.0	1.4	1.0	.8	1.0	1.2	1.4	1.28	1.56		
+7.8	+0.4	+9.6	+1.2	+9.6	+1.8	+9.4	+1.2	+8.8	+0.8	+7.2	1.04	6.98	+1.04	+6.98
1.8	1.2	1.4	.8	1.8	.6	1.8	1.2	2.6	.8	3.2	.88	1.86		
+8.2	+2.0	+8.8	+1.4	+8.2	+2.2	+7.2	+2.4	+5.4	+2.2	+5.2	2.06	6.52	+2.06	+6.52
1.4	.8	1.4	1.0	2.4	.6	2.0	.4	1.2	.2	.8	.78	1.62		
+8.4	+2.0	+10.8	+3.0	+10.4	+2.4	+10.2	+2.8	+10.0	+2.8	+8.6	2.46	7.98	+2.46	+7.98
1.6	1.2	1.8	1.4	1.4	1.2	2.2	1.2	2.2	1.6	2.6	1.18	1.76		
+8.6	+1.0	+7.4	+1.4	+7.4	+1.0	+5.0	+1.8	+6.2	+1.4	+4.0	1.20	5.66	+1.25	+5.66
1.8	1.4	2.8	1.0	1.2	1.0	1.4	1.0	2.4	.6	.8	.92	1.48		
+5.6	0.0	+4.4	+1.6	+5.6	+1.2	+6.0	+0.6	+4.8	+2.2	+2.4	.94	4.36	+.86	+4.36
2.0	1.2	2.2	2.4	1.4	1.6	2.8	1.0	1.4	2.2	2.8	1.74	2.38		
+7.4	+2.0	+8.2	+2.6	+8.8	+2.2	+9.2	+2.6	+5.8	+1.6	+2.6	1.98	5.28	+1.98	+5.20
1.8	1.2	1.6	1.0	2.6	2.2	2.4	1.4	3.2	1.6	2.6	1.38	2.50		
+0.8	—2.2	+2.0	—3.2	+2.4	—3.0	+0.6	—2.8	+1.2	—2.0	—2.2	2.82	2.04	—2.82	—.04
8.0	1.8	5.8	2.4	4.2	1.8	4.2	2.0	4.2	2.8	3.0	2.78	5.86		
+5.4	—0.8	+4.6	—0.4	+3.0	—1.4	+4.4	+0.4	+2.4	+0.4	+1.2	.46	3.68	—.06	+3.68
2.2	.8	2.4	1.2	2.0	1.4	.8	1.2	1.8	.4	1.2	1.02	1.77		
+3.6	—1.2	+5.6	—1.4	+6.0	—1.8	+5.4	—0.6	+5.8	—1.4	+3.4	1.10	3.92	—1.10	+3.64
2.8	1.2	3.0	1.0	2.6	1.0	2.2	1.4	2.4	1.0	1.0	1.18	2.06		
+4.2	+2.6	+4.2	+3.0	+4.8	+3.2	+4.4	+2.8	+4.2	+3.4	+4.4	2.40	4.22	+2.40	+4.14
4.2	1.0	3.6	1.8	2.6	1.6	1.6	.8	1.6	1.4	1.2	1.00	2.64		
+2.6	+0.4	+3.8	+1.4	+3.8	+1.4	+3.2	+0.8	+3.2	+0.4	+2.0	.89	3.10	+.89	+3.10
3.4	1.6	1.6	1.0	1.2	1.0	1.2	.8	1.4	.8	.8	1.10	1.68		
+5.4	+1.0	+4.0	+1.4	+3.2	+2.6	+3.6	+2.8	+2.8	+2.6	+3.0	1.50	4.30	+1.50	+4.30
3.8	1.4	4.4	1.4	2.5	1.5	1.9	1.6	1.7	2.1	2.1	1.59	2.52		
+10.4	+4.2	+9.8	+3.6	+9.6	+3.4	+8.8	+4.4	+8.6	+3.2	+7.0	3.92	7.94	+3.92	+7.94
2.8	1.4	4.8	1.2	3.8	1.8	2.4	1.2	3.2	1.2	2.0	1.28	2.53		
+3.4	+0.8	+3.0	—3.8	+2.4	—2.2	—0.4	—3.6	—1.4	—3.0	—1.0	2.09	1.72	—2.01	+.84
3.4	2.0	4.0	2.6	4.1	2.2	3.4	1.6	3.2	1.7	3.5	1.70	3.10		
+3.8	+0.6	+4.6	+0.8	+4.0	+1.2	+3.4	—0.2	+3.4	+0.2	+2.4	.56	2.84	+.28	+2.84
2.6	2.2	2.0	1.6	1.8	1.6	1.8	2.2	1.6	.6	1.6	1.40	2.18		
+6.8	+2.2	+6.8	+1.8	+7.2	+2.6	+7.4	+2.2	+6.6	+2.4	+4.6	2.10	4.82	+2.10	+4.82
1.2	1.0	1.4	.2	1.8	.6	1.8	.2	2.0	.8	1.4	.62	1.86		
+4.4	+0.8	+4.6	+0.8	+3.8	+0.4	+3.8	+1.2	+3.8	+1.2	+3.2	.70	3.88	+.70	+3.88
3.2	1.6	2.4	1.6	1.6	.4	1.8	1.2	2.8	1.2	2.0	1.22	2.06		
+4.2	+5.2	+4.6	+4.2	+4.6	+4.8	+5.2	+4.6	+5.6	+5.0	+6.0	4.46	3.58	+4.46	+2.94
1.4	.8	2.4	1.0	1.2	1.6	1.2	1.4	1.4	1.8	1.6	1.42	1.76		
+6.2	+2.2	+6.6	+1.2	+7.8	+1.4	+9.0	+0.4	+6.2	+0.2	+7.2	.80	6.44	+.72	+6.44
2.6	1.4	5.2	1.2	3.6	1.8	3.4	2.0	3.2	2.2	2.0	1.36	3.06		
+6.4	+1.2	+7.0	+1.2	+5.6	+1.6	+5.8	+2.0	+4.4	+2.2	+5.2	1.56	4.80	+1.56	+4.80
1.2	.8	2.4	1.2	2.2	.8	1.8	.4	1.0	.2	.8	.88	1.58		
+8.0	—1.0	+7.0	—0.6	+6.2	—0.6	+5.4	—0.4	+4.8	—1.0	+4.2	.60	5.12	—.20	+5.12
2.0	1.8	2.0	1.8	2.4	1.4	1.8	1.2	1.4	1.8	1.4	1.64	2.08		
+5.4	+0.6	+6.6	+0.4	+5.2	+1.0	+3.8	+1.0	+3.0	+1.6	+2.8	1.20	3.54	+1.20	+3.54
1.4	1.0	.8	1.2	1.8	1.0	2.6	1.4	1.2	.8	1.2	.76	1.36		
+7.2	+1.6	+8.2	+2.2	+6.8	+2.2	+4.8	+1.6	+3.8	+2.0	+2.8	1.62	4.64	+1.62	+4.64
3.6	.4	2.8	1.0	3.8	.6	2.8	1.2	2.0	1.2	2.4	.92	2.86		
+1.2	+0.8	+4.6	+2.2	+6.0	+2.2	+6.6	+2.0	+6.4	+1.6	+6.2	1.48	4.20	+1.48	+2.04
2.0	1.6	1.6	1.8	2.2	1.4	1.8	1.2	2.2	.8	2.6	1.24	2.24		
+6.0	—0.8	+7.2	—.10	+4.8	—1.0	+5.2	—0.4	+6.2	—0.4	+3.8	.66	4.82	—.06	+4.82
3.2	2.8	3.8	2.6	1.4	3.0	1.2	2.8	2.4	2.8	1.4	2.62	2.40		
+10.0	+1.6	+9.8	+1.0	+10.0	+1.0	+7.4	+1.4	+6.4	+1.0	+5.0	1.44	6.96	+1.44	+6.96
1.6	.8	3.2	1.0	5.0	1.0	4.2	1.0	4.2	1.0	3.4	.68	2.90		
+6.4	+1.2	+5.4	+1.4	+4.6	+1.4	+4.8	+1.6	+3.8	+1.2	+3.2	1.32	+3.70	+1.32	+3.70
1.6	1.2	1.6	1.0	1.6	1.0	2.0	.8	1.6	.8	.8	1.04	1.52		
+8.0	—0.2	+5.8	0.0	+5.4	+0.6	+5.6	+0.8	+4.2	+0.4	+5.6	.47	5.04	+.43	+5.04
2.8	1.0	4.4	.4	4.4	.6	3.6	1.2	3.6	1.2	3.2	.92	3.00		
+9.6	—0.6	+9.0	—0.6	+10.0	—0.6	+8.0	—0.2	+7.6	—0.2	+4.6	.32	6.56	—.28	+6.52
4.4	1.0	4.4	.6	3.8	1.0	4.0	2.2	4.2	.2	4.6	.88	3.18		
+6.6	+1.6	+7.4	+0.6	+4.2	+1.0	+5.6	+1.4	+3.6	+0.8	+4.2	1.18	4.44	+1.18	+4.44
1.4	1.6	1.2	1.0	1.6	1.0	2.0	1.4	1.8	1.2	1.8	1.10	1.62		
+8.2	+0.8	+9.0	+0.2	+7.6	+1.0	+8.0	+0.8	+6.6	+1.4	+4.0	.90	6.14	+.90	+6.14
1.8	1.6	2.8	.6	3.0	1.0	2.4	1.2	2.4	1.0	1.6	.98	1.98		
+3.8	+1.4	+2.8	+1.4	+3.4	+1.8	+3.2	+1.4	+3.8	+1.8	+3.0	1.48	2.74	+1.48	+2.74
1.8	1.4	1.8	.8	1.2	.2	1.2	1.0	1.2	1.0	.6	.78	1.28		
+6.6	—1.6	+6.4	—0.2	+7.0	0.0	+5.8	—0.6	+8.8	—0.6	+1.8	.60	5.00	—.24	+5.00
1.0	1.2	2.6	1.0	2.0	1.2	2.2	1.4	2.2	1.4	1.8	1.12	1.86		
+6.4	0.0	+8.0	—0.8	+9.4	+0.8	+6.0	+0.8	+4.2	+0.8	+4.6	.60	4.90	+.44	+4.78
2.4	1.2	3.0	1.6	1.6	1.6	2.4	.4	2.4	1.2	2.6	1.16	2.24		
+7.2	+2.0	+8.6	+2.2	+9.6	+2.0	+8.8	+1.6	+8.0	+2.4	+6.2	1.86	6.52	+1.86	+6.52
1.2	.4	2.4	.2	2.6	.4	1.6	.8	1.8	.4	1.0	.54	1.54		
+7.4	+0.6	+6.4	+1.2	+6.6	+0.8	+5.2	+0.4	+5.6	+0.6	+4.8	.74	5.82	+.74	+5.82
1.4	1.0	1.4	2.0	.8	.8	1.6	.8	1.4	.6	.8	.98	1.36		

211	+2.8	+2.4	+2.6	+6.6	+2.0	+8.2	+1.8	+11.2	+1.6	+11.8	+1.4
	1.2	1.6	1.0	2.4	1.2	2.4	1.0	2.0	1.2	2.2	1.0
213	+1.2	+2.0	+1.8	+1.8	+0.6	+3.2	+1.2	+3.6	+1.0	+4.8	+1.2
	.8	1.2	.6	1.2	1.0	1.4	.8	1.6	1.4	1.2	1.2

TABLE VIII.

	0-30		0-23		0-17		0-12		0-8		
Ave.	128	158	128	151	128	145	128	140	128	136	128
M.v. (v.d.)	1.49	2.38	1.53	3.01	1.47	3.26	1.61	3.43	1.53	3.28	1.54
% + C.E.	33	68	32	83	33	88	32	96	31	100	32
% — C.E.	66	32	61	16	66	9	65	3	64	0	67
% o C.E.	1	0	7	1	1	3	3	1	5	0	1
C.E. (v.d.)	1.36	2.59	1.53	3.28	1.47	4.18	1.55	5.34	1.59	6.81	1.47
G.C.E. (v.d.)	—.38	+1.00	—.47	+2.56	—.51	+3.98	—.53	+5.31	—.63	+6.81	—.53
	256	286	256	279	256	273	256	268	256	264	256
M.v. (v.d.)	1.23	1.63	1.27	2.10	1.25	2.30	1.25	2.32	1.28	2.50	1.35
% + C.E.	76	73	74	90	75	97	66	98	65	98	65
% — C.E.	23	24	23	10	22	3	31	1	34	2	30
% o C.E.	1	3	3	0	3	0	3	1	1	0	5
C.E. (v.d.)	1.56	1.69	1.57	3.13	1.65	4.23	1.54	5.48	1.52	6.36	1.59
G.C.E. (v.d.)	+.99	+1.04	+.91	+2.86	+.92	+4.12	+ .74	+5.46	+.71	+6.29	+.80

.8	+1.4	+11.6	+1.0	+10.6	+0.8	+10.8	+1.4	+9.6	+0.6	+8.2	1.60	9.10	+1.60	+9.10
.2	1.0	4.1	1.4	3.6	1.6	4.4	1.0	3.0	.6	3.4	1.12	2.91		
.8	+1.2	+5.8	+0.6	+5.8	+1.0	+6.0	+0.8	+5.8	+0.8	+4.6	1.02	4.34	+1.02	+4.34
.2	1.2	1.2	1.4	1.6	1.4	1.2	1.6	.8	1.6	1.4	1.18	1.28		

LE VIII. Summary from Table VII

	A. Men										Average			
	0-5		0-[illegible]		0-2		0-1		0-.5		Arithmetic		Algebraic	
											O	V	O	V
36	128	133	128	131	128	130	128	129	128	128.5	128	138	128	138
28	1.54	3.21	1.61	3.25	1.53	3.10	1.62	2.89	1.54	2.72	1.54	3.05		
00	32	99	31	100	34	96	35	93	34	89	33	91		
0	67	0	66	0	63	4	64	5	64	11	65	8		
0	1	1	3	0	3	0	1	2	2	0	2	1		
81	1.47	7.15	1.55	6.97	1.62	6.35	1.66	5.64	1.61	4.85	1.54	5.32		
81	—.53	+7.15	—.51	+6.97	—.57	+6.27	—.57	+5.55	—.64	+4.71			—.53	+5.03
	B. Women													
64	256	261	256	259	256	258	256	257	256	256.5	256	266	256	266
.50	1.35	2.59	1.28	2.45	1.27	2.29	1.38	2.09	1.39	1.92	1.29	2.21		
98	65	98	63	98	63	97	60	97	68	94	67	94		
2	30	2	32	2	36	3	37	3	30	5	30	5		
0	5	0	5	0	1	0	3	0	2	1	3	1		
.36	1.59	6.71	1.73	6.26	1.81	5.74	1.79	4.98	1.77	4.06	1.65	4.86		
.29	+.80	+6.56	+.70	+6.10	+.66	+5.59	+.64	+4.95	+.69	+3.85			+.77	+4.68

(C. E.) and the mean variations (m.v.) on both standard and variant are presented in the two columns headed O. V. (Arithmetic). The algebraic averages for the constant errors (C. E.) on both standard and variant are given in the two O. V. columns at the extreme right of the table.[14]

The consolidated footings for Table VII, section A and B, are given in Table VIII. The top notation is thus the same as in Table VII. A contains the final footings for men and B for women. The footings are set out as follows: Ave. m.v. is the average mean variation for the respective points in terms of vibrations; C. E. % +, the per cent. of individuals who made a constant error in the direction of a sharp; % —, the per cent. of those who flatted; % O, the per cent. of those who made no appreciable constant error in the ten trials; C. E., v.d., the average magnitude in vibrations of the constant errors, without regard to sign; and G. C. E. v.d. the tendency of the constant erors for the group, the algebraic mean. At the right, the grand averages for both groups are presented under the headings designated above.

Comparison of the abilities of men and women

The most striking general feature of these experiments is the the fact that women show the same ability as men, vibration for vibration, although the women sang an octave higher than the men.

The data on which this assertion is based may be traced most readily in the curves, Figs. 5-8, 10. In Fig. 5 C. it is seen that the curves for the average constant errors on the standard as well as on the variant practically coincide. On the standard they are almost straight lines, the variation for the men being from 1.36 v.d. to 1.66 v.d. with an average of 1.54 v.d., while in the case of the women the variation of this measure is from 1.52 v.d. to 1.81 v.d. with an average of 1.65 v.d. The curves for the variants do not come so near coinciding; they are of the same form, but the women have the advantage, their range of C. E. falling between 1.69 v.d. and 6.71 v.d. with an average of 4.86 v.d., while that for the men lies between 2.59 v.d. and 7.15 v.d. with an average of 5.32 v.d. As further confirmation of the fact that the average con-

[14] There would be little gained by placing E., the crude error, in our table as this measure is something of a cross between C. E. and m.v. and serves simply to indicate the distribution of the constant errors.

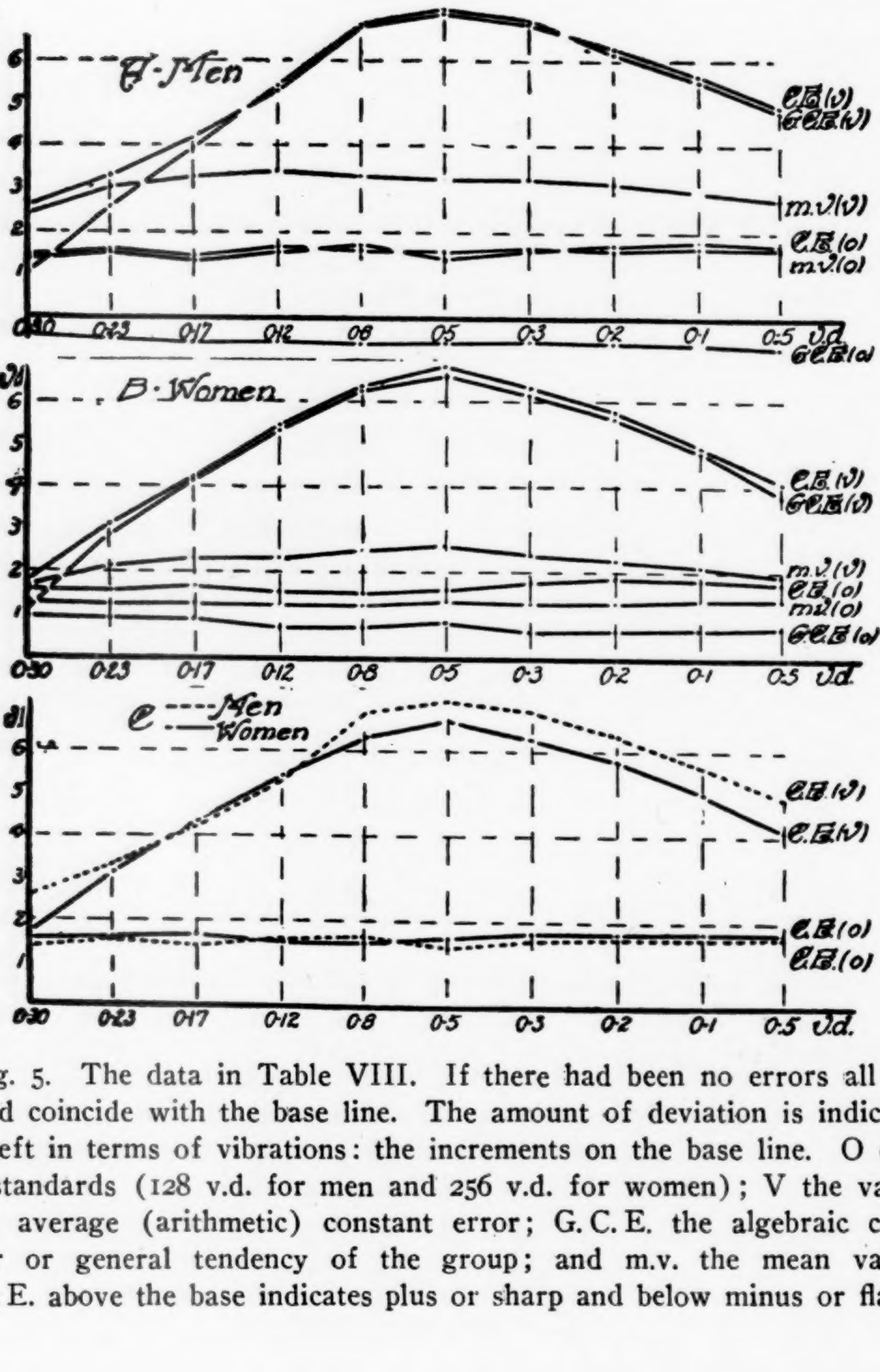

Fig. 5. The data in Table VIII. If there had been no errors all curves would coincide with the base line. The amount of deviation is indicated at the left in terms of vibrations: the increments on the base line. O denotes the standards (128 v.d. for men and 256 v.d. for women); V the variants; C. E. average (arithmetic) constant error; G. C. E. the algebraic constant error or general tendency of the group; and m.v. the mean variation. G. C. E. above the base indicates plus or sharp and below minus or flat.

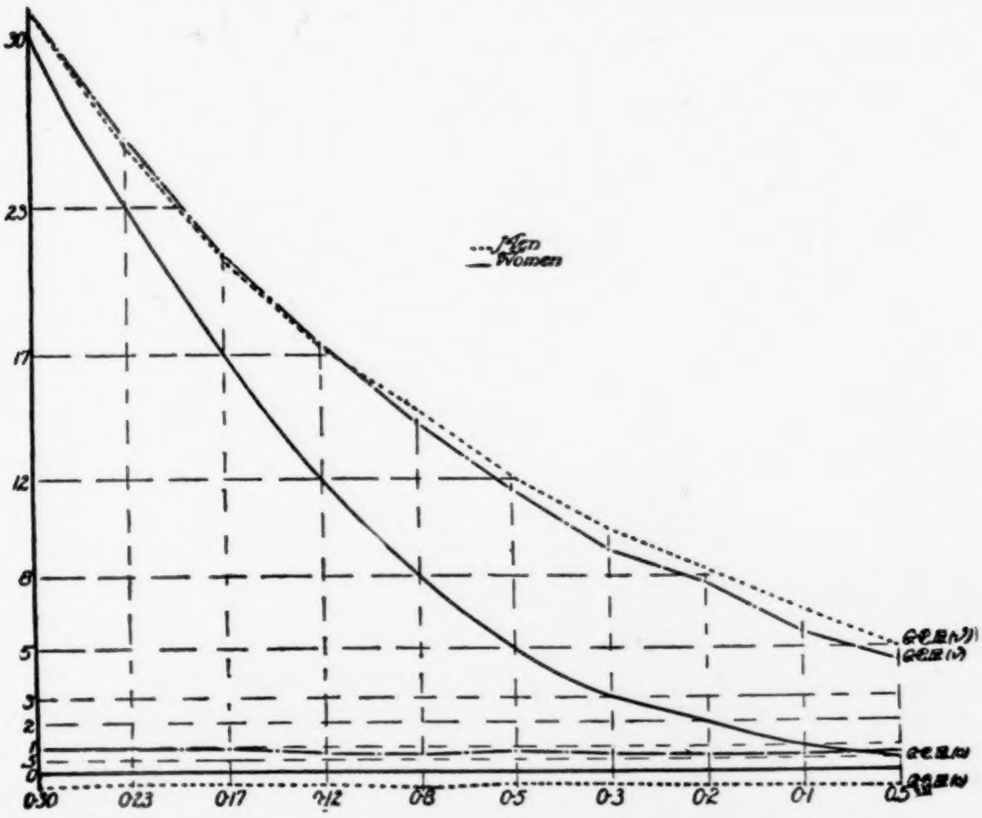

Fig. 6. Intervals as sung. (Table VIII). The distribution of the group constant errors (G. C. E.) for the standards (128 and 256 v.d.) and the variant in each interval. The intervals represented by the forks are shown in the heavy solid curves with which the other curves would coincide were there no errors in singing.

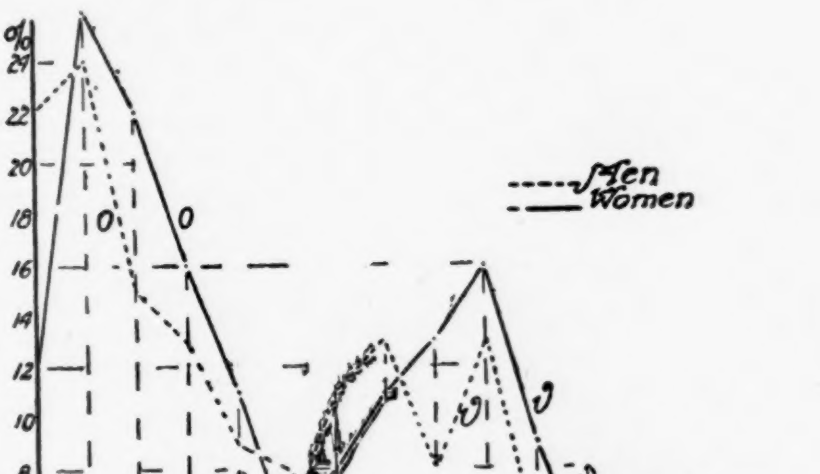

Fig. 7. The distribution of the average constant errors of all intervals for each observer with reference to the magnitude of the error. The data for this figure are found in the columns headed Arithmetic Average in Table VII. O, the standard tone: V, the variant.

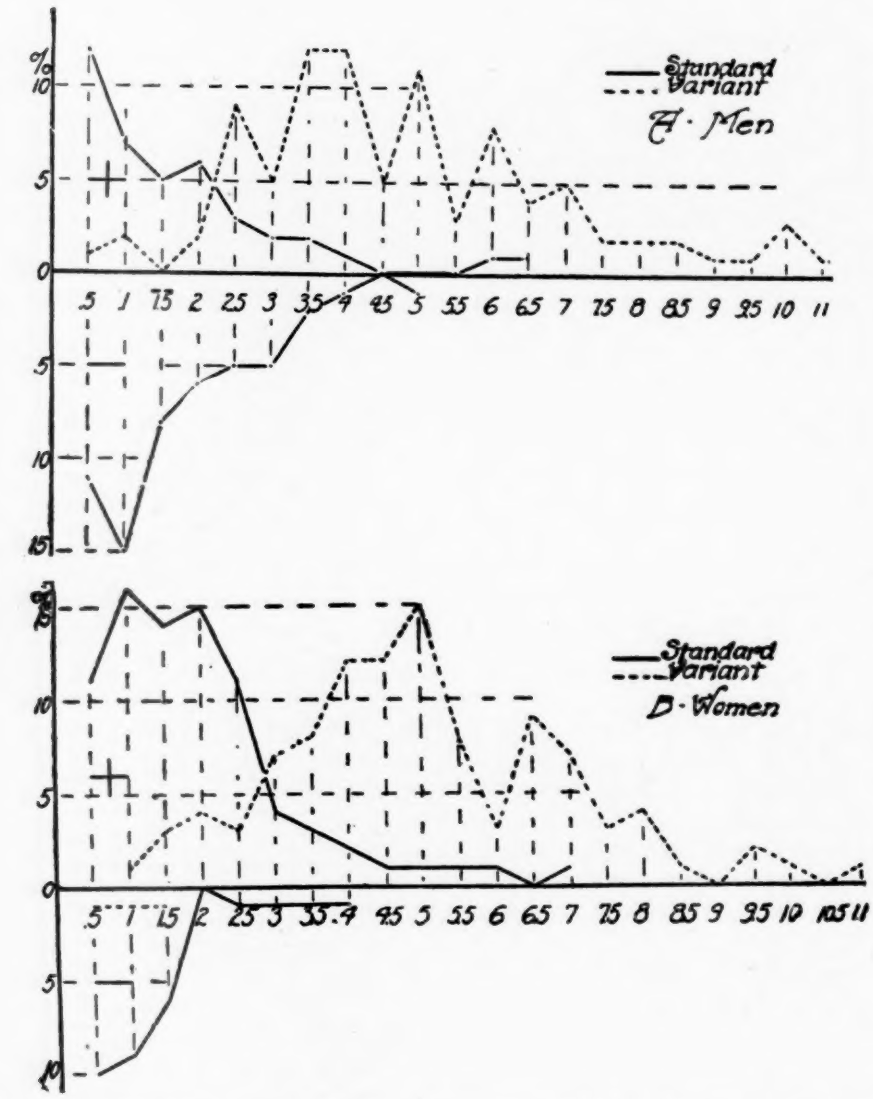

Fig. 8. Distribution of constant error, flat being denoted by — below the base and sharp by + above.

stant errors for both men and women represent approximately equal magnitudes attention is called to Fig. 7 in which is presented the distribution of the average constant errors of all intervals for each observer with reference to the magnitude of the error. The men have a slightly better record on the O. but the women have a more than compensating advantage on the V.

A corresponding agreement in the records for men and women is seen also in the constant tendency for the group (G. C. E. Fig. 5 A and B, 6, and 8 A and B). While the women tend to sharp and the men to flat on the standard (see Fig. 6) the amount is not far from equal in the two cases. (Cf. Table VIII, 65 per cent. of men flat on O while 67 per cent. of women sharp). In view of the general tendency of both men and women to sharp on the variant this difference in the tendency on the standard gives an advantage to the women as regards accuracy in the singing of the interval. An advantage which amounts to an average of over 2.0 v.d.

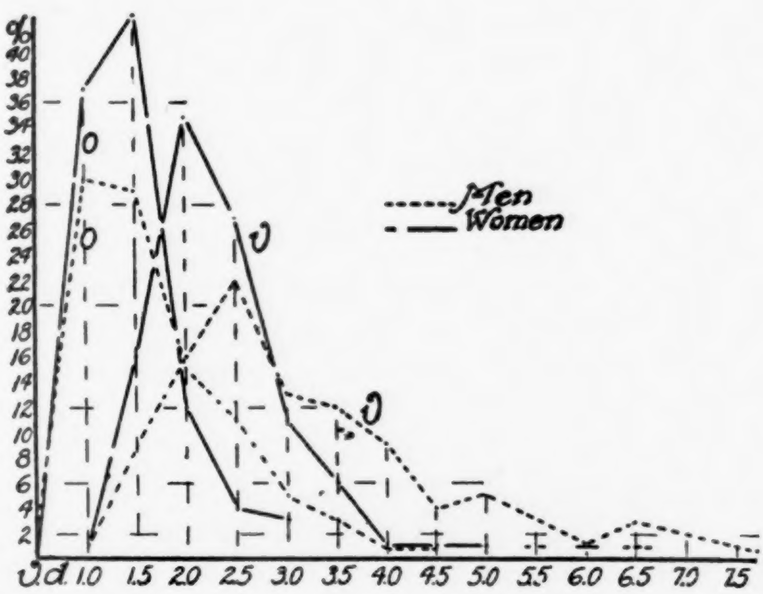

Fig. 9. The distribution of the mean variation (m.v.) for individuals (Table VII, average m.v. at right) with reference to the magnitude of the variations. O, m.v. of the standard tone; V, m.v. of the variant.

In the mean variation (Figs. 5 and 9), which is an important criterion, the advantage is more clearly in favor of the women, particularly in the singing of the variant. There are more men than women with a relatively large variation: but the mode in the case of O is slightly better for the men than for the women. The averages

of the men (Table VIII) are 1.54 and 3.05 v.d. as against 1.29 and 2.21 v.d. for women.

Taking all the data into account the general balance of all scores results practically in a draw:[15] men and women sing with equal accuracy (in terms of number of vibrations of error) although the former sing at 128 v.d. and the latter at 256 v.d. If on the other hand we count the error in relative parts of a tone instead of vibration for vibration, the women sing twice as accurately as the men. It may, however, be shown that the former statement represents the more logical point of view.

This result is in harmony with the results found in Series I with reference to accuracy within the tonal range. It was there found that so long as the singer was certainly within his natural range the man could sing the two tones here considered, 128 v.d. and 256 v.d., with nearly equal accuracy, in terms of vibrations and that, therefore, he tended to sing the higher twice as accurately as the lower. The difference here discussed is therefore not peculiarly a sex difference, but distinctly a matter of psycho-physic law of voice control within the tonal range. Men and women have equal ability in pitch discrimination (reference 21 p. 44), so also in voice control they have equal ability level for level within the tonal range. The fact however remains that women's voices are pitched in a higher register than men's voices and therefore, from the musical point of view, they can sing their tones relatively more accurately.

This result is, after all what we should expect for the principal limit upon accuracy in singing is accuracy in hearing and we know that both men and women can hear a difference of, *e.g.*, 1 v.d. as easily at 256 v.d. as at 128 v.d.

The mean variation

Fig. 5 shows that the mean variation is larger for the variants than for the standards. This is because the former are more difficult. It should be noted that this difference in the mean variation is a measure of the relative difficulty of the two tones as felt

[15] The following facts are significant: (1) there are fewer poor observers among the women; (2) women have smaller mean variations than men; and (3) women more nearly reproduce the intervals. It seems quite likely that in a mixed college group such as we have here, the women give more attention to vocal music than do the men, which may account for their superiority in this test.

and would also be a measure of the relative degree of accuracy in the singing of them were it not for the operation of the two motives for sharping the variant about the middle of the series of the increments. The fact that the mean variation is unaffected by the operation of these two motives is an indication of their fairly rigid operation.

The constant error

Figs. 5 and 6 show that the singing of the standard tone is not affected by the magnitude of the increment to be sung. The constant error is small and uniform. This is due partly to the fact that the standard tone was the same in all trials and therefore tended to become more or less automatic, and partly to the fact that the standard was sung first and that therefore the difficulty in marking off the interval would tend to crop out in the variant tone.

The singing of the variant follows the law that (1) all these small increments are overestimated and that (2) this overestimation increases gradually from the largest interval (0-30) and reaches a maximum in the cases of both men and women (Fig. 5, A and B) at the 5 v.d. interval from which it gradually again diminishes.

There are probably several motives operating to produce this overestimation; the fact that the maximum falls in the increment 5 v.d. points to a relationship between the hearing and the singing of the interval. The median for the least perceptible difference in pitch for this same group of individuals falls on 3 v.d. The increment 5 v.d. in singing would therefore represent one of the smallest increments actually heard. The distribution around this would be analogous to the distribution of the records in pitch discrimination for this group.

It is probable that, as in visual perception of space, all small angles are overestimated, there is in hearing of pitch a tendency to overestimate the smallest increments perceived. If we represent the uniformly increasing series of increments of pitch difference as a sharp wedge the apparent magnitude would be represented by a wedge blunted and thickened.

The operation of such a principle has been demonstrated for hearing in the matter of localization of sound. Starch (23) found that when a correction is made for the least perceptible change in the direction of the source this correction is always overdone.

The lack of fine control of the voice to reproduce the smallest

differences that are heard is another element involved. This factor is partly due to lack of knowledge and practice in this kind of voice control. The small differences which are actually heard larger than they really are, are sung still larger on account of this general lack of control for the making of fine shadings in pitch. This overdoing of a difference may perhaps be regarded as another phase of the same principle as the overestimation of small differences in pitch in hearing. At any rate the enlarging of the small discriminated increments is without doubt much increased in the singing. These small increments are overestimated in hearing (when heard) and are again overdone in the singing; and that this enlarging is proportionate up to the threshold for pitch discrimination.

In applying these principles to the interpretation of the relative magnitude of the errors in the singing of these increments we must bear in mind that where the small differences are not heard there would be a tendency to repeat the standard in trying to sing the variant—this happens not only because the difference is not heard, but even when an effort is made to sing an imperceptible sharp theoretically known to exist there is a tendency for the voice to "fall into the groove" of the standard tone which has been sung immediately before.

On the other hand it seems reasonable to take account of the fact that in this test we are asking the observer to do something with which he is almost entirely unfamiliar. In the larger intervals he recognizes differences but overestimates and oversings them. This overestimation increases regularly from the largest interval, 0-30, to 0-5, as was above noted. At 0-3 most of the observers fail to hear the difference because the conditions of the test do not provide the immediately successive presentation which is most favorable for the discrimination of pitch differences. Therefore, at 0-3 failing to hear the second fork higher, recognizing that he has not yet reached the smallest possible interval, and knowing that the second fork is higher than the O, our observer concentrates his attention, trying harder and harder until the last interval is sung. He is in large measure freed from the factor of overestimation in hearing for he hears no difference. He will very likely tell you that the forks sound just alike, but he knows and is reminded that the second one of each pair is higher. This knowledge forms the basis of his control of the voice. Quite naturally under the circum-

stances he resorts to the tendency (noted above) to take his cue for the second tone not from the fork but from his own previous tone. He "falls into the groove", however, just long enough to get his bearings, then sharps from this point, the magnitude of the sharp being governed roughly by the subject's pitch discrimination ability. In about 8 per cent. of the individual records of 0-.5 the records on the .5 v.d. are not sharp or may be slightly flat; in other words, the observers took the risk of making no sharp.

Applying these factors in the interpretation of the error in the singing of these small intervals of different magnitude, we find that, (1) the average overestimation is relatively small for the smallest increments because in many cases the difference is not heard and in singing a very small interval the voice uses its previous reproduction as the standard, sharping from it, and (2) the overestimation of the small increment is greatest for the smallest increments perceived and gradually diminishes as the increments grow larger so that it tends to disappear on the average when the magnitude of a half-tone is reached. Therefore, our test seems to have met the conditions for measuring the minimal producible change in the pitch of the voice. The increments from 0-30 to 0-5 serve to work down the voice, to make clear to the observer what is to be done, and to center his attention for most careful control. The four smaller increments, 0-3 to 0-.5 are the place where the "ability to make faint shadings" is really tested and under usual conditions the reproductions on the smallest increment, 0-.5, would seem to give the best measure.

If from the records on 0-.5 (algebraic C. E. or G. C. E.) we compute the magnitude of the smallest interval as *actually produced* by the individual observers and distribute these magnitudes according to their frequency, we have the curves of Fig. 12. The median value of the measures represented in Fig. 12 is 4.0 v.d. for women and 4.5 v.d. for men. There are more extremely poor observers among the men so that the average smallest intervals produced are 5.6 v.d. and 3.7 v.d. for men and women respectively.[16] These median values are in harmony with the results for pitch discrimination and may be taken as measures of the ability to produce minimal changes sharp or flat in the pitch of the voice.

[16] A part of this difference between men and women is to be accounted for in the fact that on the average the men flatted the O.

Dr. D. A. Anderson made a test on "minimal change in the pitch of the voice" in the Iowa Psychological Laboratory in 1909. His observers imitated the pitch of one standard fork and then sang the tone the least possible sharp or flat according as directed, making ten successive trials in each direction. There were 115 women and 65 men in the group tested. From the unpublished results of this test we learn that the average minimal producible change for men was 5.5 v.d. and for women 4.6 v.d as against 5.6 v.d. and 3.7 v.d. in our test. In comparing these results it must however be noted that 45 of Professor Anderson's poorest observers, most of them men, made no records which entered into his averages.

Seashore (19) reports the results of some tests of "minimal producible change" given to a small group of observers. The average records for six men on five successive days are as follows; 3.4, 3.5, 3.0, 2.6, and 2.7 v.d. Evidently the factor of practice entered here. However, the average of these results, which represents the only other available data on this ability in voice control, falls on the mode of our curve (Fig. 12) for men.

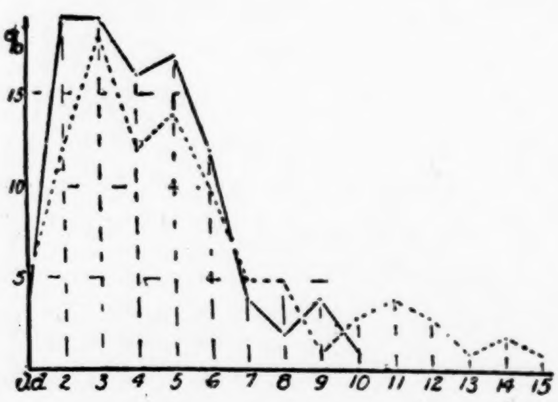

Fig. 10. Distribution of the magnitudes of the smallest interval actually produced by men and women. The method of computing the average magnitude of the smallest interval produced by each observer is illustrated in the following example: if the C. E. on O is —.9 v.d. and on V is +1.4 v.d. then the produced interval would equal the difference between —.9 v.d. and +1.4 v.d. plus .5 v.d. (the real step between the forks) = 2.8 v.d. If the C. E. on O is plus it is of course subtracted from the sum of C.E. on V and .5 v.d. Men dotted line; women broken line.

The average constant error (C. E.) on the standard is small and uniform, as is also the mean variation and the constant tendency for the group, (G. C. E. on the standard). Accuracy in the standard is not influenced by any difference in the magnitude of the in-

crements. This is chiefly because the standard tone was sung before the variant was sounded, and partly because a sort of "rut" was formed for the singing of the repeated standard.

The researches previously reviewed contain scattered measures on this ability. Klünder (1) found that he could reproduce an organ tone of 128 v.d. with an average crude error of .47 v.d. He rejected however the records of some other observers who showed larger errors. Cameron (4) worked with seven observers and tried a number of organ tones. The records by three of these observers gave an average error of about 6.6 v.d. Berlage (2), whose three observers reproduced voice tones, does not give the pitch of the standards. The average error for the three men, singing with an interval of from 1 second to 2 seconds, is .50 v.d.[17] Seashore (19) gives 1.2 v.d. as the average error of 100 trials by each of six men, on standard 100 v.d. Sokolowsky (22) with his seven professional singers finds an average error of 1 v.d. at the average pitch of 251 v.d.

The group constant error

Throughout the previous pages there have been references to the tendency of both men and women to sing sharp when reproducing a tone. The difference in the direction of this error in the standard for men and for women is so constant that, while small, it points to some motive in the character of the tone, the mode of singing, or some tendency characteristic of a given pitch level. The distribution seen in Figs. 7 and 8 shows that the sharps and the flats are not far from equal both in the number and the magnitude for men; for women the sharps predominate in both magnitude and number.

Cameron (4) noticed this tendency and called attention to it. In his experiments it appeared especially in sustained tones.[18] We have not worked with sustained tones but have found the same

[17] Berlage's tables are needlessly complicated by his using the signs with opposite from the usual meaning.

[18] Berlage (2) did not find this tendency to sharp and was surprised, but we must remember that he worked with voice tones for standards (the richest tone possible) and our experiments seem to show that with rich standard tones the sharping of the constant error is considerably decreased. Sokolowsky's (22) results are also negative as regards any general tendency for both sexes to sing sharp. The errors on the twenty tones sung by women, however, *show an algebraic average of +1.03 v.d.*, although eleven of these tones were sung flat.

tendency with reproductions of one and two seconds in length. Reference to our tables (G. C. E.) will show that almost without exception sharping is the predominant direction of the constant errors in all six series of our experiments. The tendency to sing sharp is not materially affected by the level of the pitch so long as the tone remains within the range of the voice; it is increased by loud volume of voice, weak volume of standard, certain vowel formants such as are found in "e" and "i", and by purity of the standard tones.

The best cases

The question naturally arises, to what extent the presence of a few cases of very large error affect the averages. To cast some light on this and also to gain an idea of the performance of the best observers in the group the author made a selection of twenty-five persons of each sex. The selection was made chiefly on the basis of a small Ave. m.v. in the standard (o). The size of the Ave. C. E. of o and the Ave. m.v. for the increments, were used as secondary criteria. There are some records, for example N. 9, which from the standpoint of the constant errors alone are very near the ideal curves, but because of rather large mean variations must be omitted from these selected groups. The selection of women was as follows: Nos. 1, 3, 13, 15, 21, 55, 61, 63, 77, 85, 93, 97, 105, 107, 113, 117, 125, 153, 159, 169, 177, 181, 183, 201, and 209. The men's records chosen were: Nos: 6, 8, 10, 12, 16, 28, 50, 62, 68, 72, 82, 88, 102, 106, 110, 114, 120, 126, 128, 144, 146, 148, 154, 156, and 164.

The separate tabulation of these fifty supposedly best cases reveals the presence of the same general tendencies in these selected groups as have been noted in the large groups, with the difference that they are not so pronounced and that here the men in a relative comparison make a better showing than the women, in that their overestimation especially of the smaller pitch increments is less. Therefore blame for the large errors (overestimation of intervals) can hardly be shifted to a few individuals as indeed we might have shown by referring to Figs. 7 and 8 which demonstrate that the distribution of the errors forms fairly normal frequency curves.

Correlation of singing with pitch discrimination

Pitch discrimination records are available for eighty-two of the men, and one hundred and four of the women who acted as observers

in our tests. The well-known formula of the Pearson "Product-Moments" was employed and resulted in the following correlations:

			r	P.E.r
Men:	Size of ave. smallest interval produced	with Pitch. Disc.	+.21	.072
	" " " m.v. on O.	" " "	+.04	.074
	" " " C.E. on O.	" " "	+.08	.074
	" " " m.v. on V.	" " "	+.33	.066
	" " " C.E. on V.	" " "	+.15	.073
Women:	" " " smallest interval produced	" " "	—.11	.065
	" " " m.v. on O.	" " "	+.27	.061
	" " " C.E. on O.	" " "	+.11	.065
	" " " m.v. on V.	" " "	+.51	.048
	" " " C.E. on V.	" " "	—.07	.065

It will be recalled that in order to be satisfactory a coefficient "should be perhaps three to five times as large" as its probable error. This rule liberally applied to our results leaves us the coefficients +.33 and +.51 both of unquestionable reliability. These coefficients represent the correlation between pitch discrimination and the average mean variation in singing the intervals, for men and women respectively.

The test of 1910

A series of musical tests, given by the writer in the Iowa Psychological Laboratory, during November and December of 1910, included one on Accuracy in Reproducing Tones. There were ninety men and one hundred and seven women, members of the elementary psychology classes who took this test.

The apparatus besides the tonoscope consisted of five large forks with pitches as follows: 128, 256, 320, 384, and 512 v.d. The experimenter instructed the observer to take the 256 v.d. fork, strike it gently, bring it to his ear, listen carefully, and then to reproduce the same pitch. This he repeated with fork 256 v.d. Then taking the 320 v.d. fork he proceeded as described. The last four forks were gone over five times in this manner, which gives ten trials on each tone, forty trials in all. The test is thus very simple, the reproduction of four tones (two successive trials on each) which are at the same time natural musical intervals: major third, fifth and octave. No restrictions were placed upon the observer in the matter of humming or singing with the standards. As the fork was in his hand he sang with it or after it as seemed best to him. About one-half the observers preferred to take the fork away from the ear before beginning to sing. The men sang the tones one octave below

the pitch of the fork. When it was difficult for them to commence doing this, the 128 v.d. standard was used for orientation.

The results for this series of 8,000 reproductions are given in Table IX. The notation is the same as in Table VIII. The test of 1910 was complicated by the factor of natural musical intervals, it was also considerably shorter and simpler than the one of 1913 but in comparing it with the latter we find the results in practical agreement on some points.

(1). There is a uniform tendency for the majority of observers to sing sharp. Here again the tendency appears to be greater for women than for men, the G. C. E. for men being + .26 v.d., for

TABLE IX. Accuracy of singing: test of 1910

90 men	128 v.d.	160 v.d.	192 v.d.	256 v.d.
Ave. m.v.	1.42	1.36	1.43	1.79
% + C. E.	47	63	51	63
% — C. E.	53	37	49	37
Av. C. E.	1.62	1.62	1.72	2.70
G. C. E.	+ .26	+ .65	— .06	+1.06

107 women	256 v.d.	320 v.d.	384 v.d.	512 v.d.
Ave. m.v.	1.89	2.15	2.07	2.83
% + C. E.	81	90	84	86
% — C. E.	19	10	16	14
Av. C. E.	2.59	3.91	3.47	5.90
G.C.E.	+2.39	+3.36	+3.11	+4.76

women + 2.39 v.d., a difference of 2.13 v.d. as contrasted with 1.30 v.d. in the previous measurements. In the test of 1910, as mentioned, the men and women used the same forks, the men singing the standards one octave low. Therefore the tendency for men to sing less sharp than women in the 1913 experiments can hardly be attributed to a timbre or sound volume difference between the sets of forks. The men are much more evenly divided between the sharping and flatting tendencies than the women, for example on 256 v.d. the one tone which both sexes had in common, the percentages in favor oi sharping are 63 and 86 for men and women respectively. (2) The average constant error (arithmetic) on 128 v.d. is slightly larger in 1910, 1.62 v.d. as against 1.54 v.d. The mean variation for 128 v.d. are 1.42 v.d. (1910) and 1.54 v.d. These differences are rather slight. (3) Men and women sing their one common tone (256 v.d.)

with equal accuracy: m.v. 1.79 v.d., Av. C. E., 2.70 v. d. (men) to m.v., 1.89 v.d., Av. C.E. 2.59 (women). It would seem from a comparison of available norms for voice range in the sexes (Helmholtz (9) and Zahm (27) that 256 v.d. should be about as high for men as it is low for women, and that it is well within the average range of both. We have here therefore a confirmation of our previous conclusion, *i.e.*, that men and women sing with equal accuracy vibration for vibration. However the errors in this case under consideration (1910) are much larger than the results of Series VI would lead us to expect. This is true of all the tones sung by the women and renders them incomparable with the previous results.

Recommendations toward a standard test

The recommendations which follow must be considered as preliminary and as applying simply to the two measures of singing ability considered throughout this study, *i.e.*, the ability of the voice to reproduce pitch, and the ability to produce voluntarily small changes sharp or flat in the pitch of the voice.

1. The two factors may be tested together with advantage. They are closely related phases of the same thing. Neither of them should be taken in combination with such factors as accuracy of tone memory, or judgment for musical intervals.

2. Use a graded series of standard tones similar to that commonly employed in testing for pitch discrimination. Such a series has obvious advantages over the use of a single standard; (1) If several observations are to be made at a single sitting the effects of practice are not so great. (2) The small pitch intervals make clear to the observer what he is expected to do with his voice. (3) The variety of standards (and hence degrees of difficulty) reduce monotony and fatigue. A graded series furthermore has advantage over any other series: (1) it keeps the test comparatively free from complication with the singing of musical intervals, and (2) when the standards represent small steps of pitch difference the observer discriminates more carefully and is not so likely to be satisfied with a mere approximation.

3. Use tuning forks for standards. They are very easily manipulated, are not subject to certain sources of error commonly met in the control of reeds, pipes and strings, and are readily arranged

into a graded series as recommended above. Any disadvantage, if indeed it may be so called, from the standpoint of the purity of the fork tone seems more than compensated for in having a definable quality and a standard on which all observers are equally unpracticed.

4. Begin with the largest pitch increments and proceed to the smallest and then in reverse order back to the largest. This will economize effort, provide the best practice, and help to control the attention. For general testing ten intervals representing as many degrees of difficulty, ranging from 0-30 to 0-.5 are not too many. For extensive testing of one observer or in working with highly practiced observers the increments which are distinctly above the threshold for pitch discrimination may be omitted.

5. Give the tones in pairs, presenting the variant tone immediately after the reproduction of the standard, thus securing a rapid adjustment which favors discrimination in the kinaesthetic sensations from the larynx. As an alternative procedure the two tones might be presented in immediate succession as in the pitch discrimination tests, the observer carrying the standard in mind while listening to the variant, and then singing them in quick succession.

6. Control conditions: (1) The forks should be presented before resonators which are some distance from the observer and care must be exercised to present them with uniform intensity. (2) The observer should use a medium volume of voice in singing the tones, (3) The experimenter should select the vowel to be sung and insist on a good quality. (4) If time intervals are used between standards and reproductions they should be short, not longer than two seconds at most. (5) Time intervals should be introduced between pairs of tones. These should be at least 2 seconds in length. Longer intervals would doubtless be better as the voice could the more easily be kept out of a "rut" in reproducing the standard. (6) Secure effort on the part of the observer who is too easily satisfied with his own performance.

Our test is one of motor control. As a musical test it bears the same relation to the motor side as pitch discrimination does to the sensory side. In fact it is in a practical way the motor pitch discrimination of the singer, and as far as singing is concerned it is more important than simple sensory pitch discrimination.

SUMMARY OF CONCLUSIONS

Among others the following general conclusions may be gleaned from the foregoing experiments.

1. The human voice is about equally accurate, in terms of vibration, at all points well within its range; therefore, the high tones are sung relatively (per cent.) more exactly than those which are low.

2. A strong standard tone (especially with low forks) is reproduced as decidedly lower than a weak standard.

3. The voice can most easily reproduce pitch for those standard tones which have a rich timbre, such as the organ tone.

4. Measured in terms of average error the voice is less accurate when its volume is large.

5. Vowel quality affects the accuracy of vocal reproduction of tones. The *"i"* (as i in machine) is reproduced the highest, *"o"* the lowest, and *"a"* occupies a middle position.

6. Men and women sing in their representative ranges with equal accuracy vibration for vibration of error.

7. Women show better relative voice control than men, if judged on the basis of their mean variation.

8. With women there is a general tendency to sing sharp. Men are about equally divided in this regard, sharping however being slightly more frequent.

9. The average error of the voice in reproducing a tone given by a fork is 1.5 v.d. for men at range 128 v.d., and 1.5. v.d. for women at 256 v.d. in a representative group of students.

10. A small perceptible pitch difference between two tones is overestimated in the signing.

11. The average minimal producible change of the voice for men at 128 v.d. is about 5.5 v.d., and for women at 256 v.d. it is 3.5 v.d.

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THE EFFECT OF TRAINING IN PITCH DISCRIMINATION

BY

FRANKLIN ORION SMITH

CONTENTS

The present investigation forms a part of a series of researches in the Iowa laboratory,[1] upon the tonal hearing. The problem was to determine the effects of training in tonal hearing, considering age, sex, musical education, general intelligence, and kinship.

The investigation consisted of a preliminary training series, a ten days' practice series and the correlation of the results with those of other researches. The experiments were conducted in the University and in the public schools of Iowa City and Cedar Rapids in 1908-1912.

Method of procedure

The tuning forks and accessories which were employed in this research are fully described by Professor Seashore in his report for the American Psychological Association on the standardizing of pitch discrimination tests.[2] The experimental precautions, both subjective and objective, were observed as set forth in that report. The only change made in the apparatus consisted in using two resonators instead of one, which is a decided improvement because one resonator alone does not speak sufficiently well at the extremes where increments as large as 23 or 30 v.d. are used. The methods of procedure recommended in the above named report were followed, as

[1] The writer wishes to acknowledge his manifold indebtedness to Professor Seashore for his supervision and coöperation, which have made this research possible. To Dr. Mabel C. Williams and to companions in research who are working upon related problems in the laboratory, he expresses his grateful appreciation for assistance.

[2] Psychol. Monog. No. 53.

described on pages 39-43 of that report. The "heterogeneous" method was used in all preliminary experiments and with unclassified groups. This consists in presenting the increments 30, 23, 17, 12, 8, 5, 3, 2, 1, and .5 v.d. in the order named a number of times and finding at what level in that series the threshold falls in from ten to twenty trials. The mean variation of the records for all such sets is then computed by the method described on page 42 of the above named report, as follows:

"For ordinary work we therefore recommend as a measure of variation in the record the use of the mean variation (m.v.) computed as follows: Regard the difference between successive steps as equal psycho-physic steps and, with the increment which is nearest to the median as a base, multiply the number of cases which are one step from this base by 1, the number that are two steps away by 2, the number that are three steps away by 3, etc.: divide the sum of these products by the total number of cases (sets)."

The homogeneous method is the ordinary method of right and wrong cases or constant stimuli, counting the threshold at 75 per cent. correct cases. This method was used in dealing with individuals or groups formed on the basis of preliminary tests.

The preliminary training consisted of two tests which are designated as the first and second preliminary tests respectively. The observers consisted of pupils in the elementary and high schools, and students in the University. The ages vary from nine years to maturity. Most of the observers were unmusical in the sense that they had received no special training in music. These tests were made in the schoolrooms under good conditions. The temperature and ventilation were regulated by automatic systems (except in two small grade schools). The regular teacher remained in the room during the experiment maintaining normal conditions of order and school spirit. These general conditions did not differ materially among the schools nor among the different rooms of the same school. The tests were carried on in the morning between nine and twelve o'clock, each test lasting twenty to twenty-five minutes.

Since it was not practicable in all cases to employ the homogeneous method, all the group tests were made by the heterogeneous method. In figuring the results the nearest whole vibration (except 0.5 v.d.) was taken. The increments in the series of tones used (0.5, 1, 2, 3, 5, 8, 12, 17, 23 and 30 v.d.) are referred to as units and are con-

sidered equally difficult to distinguish. That is, 23 to 30 v.d. is assumed to be as difficult for one whose threshold is 23 v.d. as 1 to 2 v.d. is for one whose threshold is 1 v.d.

In case of defective hearing the pupil was seated where he would be certain to hear; or, if the deafness was serious, he was excused from the test. The rhythm of the work period was not so easily controlled. The tests were comparatively short and every effort was made both by the experimenter and the attending teacher to keep the effort up at a high pitch throughout the test. Indifference is perhaps the largest source of error in the few cases where it was manifest. This could be recognized directly at the time of the test and usually also by the distribution of errors in the records.

One of the most striking and yet perplexing facts about pitch discrimination is that there is often no relation between the feeling of certainty and the correctness of the judgment. The judgment is often based upon a clear illusion. This illusion of hearing in the case of wrong judgment aids much in the encouragement to sustained effort.

Anticipatory judging is a fruitful source of errors. Under the influence of expectant attention the observer anticipates the second tone the moment he hears the first. The experience is analogous to the illusion of lifted weights. With a strong expectation of hearing the second tone high, or low, the organism is set to make the appropriate response and this has marked influence upon the judgment. Closely related to anticipatory judging is the tendency to compare the present tone with the preceding pair. In fact this tendency often leads to anticipatory judging especially when the first tone of the present pair is compared immediately with the last tone of the preceding pair.

The confusion of pitch and intensity is a troublesome source of error, particularly with unpracticed observers. Making the tones actually objectively equal in intensity does not always allay the difficulty as disturbing associations may tend to create confusion. High tones are intrinsically louder than low tones. A slight difference in intensity is often interpreted as a difference in pitch.

In computing the characteristic figure of a record it is necessary to take account of internal evidences and make a "correction" as is explained in the report of this test referred to above, pages 45-48. This must always be a matter of "good judgment" and can not be

done mechanically. Certain factors may however be quite clear and exact. The distribution of the records in the heterogeneous test with respect to the operation of the laws of chance is one of the most telling. A record of, *e.g.*, 8 v.d. may on examination of the distribution of the errors be found to contain indisputable proof of a threshold of 2, or 1, or .5 v.d. as the case may be.

Sometimes when a source of error has been noted a study of the distribution may show where it operated and where it did not operate. A small mean variation, *e.g.*, 1.0 or less is almost certain proof of the reliabliity of the actually computed median. The study of the internal evidences therefore has its principal use in cases showing a large mean variation. All our records were examined with reference to internal evidence of error in the computed median and, it must be frankly admitted, wherever such evidence was found the correction was made. All the records here used in the heterogeneous method are therefore "corrected" records. Fig. 1 shows, it will be seen, that the tendency of the correction is to lower the record and that most of the corrections are made for those who have poor records.

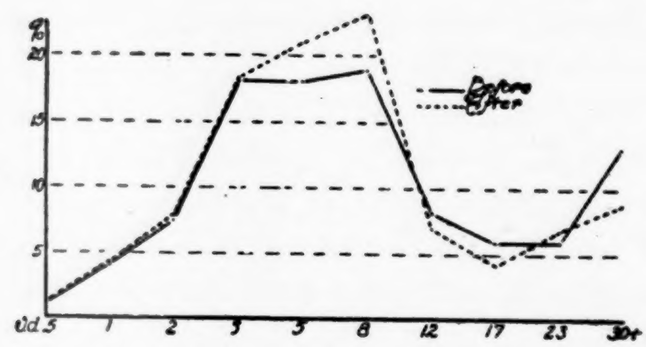


Fig. 1. Distribution of 476 pupils for one day's practice before and after the records had been corrected.

The effect of instruction

As a preliminary to the training series two tests of about 25 minutes each were given to 1980 pupils by the heterogeneous method in their regular class divisions. In the first period the test was begun without any explanation beyond what was necessary to direct them about reporting "higher" or "lower". The second period was opened with simple and diversified explanations and illustrations of what pitch is. This explanation was based upon a previous study of the kinds of difficulties encountered. Pitch was differentiated

from intensity, duration, volume, timbre, etc. in familiar talk and by different instruments.

Unfortunately the two factors of instruction and experience, or direct observation resulting in a growing familiarity with the problem, are not isolated. We have simply the records for the two periods and must interpret the gain as due to both of these factors, which are, of course, inseparably associated.

To facilitate comparison the observers were divided on the basis of these tests, into A, B, and C grades in accordance with the possession of a good, medium, or poor ear. Grade A includes those who

TABLE I. Distribution of those who improved in the preliminary test

	30	*23*	*17*	*12*	*8*	*5*	*3*	*2*	*1*	*0.5*	*A*	*B*
30+	11	11	10	15	6	3	2	1	0	0	59	
	23	*17*	*12*	*8*	*5*	*3*	*2*	*1*	*0.5*			
30	13	13	11	17	7	3	0	0	0		64	11
	17	*12*	*8*	*5*	*3*	*2*	*1*	*0.5*				
23	13	12	15	2	3	0	0	0			45	24
	12	*8*	*5*	*3*	*2*	*1*	*0.5*					
17	17	38	14	4	1	1	1				76	36
	8	*5*	*3*	*2*	*1*	*0.5*						
12	25	30	16	3	1						75	55
	5	*3*	*2*	*1*	*0.5*							
8	103	43	15	10	0						171	101
	3	*2*	*1*	*0.5*								
5	126	35	8	8							177	159
	2	*1*	*0.5*									
3	122	45	13								180	197
	1	*0.5*										
2	44	7									51	177
	0.5											
1	9										9	147

Italics designate increments; the other figures give the number of cases for each of the respective degrees of improvement; thus, of those who had a record of 30+ in the first test, 11 went to 30—, 11 to 23, 10 to 17, 15 to 20, 6 to 8, 3 to 5, 2 to 3, and 1 to 2 in the second test. *A* shows the total number of cases at each increment in the first test; *B* same in the second test.

hear differences of less than 3 v.d.; grade B those who hear differences of 3 to 14 v.d.; and grade C those who hear differences of 14 to 30 v.d. or above.

The records show that 54 per cent. made no improvement in the second test; 46 per cent. of all observers made better records in the second preliminary test than in the first. The amount gained varies from 1 to 8 units. The average amount gained varies from 3.8 v.d. at nine years of age to 0.3 v.d. at maturity.

Table I analyzes the distribution and the amount of gain by the

cases (46 per cent.) which improved with the instruction. Of the 46 per cent. who improved, only 7 per cent. changed from grade C to grade A in the second test. Of the 425 pupils (22 per cent.) who improved and were in grade B at the beginning, 255 (60 per cent.) changed to grade A in the second test. Measured by the first test 26.5 per cent. of those who improved were in grade A. Measured by the second test 70 per cent. were in grade A. Of the changes to grade A, 96 per cent. were from grade B; and 91 per cent. of the changes to grade B were from grade C.

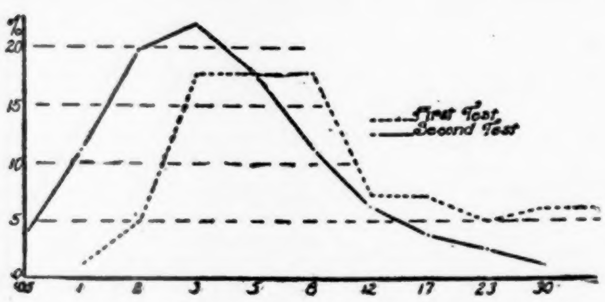


Fig. 2. The effect of instruction. Distribution of 907 pupils who made improvement from the first to the second preliminary test.

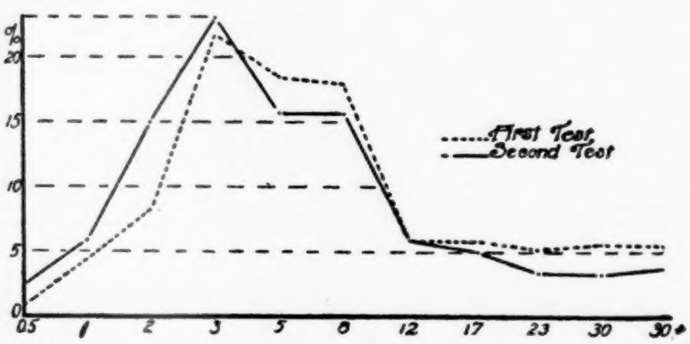


Fig. 3. Distribution of entire group, 1980 cases, in preliminary tests.

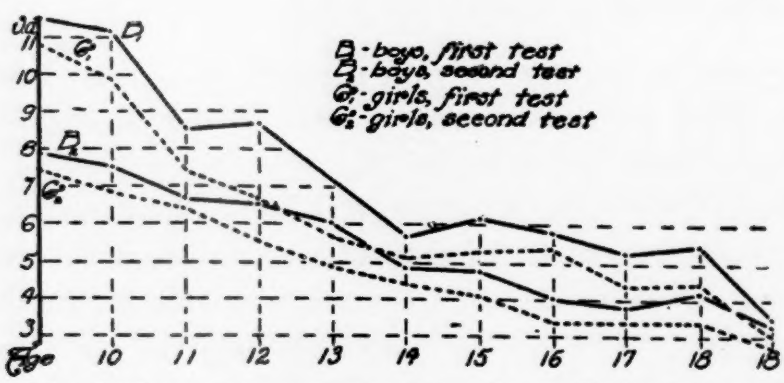


Fig. 4. Distribution of improvement in the preliminary tests by age and sex, 417 boys and 490 girls.

The effect of the instruction and experience thus gained from the first to the second test is shown in Fig. 2 which represents only the 46 per cent. of cases in which improvement was made. Fig. 3 shows the effect upon the whole group of 1980 cases. Fig. 4 shows the distribution of improvement by age and sex.

TABLE II. Distribution of forty-seven out of fifty-four university students who improved with individual instructon

	23	*17*	*12*	*8*	*5*	*3*	*A*	*B*
30	1	1	1	2	1	0	6	
	17	*12*	*8*	*5*	*3*	*2*		
23	0	0	1	1	0	0	2	1
	12	*8*	*5*	*3*	*2*	*1*		
17	0	0	4	1	1	2	8	1
	8	*5*	*3*	*2*	*1*	*0.5*		
12	2	2	1	1	0	2	8	1
	5	*3*	*2*	*1*	*0.5*			
8	3	6	2	1	0		12	5
	3	*2*	*1*	*0.5*				
5	4	1	4	2			11	11
Below 5								28

Notation and plan of this table same as in Table I.

A similar test of the effect of instruction was made in a class of 200 adults. After two preliminary tests, one heterogeneous and one homogeneous, the poorest one-fourth of the group were taken and instructed individually as to the actual nature of pitch hearing. An effort was made to find out what particular difficulties they were encountering, and explanation and illustration were based progressively upon this information. As a class these had made but little improvement in the second preliminary test, both the first and the second having been given "without instruction". But as a result of this personal instruction all but 7, *i.e.* 47 out of the 54 made rapid improvement. The change in the record for the group is shown in Fig. 5 by giving the distribution at the beginning and at the end of the period of individual instruction. The distribution of the gain is analyzed in Table II.

The fact that these were adults familiar with the class room and trained in many psychological experiments, yet made such marked response to the instruction and individual help, doubly emphasizes the importance of thoroughness and individual attention in the instructions if the records are to be entirely reliable.

One of the best experimental proofs that we have showing the efficacy of individual care and instruction is found in the un-

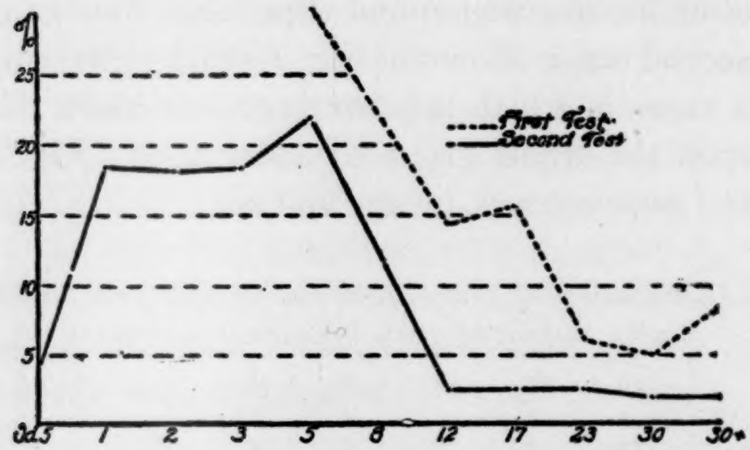


Fig. 5. Distribution of 54 university students in individual tests (Table II).

published experiments of Dr. H. S. Buffum, which have been summarized in the above mentioned Psychological Association report by Professor Seashore, as follows:

Dr. Buffum experimented on twenty-five eighth grade pupils in a grammar school room. He first made a fifteen minute individual test of each pupil and classified them on this basis into three groups with modes at 3, 8, and 17 v.d. respectively. The object was two-fold: (1) to determine the effect of practice and (2) to determine the success of the preliminary examination. For this purpose he gave them twenty forty-five-minute periods of training.

The results show (1) that for no group is there any evidence of improvement with this practice, and (2) that all except two children remained throughout the whole practice series within the group to which they had been assigned. Of these two, one who had been assigned to group III was immediately found to belong to group I as there had been a failure to understand the preliminary test; and the other, although retained in group II, proved really to be near the dividing line and could have been classified in group III. Evidently the physiological threshold had been reached in twenty-four of the twenty-five cases in the preliminary test."

In Dr. Buffum's experiment the fifteen-minute preliminary classification was so efficient as practically to eliminate poor records due to ignorance of the test.

The significance of instruction is further proved by the records in successive classes in the university for a period of years. It is found that the average record has improved slightly from year to year. There is no reason for believing that this is due to anything

but improved skill and technique and increased care in the instructions and charge to those about to be examined.

In the above records we have conclusive evidence that effective instruction is of the greatest importance in making records on pitch discrimination. It is not a poor ear, but ignorance that accounts for the bulk of poor records in a first test. Those who made a fine record in the first test are, of course, not subject to this source of error; and those who have poor records but show no improvement after instruction or prolonged training may also be free from this source of error. It is a safe rule to say that all tests should be preceded by efficient instruction; if this can be made individual, so much the better; and all who show poor records must be subjected to more intensive and searching instruction before the record can be accepted for serious purposes.

The effect of practice

The first of the two extensive experiments in practice was a series of group tests by the "heterogeneous" method covering a period of ten days. The second was a series of individual tests on adults by the "homogeneous" method. In addition to these certain special training series will be described.

The group tests were made on 476 pupils (215 boys and 261 girls) in two elementary schools selected from those in which the preliminary tests had been made. These practice tests were conducted in the same manner and under the same conditions as the preliminary tests except with regard to instruction. Each test was preceded by a brief warming-up exercise in which the pupils answered orally. This also helped to keep interest alive. A short rest period was taken at the middle of each test. At this time opportunity was given the pupils to ask questions about the test.

Running parallel with the class tests were certain individual tests which were carried on in the afternoon following a given set of class tests. At the noon intermission the records of one or two grades were checked up and pupils whose threshold for that day was between 20 v.d. and 30 v.d. were given individual practice. The object of these individual tests was to give special assistance to backward pupils, aiding them to distinguish different tone qualities and to form right habits of attention. These tests include 71 boys and 35 girls constituting the poorest in the group tests.

With regard to the general musical preparation of these pupils it may be said that music was taught systematically in all the grades, and that the schools were provided with Victor graphophones in which high grade selections were played regularly.

For comparison the cases under observation may be divided as follows: Group I, those who made no improvement either with instruction or practice; Group II, those who made no improvement in the practice; Group III, those who made little (1-3 v.d.) improvement in the practice; and Group IV, those who made marked improvement (3 v.d. +).

The records of these practice series on children are set forth in Tables III-IX and Figs. 6-10. Table III gives the daily average threshold for the twelve days of training by ages, section A showing those who do not improve with training and B those who do improve with training. Table IV gives the daily average threshold for those who improve with training regardless of age for the four

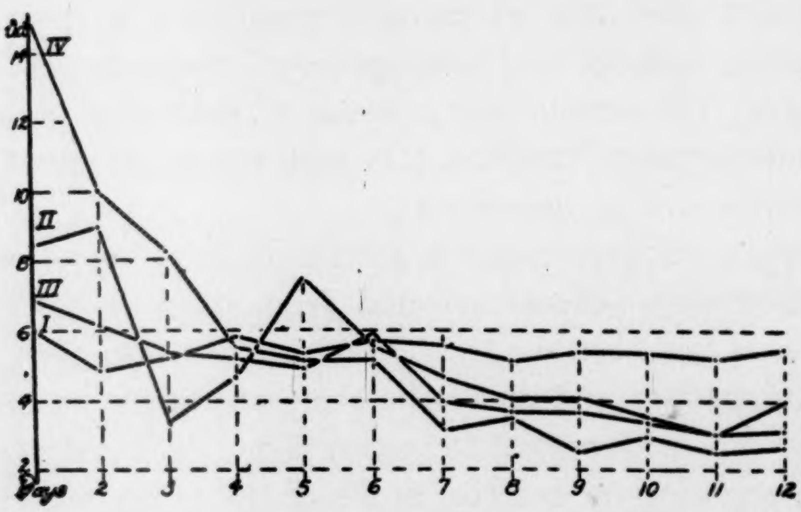


Fig. 6. Daily average, by groups, of those in the practice series (Table VI).

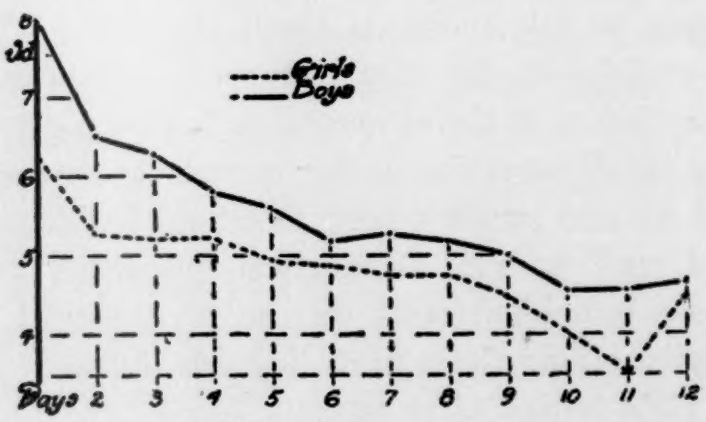


Fig. 7. Daily average by sex (Table V).

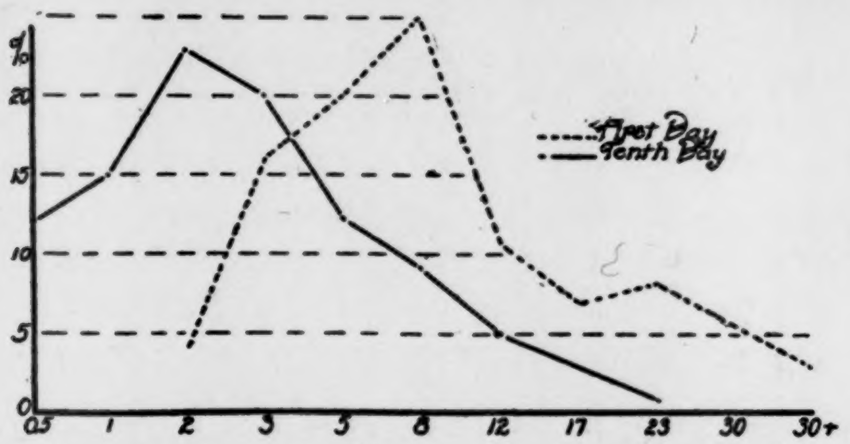


Fig. 8. Distribution of 270 pupils who improved with practice (Table VI).

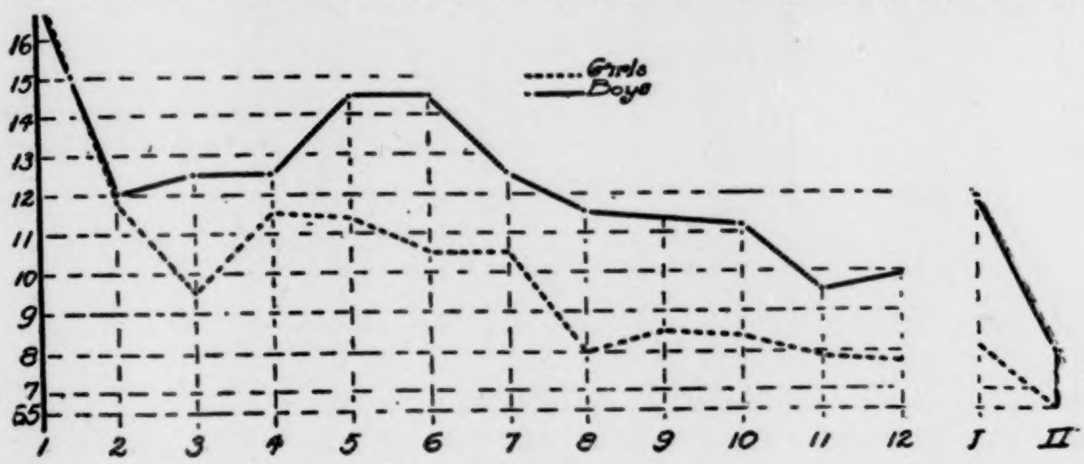


Fig. 9. Daily average record of those who were given special individual help (Table VII).

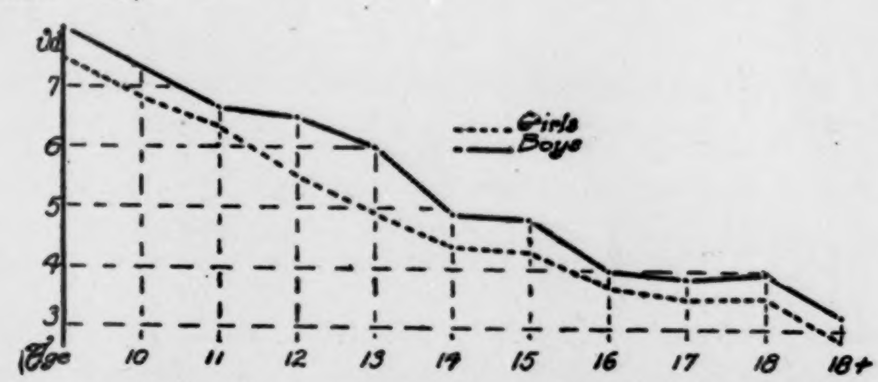


Fig. 10. Comparison by ages of the average (median) abilities of boys and girls.

groups. Table V gives the averages of the same separately for the boys and girls, Groups II, III, and IV combined. Table VI gives an analysis of the distribution of those who improve with practice. Table VII (Fig. 9) contains the record of those who were given individual tests or help during the practice, showing the daily record and the record of two individual tests in the average for the group. Table VIII gives a comparison of the mean variations with reference to sex and age. Table IX gives the distribution of those who attain the approximate physiological threshold in different days of the practice.

TABLE III. Daily average threshold, by age

A. Group I: those who made no improvement with training

Age	1	2	3	4	5	6	7	8	9	10	11	12	Number
9	6	6	6	6	8	7	8	10	9	8	7	8	29
10	6	6	4	7	8	8	6	7	7	8	6	8	31
11	8	4	6	7	5	7	6	7	5	6	7	8	47
12	8	8	8	5	7	8	7	7	8	8	8	5	23
13	5	5	5	5	4	5	5	5	4	4	5	5	29
14	6	5	5	5	4	4	5	4	4	4	4	5	32
15	5	6	5	5	4	4	4	4	4	4	4	4	15

B. Groups II, III, IV combined: those who made improvement

Age	1	2	3	4	5	6	7	8	9	10	11	12	Number
9	7	5	5	5	4	4	4	4	4	3	3	2	16
10	7	7	5	5	4	4	5	3	3	3	3	3	43
11	10	10	8	7	8	5	5	5	6	5	4	5	43
12	7	5	4	3	3	3	2	2	2	2	2	2	53
13	8	8	5	5	6	4	4	4	3	2	2	2	51
14	7	7	5	5	5	5	4	3	4	3	3	3	34
15	7	8	7	5	5	5	4	3	3	3	4	3	30
Ave.	7.5	7.1	5.7	5.0	5.0	4.3	4.1	3.4	3.5	3.0	3.0	2.9	
% gain		9	30	15	0	15	4	15	0	9	0	2	

TABLE IV. Daily average by groups

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	Number
Group I	5.9	4.8	5.2	5.8	5.1	5.5	5.4	5.0	5.4	5.2	5.1	5.4	206
Group II	8.5	9.0	3.3	4.6	7.6	5.3	4.7	4.0	4.0	3.5	3.0	5.0	52
Group III	6.9	6.2	5.4	5.1	5.0	5.8	4.5	3.5	3.5	3.4	3.1	3.0	172
Group IV	15.0	10.0	8.2	5.5	5.1	5.1	3.1	3.4	2.5	2.9	2.6	2.7	46

TABLE V. (Fig. 7) Daily average by sex for Groups II, III, and IV

	Number	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
Boys:	215	8.1	6.5	6.3	5.8	5.6	5.2	5.3	5.2	5.0	4.6	4.6	4.7
Girls:	261	6.3	5.3	5.2	5.2	4.9	4.8	4.7	4.7	4.5	4.0	3.5	4.5

TABLE VI. Distribution of those who improve with practice

	30	*23*	*17*	*12*	*8*	*5*	*3*	*2*	*1*	*0.5*	*A*	*B*
30+	0	1	1	2	2	1	0	0	1	0	8	0
	23	*17*	*12*	*8*	*5*	*3*	*2*	*1*	*0.5*			
30	2	3	2	4	1	2	1	1	0		16	0
	17	*12*	*8*	*5*	*3*	*2*	*1*	*0.5*				
23	4	5	4	4	2	1	1	0			21	3
	12	*8*	5	*3*	*2*	*1*	*0.5*					
17	5	7	3	2	1	0	0				18	8
	8	*5*	*3*	*2*	*1*	*0.5*						
12	8	10	8	3	1	1					31	14
	5	*3*	*2*	*1*	*0.5*							
8	14	26	15	6	6						67	25
	3	*2*	*1*	*0.5*								
5	15	26	10	4							55	33
	2	*1*	*0.5*									
3	12	16	14								42	55
	1	*0.5*										
2	4	8									12	59
Below 2												73

Notation and plan of this table same as in Table I.

TABLE VII. Daily average record of those who were given special individual help

Days	1	2	3	4	5	6	7	8	9	10	11	12		
Boys:	17.3	12	12.5	12.5	14.5	15.5	12.5	11.4	11.3	11.2	9.5	9.8	*12.*	*8.*
Girls:	17.7	11.8	9.5	11.5	11.4	10.5	10.5	8.	8.5	8.4	7.9	7.8	*8.2*	*6.5*

Italics, average record on the first and the second individual tests respectively.

TABLE VIII. Average mean variation from the individual records in the preliminary and final tests.

	Boys (215)		Girls (261)	
Age	Prelim.	Final	Prelim.	Final
9	1.82	1.93	1.99	1.97
10	1.66	1.81	1.76	1.60
11	1.63	1.71	1.68	1.82
12	1.70	1.53	1.51	1.68
13	1.47	1.52	1.54	1.60
14	1.45	1.48	1.65	1.65
15	1.64	1.59	1.53	1.38
Total	1.61	1.65	1.65	1.69

TABLE IX. Distribution of those who reach the approximate physiological threshold on different days of practice.

Days	1	2	3	4	5	6	7	8	9	10
Per cent.	6	8	9	9	9	13	12	13	8	7

Of the 476 children 206 (43%) fall in Group I; *i.e.*, so far as the instruction and practice are concerned, these made no improvement that could be traced in the records, due allowance being made for daily variable errors. The number of those who do not improve with practice is relatively greater for the younger than for the older children.

Classifying these 206 on the basis of record into Grade A, those whose threshold is 4 v.d. or less; Grade B, those whose threshold is between 4 v.d. and 14 v.d.; and Grade C, those whose threshold lies above 14 v.d., we find 40 per cent. in Grade A, 51 per cent. in Grade B, and 9 per cent in Grade C. Of the 270 cases (57 per cent.) which show improvement with practice 19 per cent. are in Grade A, 64 per cent. in Grade B and 17 per cent. in Grade C.

Relatively the largest number of cases of improvement occur among those who start out with a very inferior record. This can be shown by comparing the distribution of cases which make improvement for each of the increments as set out in Column A, Table VI with the normal distribution of thresholds for the entire group.

Of those who did not improve ten were unable to hear any of the increments used and judge as required. It was however found upon

making private examination of the seven of these who were available that they could hear tone differences. Two of these could distinguish between A and B on the piano. Two of them seemed unable to grasp the concepts "high" and "low" with reference to the naming of pitch. One of these—a scatter-brain—could, however, sing a half-tone correctly when played on the piano. The other—retarded about five years—could sing a fifth fairly accurately with the piano. Three were able to imitate a pitch difference in the forks of 3 v.d. by singing enough to show whether the second of the two tones was sharp or flat. The other three were, unfortunately, not available for special tests. Thus, of the 476 cases not a single case of so-called tone deafness was found.

The last line in the footings of Table III, B shows that the gain of those who do improve is most rapid in the first part of the training series, 54 per cent. of the gain being made in the first three steps. The further analysis of these figures in Table IV, illustrated by Fig. 6, shows that this principle is true for all three of the groups which show improvement.

All the observers included in Table VII took the first individual test which occurred on different days, from the third to the seventh day. Most of these tests were given early in the practice series. The second test began on the fifth day and extended over the remainder of the practice series. Only 26 boys and 9 girls needed to take this test. A very few were given a third test near the end of the practice but the results were not included in the table. Not only does the individual test yield a lower median than the group test in a majority of cases, but the individual test often influences the later results of group practice. In this experiment 6 boys and 4 girls made immediate and permanent improvement after the first individual test which it will be remembered was accompanied by instruction. In one case the gain was from 30 to 9 v.d.; in another from 23 to 5 v.d. and in a third from 24 to 10 v.d. In some cases improvement did not begin until after the second test, and in the case of 29 boys and 13 girls no improvement was made. Of these only 2 (both girls) made better records in the individual tests.

The average amount of improvement for all cases at each increment decreases with the diminishing of the increment. This is seen in Table VI, and may also be seen graphically in Fig. 8. It must be remembered that this figure does not represent the whole group but only those who improved.

The series is not long enough to guarantee that any or all reached the physiological threshold.[3] The main difficulty in determining this lies in the fact that persons often come to a "plateau" in the record which is due to some motive or condition which may be removed by instruction or training. This, however, gives trouble only when it continues to the end of the training series. Classifying the cases roughly on internal evidences of the records we find that what may be approximately the physiological limit is reached in successive days as set forth in Table IX. From the variations in the records it is quite clear that the data in this table are quite problematical. To get the actual physiological threshold it is necessary to have more favorable conditions for isolation of the observer and the elimination of disturbances. Undoubtedly there may also be several who remain on a "cognitive" plateau throughout this series and would improve under the proper impetus. Yet, due allowance being made for these sources of error, the table shows that there is a "rapid maturing" in this training; 6 per cent. reach their bed-rock level on the first day, 8 per cent. on the second, 9 per cent. on the third, etc.

After the preliminary tests the number who reach the approximate physiological threshold increases gradually to the fourth day. On the fifth day the number increases suddenly from 24 to 41 (9 per cent. to 15 per cent.) and then gradually decreases to the eighth day after which there is a rapid falling off to the tenth day. (Table IX). The results show that 47 per cent. of those who improve reach the approximate physiological threshold by the fifth day of practice.

The mean variation as given in Table VIII conveys three significant items—the result of practice, the variations with age, and the

[3] The term is here used in the sense defined by Seashore (3) page 49-50. "*The Cognitive* vs. *the Physiological Threshold.* In sensory discrimination of this sort we may speak of two thresholds: the physiological, which is set by the limits of capacity in the end organ; and the cognitive, which is set by cognitive limitations. Theoretically we always aim to reach the physiological threshold, but practically we often fall short of this and find a cognitive limit; *i.e.,* a higher threshold due to lack of information, best form of attention, interest, effort, etc.; or to disturbances of some sort. Usually inspection of a record or observations made in the test enable us to tell whether or not we have reached the physological threshold. It cannot be judged by a single rule, although a small m.v. and a well defined mode are pretty sure indications. This distinction is of greatest importance in classification, and in the theory of training."

variations with sex. It must be borne in mind that the unit of the m.v. is not the vibration but the increment, as was described above. That is, the increments increase in a geometric ratio of the second order; therefor, *e.g.*, the increment 17-23 v.d. counts one unit just as do the increments 5-8 v.d. or 1-2 v.d. It follows that as the threshold is lowered the mean variation unit remains relatively constant. Equal power of application of those who have high and those who have low thresholds should therefore show in about equal mean variations; and, conversely, unreliability in judgment will show in increased mean variation equally for the one who has a fine ear and the one who has a poor ear.

The mean variation is slightly larger in the final training test than in the preliminary. The difference is not large—only .04 units—but it is fairly constant for all ages and for both sexes. This is rather remarkable as, in the nature of the test, one would look for evidences of increasing familiarity in the lowering of the mean variation. On the other hand the fact that the procedure does not reduce the mean variation is a most telling proof of the elemental nature of the test. The test is so stripped of conditions for variablity that it is possible to be as consistent in the preliminary trial as in trials after practice.

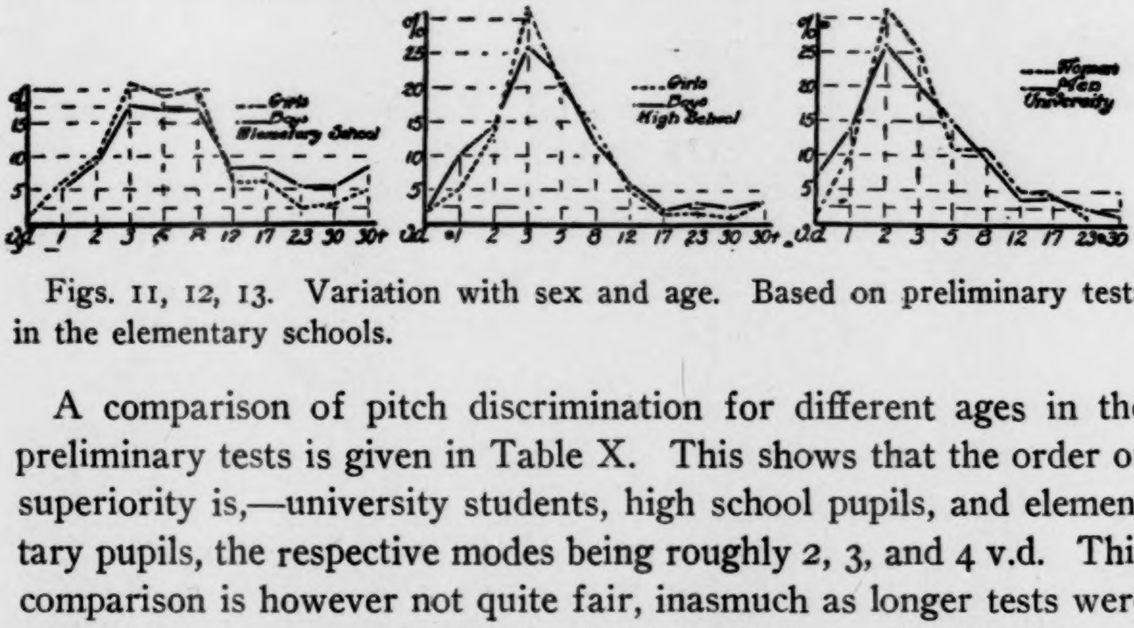
The variablity is a trifle larger for girls, both in the preliminary and the final tests. This is true for all the ages except 12 and 15 in the preliminary and 10 and 15 in the final. Were it not that this has a bearing on the much mooted question of sex difference on this point and that the data here given represent such a large number of cases (1980 in the preliminary and 476 in the final) no significance would be attached to this difference. The second decimal figure is of doubtful value in an average of this kind and, as stated, the variation is in one direction for five ages and in the opposite for two both in the preliminary and the final. On the whole our interpretation is, therefore, that practically there is no significant difference in the variability of the boys and the girls in pitch discrimination.

There is a general, though not great, tendency for the mean variation to decrease with age. This is the measure of growing reliability with age which we are accustomed to find in records of this kind.

In this practice series in the elementary schools there are two

items that count distinctly in favor of the girls. One is that of the 215 boys and 261 girls who took the practice series, 71 boys and 35 girls were considered, on the same basis, poor enough to need individual instruction and drill. This is a distinct mark of superiority in the girls. The other is that the girls in the training series, quite uniformly for all ages, have a lower threshold than the boys by from one to two vibrations. (See Table V, and Fig. 7).

This superiority of the girls over the boys is evidenced also in the preliminary tests with remarkable uniformity as is seen in Fig. 10 where a fairly constant difference is maintained throughout all the ages. The same fact is illustrated from another point of view for the elementary school in Table X and Fig. 11. This difference, however, disappears when we come to the higher ages. Fig. 12, for the high school, and Fig. 13 for the university, based on Table X reveal no recognizable superiority of either sex in the preliminary tests.



Figs. 11, 12, 13. Variation with sex and age. Based on preliminary tests in the elementary schools.

A comparison of pitch discrimination for different ages in the preliminary tests is given in Table X. This shows that the order of superiority is,—university students, high school pupils, and elementary pupils, the respective modes being roughly 2, 3, and 4 v.d. This comparison is however not quite fair, inasmuch as longer tests were given to the university students than to the high school pupils and longer to the high school pupils than to the elementary pupils; and the longer the test the more favorable the results tend to be. As will be shown later, this, together with the better control of experimental condition among the older pupils, may be ample to account for the differences here shown, so that, under equally good conditions of testing, there would probably be no evidence of variation with age.

In Table XI we see that at the age 9 the cases are about equally distributed in the three grades. Grade B remains about

constant for all ages; but the number of cases in Grade A decreases with age as the number of cases in Grade C increases. Fig. 14 shows a comparison for age only.

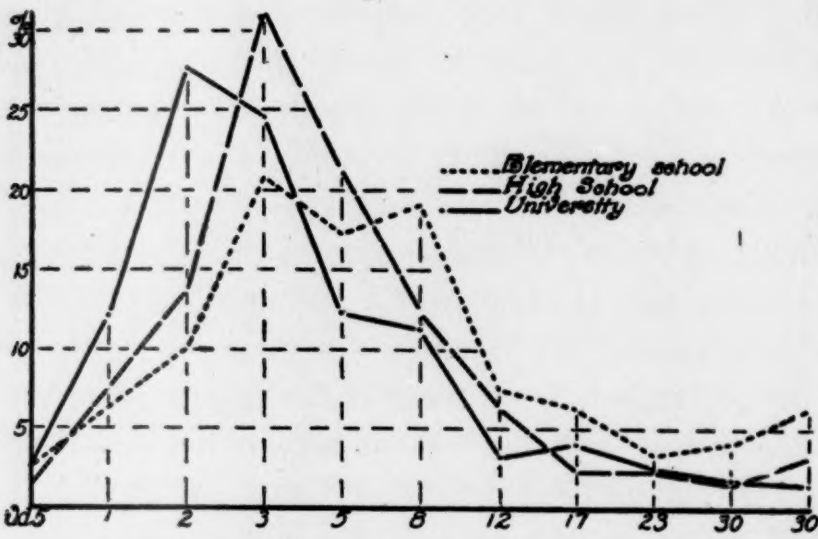


Fig. 14. Comparison of threshold of pitch discrimination for different ages.

TABLE X (Fig. 11, 12, 13 and 14). Variation with age and sex (Preliminary tests).

		Elementary School		High School		University	
	v.d.	*M*	*F*	*M*	*F*	*M*	*F*
Grade C	30+	8	5	3	3	0	0
	30	5	3	2	1	1	0
	23	5	2	3	2	2	1
	17	8	6	2	2	3	4
Grade B	12	8	6	6	5	3	5
	8	17	20	12	14	9	11
	5	17	19	21	20	13	11
Grade A	3	18	22	26	34	22	25
	2	8	10	14	13	26	32
	1	5	6	10	5	14	10
	½	0	1	1	1	7	1

M males; *F* females; numbers indicate the per cent. of cases at each step. A 4 v.d. or less; B between 4 v.d. and 14 v.d.; C above 14 v.d.

TABLE XI. Distribution by age and group in terms of per cent. of cases

Age	9	10	11	12	13	14	15	16	17	18	19+
Group A	30	24	24	16	17	12	13	12	8	4	8
Group B	38	44	40	44	41	44	40	34	34	47	26
Group C	32	32	36	40	42	44	47	54	58	49	66

The comparison of the mean variation for the three groups of ages given in Fig. 15 shows that the reliability of the records of

the high school pupils is practically as good as that of university students. Elementary pupils are slightly inferior but not so much as we would ordinarily find in other tests of discrimination.

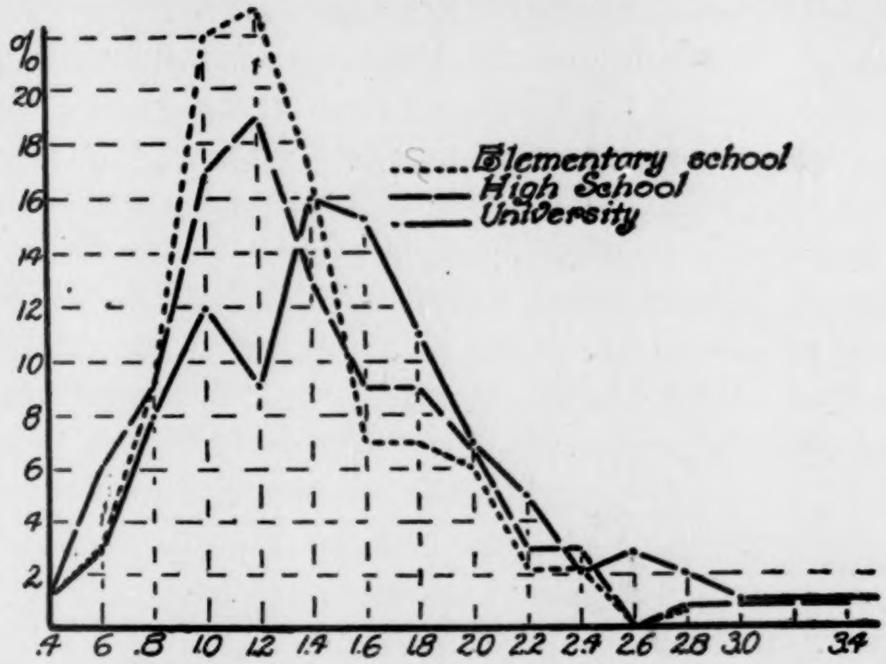


Fig. 15. Comparison of mean variation (m.v.) in the preliminary tests for different ages.

Some records of practice in pitch discrimination have been collected incidentally in this laboratory. The case of C.E.S. is presented (Fig. 16) to illustrate how variable the threshold may be aside from practice. The first practice series of twenty half-hour periods was taken in 1898 with crude methods. No resonator was used, the forks being held to the ear. This, perhaps, introduces the largest source of error in that series. Unfortunately data are not available for determining other causes of the inferiority of this record. Beginning with 1906 the Koenig resonators were used with a good quality of forks. The fact that, from this point on, the record is fairly constant would seem to indicate that the absence of the resonator in the foregoing series was the chief source of error. In 1907 the experimenter was not skilled. In 1910 the tests were made for the purpose of comparing certain conditions of environment, such as the light and sound-proof room, a class room, and out in the open air. From the 43rd to the 48th day a study was made of the effect of the duration of the tone and the time interval between the two tones. On the last four days distractions were introduced. The best record was made while the observer was intentionally tracing a maze.

Something was wrong in 1898. M. C. W. (Fig. 17) made a poor record in the twenty period practice undertaken by the same method and means as in the case of C. E. S. above. In 1908, as soon as the good resonator was introduced, her record was fine and free from fluctuations. She had, however, learned to play the violin and had gained experience in the tuning of forks in the years that elapsed since 1898. Her best records were made with distractions—tracing a maze or crocheting. These records furnish most striking evidence of the importance of reliable apparatus and technique.

In Fig. 18, characteristic results of practice, under most favorable conditions of control are shown; a, b, c, and d are the respective practice curves for four graduate students practicing one hour daily, sixteen days.

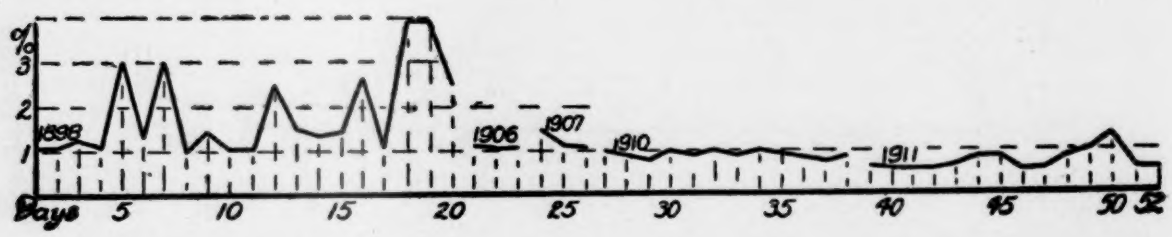


Fig. 16 Record of C. E. S.

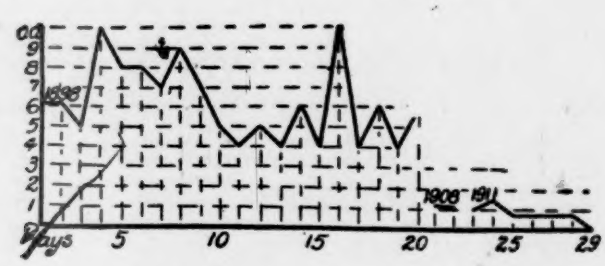


Fig. 17 Record of M. C. W.

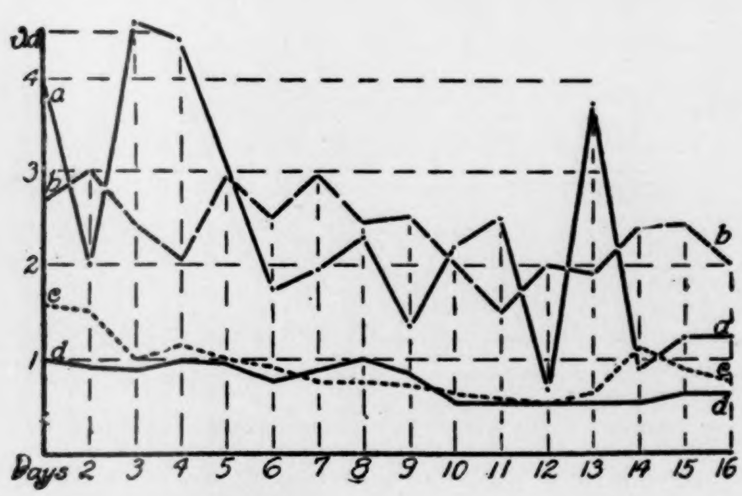


Fig. 18. The effect of training.

Factors in the development of pitch discrimination

Practice in pitch discrimination means (1) the control of a special set of cognitive factors involved in learning to recognize differences in pitch, and (2) the acquisition of skill in listening to musical tones. In most of the recent experiments on practice, such as those made by Book on typewriting, Swift on tossing balls, Bryan and Harter on telegraphy, Judd and others on handwriting, and Huey on reading, the object has been to determine the mode of acquisition of certain special habits. Of these Book distinguishes two sorts, habits of manipulation and habits of control: The latter he defines as certain general or more purely psychic habits acquired in the course of practice. It is to this type of learning that the present analysis is directed. Of these general habits or modes of control we may distinguish three types, (1) sensory control, by which is meant acquaintance with certain sensational facts, such as auditory qualities of the tones, and muscular sensations; (2) associational control, or acquaintance with memory images, as auditory, visual, and motor; and (3) control of special attitudes, as feeling of familiarity, most favorable form of attention, interest, etc.

Auditory and kinaesthetic sensations seem to play the leading rôle in judging differences in pitch. Two types of observers may be distinguished. First, there are those whose perception of pitch is chiefly in terms of tonal qualities. They learn to direct attention to the characteristic sharpness or fineness of the high tones and to the flatness or dullness of the low tones. The particular sensory quality of the tone varies with different persons. One notices that the high tone is sharper, and has a tendency to last longer in the ear than the low tone. Another describes the high tone as finer and more piercing. The lower tone is usually distinguished from the higher as being duller, deeper, heavier, and more mellow. It is also interesting to note that some observers judge altogether by the high tone, while others judge only by the low tone. Some persons seem to have an affective preference for high tones, others for low tones. This forms an apperceptive basis for the judgment.

Second, there is a considerable number who depend largely upon kinaesthetic sensations of the vocal organs in making the judgment. Regarding these Stumpf says: "If the muscular sense in the vocal organs is the same as a former tone that we have heard, we judge that it is the same tone. If we are told that a certain

tone is A, we remember that a tone giving the same sensations is A. If the muscular sense changes in a definite way when we sing two tones, we say that the tones rise. If a distance is noticeable in the change, we judge the second interval to be greater." Stricker did not think of music in terms of notes nor of auditory images, but in terms of muscular sensations in the vocal cords. He speaks of the impossibility of the reproduction of a tone in the memory without bringing into play the actual or intended use of the vocal organs. He considers the connection between tone perception and the innervation of the vocal organs a sort of reflex.

This view is in accord with many introspections in the present investigation. Some of the observers allege that they are not able to tell whether the second tone is higher or lower until they reproduce the tones either audibly or mentally in terms of vocal strain. In one individual test the observer, a university student, was not able to distinguish a smaller difference than 20 v.d. by merely listening to the tones. When he was told to hum the tones, he immediately ran down to 8 v.d. and continued to improve, reaching 2 v.d. Singing seems to enhance the power of discrimination partly on account of the timbre of the voice and partly on account of the motor elements in vocalization. "I carry the first tone over and when I hear the second I hum it to see whether I feel more or less strain in the vocal cords."

These muscular and kinaesthetic sensations are not always confined to the vocal cords. They may start in the vocal cords and spread to other organs; as, for example, "A strain starts in the vocal cords and runs up through my head." The sensations may be initiated in other organs, or they may be felt as general bodily changes. "I feel the tone as a singing in my head." "In case of the high tone, the singing is 'stronger' than in the case of the low." "The high tone seems to make a stronger impression in my ears than the low tone." By impression the observer probably means muscular strain. "I feel the tones as vibrations in the body. They seem to go all through me and cause a sort of strain." "I have a tendency to breathe more deeply for the low tones." "The high tones give me a sense of elation; I seem to mount. The low tones seem to give me an experience of gentle relaxation, a general feeling of calm." "I have a distinct tendency to move up and down according as the second tone is high or low." "Low tones seem to

drag me down; high tones seem to lift me up." "I feel an upward impulse and tend to rise with the high tone." "The mind seems to be a little more tense for high than for low tones." The affective quality of the tone is often the important element in consciousness. "When the low tone follows the high tone it seems to be more pleasing." "The high tones feel different but I can not explain the difference."

In most instances the auditory and kinaesthetic sensations combine into an auditory-vocal perception. Thus the judgment is a highly complex process conditioned by a mass of auditory and muscular sensations. The total result in consciousness, however, is a simple experience, a mark of familiarity which enables the observer to interpret the difference immediately.

In addition to sensory processes the judgment is conditioned by certain associational processes, chiefly auditory and visual images together with certain associations which are built up around these.

Many speak of carrying over the memory of the first tone and comparing the second tone with it through auditory imagery. The clearness of the image, and hence the certainty of the judgment, depends upon whether the interval is short or long. This varies somewhat with different individuals as does also the character of the imagery. Some observers associate certain familiar tones with the image of the present tone. The low tone sounds like the "hoot of an owl." The high tone is associated with the major key and the low with the minor key, or perhaps the observer imagines he hears his own voice singing the tones.

Visual imagery includes localization in space, voluminousness, and color-tone qualities. "The high tone seems to glide up at the end." "High tones seem nearer and low tones farther away." "I have a visual image of a teeter board." "High tones seem to be in the upper part of my head; low tones in the lower part." "I think of ti, do, or do, ti in the musical scale." This observer was unable to describe the tones in terms of auditory imagery. "The high tone appears to be higher up in space than the low." This reply is typical of a large number and seems to play an important rôle in the perception of difference. "The high tone has a swelling, expansive feel in the left ear and seems to have a pull upwards,—a lifting quality—almost to the point of unpleasantness in strength. The lower tone seems to be located in the right of the direction of

the head and below. The high tone is nearer the head, the low tone far away."

In the following case the method of localization is unique. The relative position of the two tones is the reverse of what is usually found. "The low tone appears to be above the high in space. It is also larger so that the two tones would be represented by a heavy above a light line, thus: "

Many observers refer the tones to a musical scale or musical instrument. M. O. thinks how she would play the violin to produce the different tones. H. S. sees her finger move up and down the violin string. O. S. says, "When I think of the second tone as higher I think of it as higher up on the piano." Another says, "I seem to see my fingers moving along the violin string."

M. C. W., a trained psychologist, locates the tone by a peculiar kinaesthetic-visual imagery. The high tones go up to the right and lie in the head, the low tones move down to the left and lie near the left side of the root of the tongue. The first sound is in the aural axis, a little to the right. All are thought of as in the head, though she knows the real source.

"High tones seem long and pointed while low tones seem big and flat." M. describes the interval as a pyramid or cone with the high tone at the apex and the low at the base. For W. "High tones are fine and sharp. They seem thin and compact: I imagine an object contracting. Low tones are relatively rich."

Colored hearing plays an important part in the judgment of some observers. Moritz Katz[4] has reported on color impressions of Schumann, Tieck, Liszt and others. In the present study the following are noted: "The high note seems to be a brighter color, the low darker." "High tones are bright and clear; low tones are dark and murky." M. always thinks of sounds in terms of color. Her impressions are remarkably complex and varied. "When I hear sounds that please me they appear violet. When I am talking with any one whose voice is pleasing, I see violet color. When I listen to a soprano solo I see a section of the rainbow. As the tones become higher they change to bright green and the very highest tones appear like little flames of fire. Low tones are reddish brown. Any rasping or disagreeable sound appears red or brown. When I hear a chorus of mixed voices or an orchestra there seems to be a large mass of violet color and from this on all sides little short

[4] Zeitschr. f. angew. Psychol., 1911, pp. 1-53.

tongues of various shades of green, yellow, and red." She does not remember when she did not translate sounds in this way. L. M., 17 years old, combines spacial and brightness qualities with tones. A very high tone appears to be a bright vertical line. As the pitch is lowered the line grows in width, but diminishes in brightness.

As regards the most favorable form of attention we have two factors, the direction of attention and the level of attention. As regards the direction of attention three modes are possible. One may attend to the beginning, the middle or the end of the tone. More than half of all observers select the middle of the tone as the critical point. The rest are about equally divided between the beginning and end of the tone. Closely connected with these modes of reaction is the snap judgment. With the organism set for a definite point in the tone, the judgment is made the instant this point reaches the focus of consciousness. This form of judgment when once brought under control almost always favors improvement.

It is also observed that it is easier to judge which tone is higher or lower if the forks are presented abruptly. If the tone swells gradually from a faint beginning, it appears to raise the pitch slightly and thus confuses the judgment.

The most favorable level of attention varies with different individuals. The introspections show that for some the closest attention to the tones is required for successful work. Others say the very keenest attention causes high nervous strain which leads to mistakes. The writer has observed this very definitely in his own case. T. F. V. says, "Much depends upon my attitude. If I hold myself in a passive attitude and answer with ease, in a reflex way, I am quite sure to be correct in my judgment; but if I get the attitude of strict attention I cannot do so well. If I can keep in a state of relaxation, I experience no difficulty in giving the judgments." Practice usually results in what Professor Welton calls receptive recognition. When one becomes familiar with tones there ceases to be that active attitude of attention which characterizes the first few tests. The two tones are not thought of separately, but the interval is grasped as a whole and is interpreted by its total effect in consciousness. The factors which enter into the judgment do not come into consciousness, but remain unconscious. All that the observer can state is that he knows the instant he hears the second tone whether it is high or low. There is no consciousness of a memory image and no comparison.

To determine the effect of distraction three series of tests were made. In the first the eyes were closed (no visual or motor distraction); in the second the eyes were kept open and allowed free movement (normal distraction); and in the third the observer was required to trace a maze while performing the test (regulated distraction).

The effect seems to be about equally distributed between helping and hindering. All but two were appreciably aided by distraction at the beginning of the series. At the close only two were especially aided and only one found distraction a hindrance. In all the other cases, when the distraction method became automatic, it ceased to influence the results. Moderate distraction seems to be an aid chiefly as a means of raising the level of non-voluntary attention. The best form of attention for a majority of observers seems to be a periodic fluctuation between sharp and instant attention to the tones and complete diversion during the interval between two pairs of tones. The problem of distraction is an exceedingly complicated one. Perhaps the most striking result of this series of tests was the demonstration that distraction enters even when we are most expected to concentrate upon a single task. Table XII shows the per cent. of right judgments with and without distraction:

TABLE XII. The effect of distraction

Observer	With Distraction	Without Distraction
1	93.9%	92.3%
2	86.2	84.5
3	89.0	91.3
4	91.2	87.4
5	90.8	87.5
6	88.7	92.6
7	86.8	91.5
8	95.8	96.3
9	92.5	91.5
10	93.3	97.3

The following extracts from introspections illustrate the general mental attitude toward the different methods. These were written at the close of the series of tests and express the observer's impression at the time.

D. A. A. "In the beginning of the test I felt distinctly dissatisfied with surroundings and was annoyed by the peculiar effect the room had on the quality of the tone. This was overcome in about twenty minutes. The maze failed to arouse interest and hence was of no assistance—quite the contrary, it was really an annoyance—a distraction unfavorable. During the time I took 100 with eyes closed I was able to inhibit any absorbing interest in anything except the discrimination of tones. I was able to concentrate

very definitely on the work in hand. Following immediately with closed eyes on a test with 0.5 v.d. I grew tired of the uniform method (eyes closed) making two mistakes in the first 50 and six in the second 50. About five of the errors were attended by a feeling of decided uncertainty and the others were caused by some annoyance. Returning to the use of the maze for the last 100, 0.5 v.d. I voluntarily renewed interest in the affair and raised my record making but four errors in the 100 judgments."

J. E. B. "At first the maze troubled me, but after going over it a number of times I could do it rather automatically so that more and more attention was given to the tones. When the eyes were open there were always numerous disturbances that would effectively distract attention. With the eyes closed there was very little to distract attention."

M. C. "Working at too great tension seemed to be my greatest difficulty during the tests, especially with eyes closed and eyes opened. The maze seems to relieve that tension though I rather expected the opposite effect. As a result of the tension I found myself confused at times and made several errors in succession. I could not notice much difference in my own attitude toward the test with eyes closed and that with eyes open. Possibly there was a greater effort to center attention on the two sounds when my eyes were open. In both of these tests there were times when I seemed to notice a difference in pitch, but could not tell which sound was higher. This difficulty seemed to disappear with the maze test."

N. E. G. "I felt that it was much easier to decide with my eyes closed than with the maze. However, my third record shows fewer errors with the maze."

P. H. H. "In the test using the maze it seemed easier to concentrate the mind; that is, the mind was concerned with two definite things: the maze and the tones, as opposed to the free associations. There was less inclination to drift to other things. The decisions in the maze test involved less conscious effort and seem to be 'felt' rather than consciously formed. Errors in the maze test often followed the effort to locate the end of the pencil line, after it was lost through the recording of the introspection. I gained better success by starting the maze line ahead of the trial."

T. F. V. "I found in this experiment that everything depended upon my attitude towards it. If I had my attention in high strain to perceive the difference between the tones and to give a correct judgment my results would be very poor. However, if I fixed my attention upon something else and gave almost passive and indifferent attention to the forks my judgments were far more certain. When my eyes were closed I attempted to focus my attention upon the retinal light and also attempted to complicate matters by means of eye-movement. That is to say, I was endeavoring to center my attention on something other than the forks. In the maze, the more intensely I worked with respect to accuracy and speed, the more clear seemed the distinction between the forks. This focusing of my attention on something else than the thing in hand was very hard to do, especially after I made one or two mistakes in close succession. If I had not been so desirous of getting correct judgments I am sure my discriminative ability would have been better."

L. E. W. "My preference is for the maze, eyes open, next and last of all eyes closed. In the latter case my mind is ever full of visual and auditory imagery, rich and prolific. One moment I am in my room and can hear the clicking of my typewriter, the next I am singing some haunting air, then I see a paper on my table I should have brought with me this morning. Sometimes I recall in auditory imagery just what the order of the last two forks was and I feel sure that I was wrong though I had unconsciously made the wrong reply. The main difficulty with keeping the eyes closed is that in so doing I can't keep a constant image or position before me; my mind refuses to remain a blank. Now, with eyes open I can fixate my eyes on some particular object and as long as this does not waver and my thoughts and attention are on the business at hand I feel secure—am so, in fact. With the maze I direct my attention to one thing continuously."

E. D. S. "Yesterday I was interested in the maze and hence was distracted by it. To-day I felt no such interest in the maze. In the test with eyes closed I became interested in the method of presenting the forks and was thinking about certain possibilities of modifying the method. This became a distraction or rather a constant object of attention and source of error."

The second function of training in pitch discrimination is the acquisition of skill in listening to musical tones. Four factors are involved. First, skill means raising the level of non-voluntary attention. The power to concentrate upon the characteristic acuteness or gravity of the tones without conscious effort usually favors correct judging and is the end to be sought in ear training. Swift found that strained attention results in distraction, and a number of observers make similar statements regarding their own experience in distinguishing tones.

Second, skill means mechanizing the conscious factors in learning to distinguish differences in the pitch of tones. The pupil has learned to image the tones as auditory, auditory-vocal, kinaesthetic, or motor qualities. In this process some one or two qualities have predominated, and the object of ear training is to form habits of listening to, *i.e.*, of thinking musical tones in terms of their dominating imagery.

The third factor in the acquisition of skill is interest. One of the chief aims of ear training should be to enlist the pupil's interest in the appreciation of musical tones and the enlargement of the scope of apperception with reference to isolated tones.

The physiological limit

The physiological limit is undoubtedly considerably lower than is indicated by the threshold which would give 75 per cent. right

cases, as here used. To demonstrate this and, at the same time, to observe the significance of the choice of a particular increment in the homogeneous method, measurements were made on seven good observers whose threshold had been recorded as being in the neighborhood of 1 v.d. Four tests were made on each of the seven observers at 1, 0.5, and 0.25 v.d. with 200 judgments at each unit in double fatigue order, or a total of 800 judgments at each unit. From the per cent. of right judgments the probable threshold with 75 per cent. right cases was computed by the Fullerton-Cattell formula.

Table XIII shows the difference threshold which was required to give 75 per cent. of right judgments for 1, 0.5, and 0.25 v.d. respectively.

TABLE XIII

No.	1 v.d.	0.5 v.d.	0.25 v.d.
	v.d.	v.d.	v.d.
1	.42	.44	.49
2	.40	.27	.23
3	1.30	3.33	1.30
4	1.10	1.47	1.10
5	1.75	1.47	1.30
Average	.82	.74	.60
6	1.00	1.00	
7	1.54	1.30	
Average	1.24	1.17	

The first five observers had more than fifty per cent. of right judgments at 0.25 v.d.; hence the threshold is calculated for the three increments of the other cases. Number 6 got only 49 and number 7 only 46 per cent. right cases on the 0.25 increment. But the significant fact is that for both of these persons the number of right judgments on 0.5 v.d. was such as to give practically the same threshold as was found on 1.0 v.d. Only one important inconsistency occurs in the above table. In the case of No. 3. the right judgments at 0.5 give a threshold of 3.33 v.d. while at 1 v.d. and 0.25 v.d. the threshold is exactly the same.

Examination of the table therefore proves that in the region of the average physiological limit the conventional threshold may be computed on the basis of observations considerably below that limit (here in five cases out of seven) and that the actual physiological limit is always considerably below the conventional threshold. This is, of course, analogous to what we find in sight; under exception-

ally favorable circumstances we may see a small, well defined object at a distance which, from the nature of the dioptric system, represents the physiological limit of acuity in vision but average records of acuity would ordinarily designate a point short of that distance.

Correlations

From the standpoint of musical training it is important to know how the ability to distinguish differences of pitch is correlated with other mental characters, as general intelligence and singing ability. In addition to these we wish to know whether brothers and sisters are more closely correlated in ability to distinguish differences of pitch than other children not related. These questions are discussed in their order.

For the purpose of the correlation between pitch discrimination and general intelligence and singing ability, the data for pitch discrimination were obtained from the final days of the practice series. No single absolute measure of general intelligence is possible. For the present purpose the teachers were instructed to mark general intelligence on the basis of two criteria, brightness and reliability, assuming these to be of equal weight. By brightness is meant quickness and accuracy of mental grasp, or, in other words, general wide-awakeness. Reliability is self-explanatory. It is the correlate of a small mean variation for daily work. For convenience of marking, these two factors may be considered as having equal weight and may, therefore, be marked independently on a scale of 10. It was explained that the markings should follow approximately the normal distribution for each age and for both sexes. The mean of the two marks was taken as the mark representative of intelligence.

In order to facilitate correlation the ten units in the series of increments used in pitch discrimination were translated into corresponding values on the scale of 10, thus: 30 v.d. corresponds to 1; 23 to 2; 17 to 3; etc. .... 0.5 v.d. to 10.

The markings on singing ability were also based on the teacher's judgment of the pupil's ability to sing correctly in pitch scale and a melody.

As regards kinship, three correlations were as follows: (1) between younger and older brothers and sisters with practice; (2) the same without practice; and (3) between the younger members

of the second correlation and other children of the same age and sex as the second members, but not related.

The correlations were determined by the Pearson product-moments method. In order to show the relative distribution of individuals for each factor correlated, each group is subdivided into five grades. This is not a quintile subdivision as there is no attempt to have an equal number of persons in each subdivision. The distribution by grades serves the purpose of comparison quite as well as the quintile or quartile method and avoids the necessity of ranking, which is practically impossible on a scale of 10 units. The method of subdivision is very simple. The scale of 10 units is is divided into five equal parts. 1 and 2 = E. 3 and 4 = D. 5 and 6 = C. 7 and 8 = B. 9 and 10 = A. An example will make clear the method. An observer gets 3 in pitch discrimination and 7 in general intelligence. He belongs to Grade D in the first factor and in Grade B in the second factor. The number who are in the same grade in each factor indicates the degree of correlation. The number who are in different grades in the two factors indicates lack of correlation or low correlation.

The results show a relatively high coefficient of correlation between pitch discrimination and general intelligence, singing ability and musical training (Tables XIV and XV). It is higher for boys than for girls and highest for both boys and girls between pitch discrimination and general intelligence.

TABLE XIV. Correlation of pitch discrimination with general intelligence and singing ability

Pitch discrimination with

(1). General intelligence	Boys	r	.70	p.e.	.023
	Girls	r	.63	p.e.	.026
(2) Singing ability	Boys	r	.71	p.e.	.023
	Girls	r	.51	p.e.	.031

TABLE XV. A. Pitch discrimination and general intelligence

Boys — Intelligence

	234	A	B	C	D	E
	A	4	15	16	3	
	B	6	31	29	3	1
Pitch	C	3	25	33	11	
	D	1	8	19	8	
	E	1	4	4	9	

Girls — Intelligence

	274	A	B	C	D	E
	A	6	14	14		
	B	12	51	27	3	
Pitch	C	11	49	37	12	
	D	4	3	17	4	
	E		1	6	3	

B. Pitch discrimination and singing ability

Boys

	234	Singing A	B	C	D	E
	A	5	7	3	2	
	B	17	22	13	8	4
Pitch	C	4	24	23	12	7
	D	2	13	16	8	6
	E	1	4	15	8	11

Girls

	274	Singing A	B	C	D	E
	A	11	21	12	1	
	B	8	46	38	10	2
Pitch	C	8	27	39	16	
	D	3	8	16	2	3
	E					

The fact of a high correlation between pitch discrimination and general intelligence favors the conclusion reached above that pitch discrimination depends partly upon the ability to learn, *i.e,* upon brightness and reliability. If this is a correct view, training in pitch discrimination is essentially mental training. It is more than reproducing tones; it is thinking tones. Another conclusion which is in harmony with what has just been said is that a child may possess a perfect ear for tones, and still be unable to distinguish differences in pitch. Musical training should begin with training in tone quality.

The coefficient of correlation between pitch discrimination and singing ability is technically high. A high correlation between these factors means that the ability to distinguish differences in the pitch of tones is an essential factor in learning to sing.

Table XVI shows that, for the groups compared, girls are superior to boys in pitch discrimination, since there are no girls in Grade E and relatively few in Grade D. But they are not shown to be essentially superior in singing ability.

TABLE XVI. Correlations for blood relationship

Correlation between pitch discrimination of

(1). Brothers and sisters:				
(a) with practice	r	.48	p.e.	.031
(b) without practice	r	.43	p.e.	.035
(2). Children not related	r	.53	p.e.	.030

The coefficient of correlation between brothers and sisters on the basis of ability in pitch discrimination is not higher than between other children. This is true both for records without practice and records after practice. Although the results are clearly negative, no sweeping conclusion should be drawn because several variables are involved, such as advantage of the knowledge which comes with age, differences in intelligence, the element of competition, etc. This is regrettable since it had been definitely hoped and planned

that this large collection of data might contribute to the solution of this interesting question. Finally, the younger member of each pair in the second correlation was compared with another child of the same age and sex as the second member, but not related. The coefficient of correlation is practically the same for the three groups. (Table XVII). No conclusions can be drawn from these meager results as regards the influence of heredity on tonal hearing.

TABLE XVII. Correlation of pitch discrimination for younger and older brothers and sisters

(1) With practice

		Older				
	129	A	B	C	D	E
	A	7	5	1		1
	B	9	32	12	5	1
Younger	C	4	17	7	4	2
	D		9	2	3	
	E		3	2	2	1

(2) Without practice

		Older				
	275	A	B	C	D	E
	A	2	13	12	7	1
	B	2	6	17	12	1
Younger	C	12	12	54	27	10
	D	7	8	25	32	5
	E	1	1	2	4	2

(3) Children not related and without practice

		Older				
	275	A	B	C	D	E
	A		4	2		3
	B	6	22	28	12	8
Younger	C	6	38	36	15	9
	D	2	15	23	4	2
	E		13	16	4	7

General conclusions

The quantitative statement and analysis of data has been presented in such condensed form that a summary of conclusions from that point of view is scarcely necessary. There is, however, need of a statement of "general conclusions" from the point of view of interpretation and application of the experimental results in the light of the quantitative data, the introspections of the observer, the daily notes of the experimenter, and a general study of the problem with the collaborators in research. Such a statement necessarily involves something of a personal equation and I am glad to acknowledge in this the co-operation of Professor Seashore whose long and varied experience in this field of research makes this interpretation possible.

The psychological limit in pitch discrimination is always below the conventional threshold (75 per cent. right cases). Thus, a person whose threshold is 1 v.d. may, under extraordinarily favorable circumstances, hear as small a difference as .25 v.d.; and it is probable that in the normal unreflective and uncritical appreciation of music the automatic "impression" of tone differences comes freely through this region of increments which are below the conventional threshold. This conventional threshold which can not be further reduced by instruction or training we have called the "approximate" physiological threshold. This is the concept of threshold that must be employed for most purposes of research and in nearly all applications of the test for practical purposes. The three factors which differentiate it from the true physiological threshold are—the convention of counting 75 per cent. right cases, the physical variation in the organ of Corti, and the failure to keep all the conditions of the measurements under control.

Success in making a true measurement on an unexperienced observer in a single sitting varies with the knowledge, keenness, and care of the observer and the many objectively favorable or unfavorable conditions of the test as well as the experimenter; but, everything taken into account, it is safe to say that when an individual test is made under favorable conditions the approximate physiological threshold may be reached in a single sitting of less than an hour for more than half of the cases of adults or children who are bright and old enough to understand the test. Even in group tests by the heterogeneous method one may reach in an hour the approximate physiological threshold of nearly half of the observers who are old enough and bright enough to observe.

A cognitive threshold, always above the approximate physiological threshold, may be due to failure in understanding what is required in the test, lack of information, defect in auditory imagery and memory, lack of application, confusions, objective or subjective disturbances, expectations, inhibitions in writing or speaking, etc. Most of these conditions are such that they may be removed by information, by inducement to use the best effort, or by learning through some experience.

There are means of determining when the approximate physiological threshold has been reached. Chief among these are the mean variation and the character of the distribution of the errors. But in

individual tests many direct observations on the character of the difficulties in judging may be helpful. In general, where a record is low (good) the chances are that the observer has no "cognitive" difficulties. The uncertainty is, of course, always with reference to the poor record. Practical advice or recommendation should therefore be cautious in the case of poor records for fear that the limit reached, although persistent, may be merely cognitive. One can not err on the side of getting too good a record; the danger is always that something has prevented a fair test of actual ability.

The sensitiveness of the ear to pitch difference can not be improved appreciably by practice. There is no evidence of any improvement in sensitiveness to pitch as a result of practice. When a person shows a cognitive threshold practice ordinarily results in a clearing up of the difficulties which in the way of a true measure of discrimination by information, observations, and the development of interest, isolation of the problem in hand, and more consistent application to the task in hand. This is, of course, not improvement in the psycho-physic ear but merely a preliminary to a fair determination of the psycho-physic limit. It follows that instruction in regard to the nature of the test and individual help are all important for the lowering of the cognitive limit and that mere practice for this purpose is a poor and uncertain makeshift. It also follows that a "cognitive" threshold is no measure at all but rather a confession that the measurement has not yet been successfully made.

Training in pitch discrimination is not like the acquisition of skill, as in learning to read or to hear overtones. It is in the last analysis informational and the improvement is immediate in proportion to the effectiveness of the instruction or the ingenuity of the observer and the experimenter in isolating the difficulty.

Reduced to its lowest terms the question of variation with age may be interpreted to mean that we have no evidence of improvement in the psychological limit of pitch discrimination with age; a young child of school age and even younger, can hear pitch fully as keenly as an adult. The amount in favor of the adult shown in all group statistics is amply accounted for by the difficulty in making a reliable test on the young and by their lack of information. This statement is based primarily on two lines of evidence,—the common occurrence of fine, irreducible records among young children, and the character of the conditions which are ordinarily overcome by instruction and training.

Pitch discrimination does not vary with sex to any significant extent. In the records here reported and in the many hundreds of other records in this laboratory in which comparisons may be made for sex, certain tendencies are shown in groups of records, sometimes in favor of one sex and other times in favor of the other sex, but on the whole, it seems certain that such differences, except so far as they are due to grouping, may be accounted for as due to the conditions of the test rather than to the sex difference in the psycho-physic capacity of pitch discrimination. Thus one of the most consistent and striking differences reported above, that of the superiority of elementary schoolgirls over elementary schoolboys may probably be fully accounted for by the prevailing trait of aloofness of the preadolescent boy toward music. These boys often regard music as a sort of frill for girls and, therefore, enter the test with less fervor than do the girls. Such interpretation is supported in part by the fact that in the high school and in the university, where the girls have had far more advantage of training than the boys, the records reveal no appreciable difference for sex.

Not a single case of tonal deafness was isolated in any of the records here reported. This would indicate that if tonal deafness exists at all in a "normal" ear, it is no so common as has usually been supposed.

We have found a high correlation between pitch discrimination and ability in singing, as judged by teachers. In the collective records there is also a high correlation between pitch discrimination and "general intelligence." This is undoubtedly due to the presence of so many "cognitive" as opposed to physiological thresholds.

Under the conditions of this test the records of members of the same family do not correlate more closely than do members of different families.

This test is elemental, *i.e.*, when applied under favorable conditions it calls forth a relatively simple and immediate sensory discrimination which does not improve appreciably with practice. It is like the minimum visible angle in visual space—the limit is set by the sense organ. We say "under favorable conditions" because the cognitive factors which condition a fair test must be recognized. As has been seen in a large per cent. of cases, we can get only a cognitive threshold in the first attempts. As elemental, this test is contrasted with, *e.g.*, a test of ability to isolate overtones

in a violin tone which represents a skill that can only be acquired through practice. It must be recognized that the test is a true and successful test, the results of which may be applied with safety, only as it is actually elemental.

The basal character of pitch discrimination in the appreciation and expression of music has become evident in many ways. Keen recognition of pitch difference is a condition of auditory imagery, auditory memory, singing or playing in true pitch. This is true as well for the affective attitudes with reference both to pitch and to timbre, for timbre is in the last analysis simply a pitch complex. It would therefore seem to be most fundamental of all tests of musical talent, although, of course, no one test by itself can be considered an adequate measure of such talent.

The educational value of this test has been strongly impressed during this work. It is unquestionably the isolation and measuring of one specific, basal factor in musical talent. It may be undertaken individually or in groups and commends itself particularly as one of the tests that should be made in schools for the purpose of vocational guidance in music, in the music studio for the purpose of learning where to place the emphasis in instruction and in adapting the course to the natural capacities of the student, and as a recurrent exercise in the schools and in the studio for the purpose of developing keenness in attention to detail of tone in ear training.

The instruments, *i.e.*, the tuning forks and resonators as here used, and the method, both the heterogeneous and the homogeneous procedure, have proved eminently satisfactory.

THE LOWER LIMIT OF TONALITY

BY

THOMAS FRANKLIN VANCE

An accurate determination of the threshold of the lowest audible tone involves a consideration of the variables which condition it. The area and the amplitude of the wave and the distance of the vibrating body from the ear of the observer are the principal objective variables. Individual differences, due largely to innate capacity, degree of practice, and ability to concentrate attention, and variations within the same individual which may be attributed to changes in physical tone and mental content, are obviously the most influential subjective variables. These variables, both objective and subjective, present particular problems which must be considered in their relation to the general problem of the lower tonal limit before the latter can be accurately determined.

No attempt will be made here to review the history of investigation on this problem. A good summary is found in Titchener's Instructor's Manual, Quantitative.

Mr. Misao Imai made a careful study of this problem in this laboratory in 1907. Inasmuch as his results have not been published and the present study is essentially a repetition of his work for the purpose of verification it is necessary to report his work in brief.

Mr. Imai's first problem was to determine the relation between the threshold and the amplitude of the wave. He produced the tones by an electro-magnetic fork 460 mm. in length and 10 mm. by 20 mm. in cross section of a prong. By differential weights five tones could be produced, namely, 35, 25, 22, 19, and 17 v.d. By varying amount of resistance different amplitudes could be secured. The test in each case consisted in determining the smallest amplitude that would produce an audible tone at a given pitch. The measurements were made on ten laboratory students. With this apparatus, he obtained the results shown in Table I.

TABLE I. The relation of threshold to amplitude

v.d.	ampl. in mm.	m.v.
30	1.30	.15
25	1.75	.30
22	2.20	.45
19	2.95	.50
17	3.45	.50

From these results he drew the conclusion that the threshold varies inversely as the amplitude; *i.e.,* increase of amplitude lowers the threshold.

With the same apparatus he conducted a second series of experiments to learn the relation between the distance from the ear of the vibrating body and the threshold. From these results he concluded that the distance at which the fundamental tone is just perceived, varies with the pitch; *i.e.,* the higher the pitch, the greater the distance may be, within given limits. Below 18 v.d., however, he found the distance uncertain as the overtones were frequently confused with the fundamental.

It then occurred to him that the area of the vibrating body might have an important bearing upon his general problem. He varied the area by means of four pairs of discs 6 mm. in thickness with diamaters of 6, 8, 10, and 12 cm., respectively, which could be attached to the ends of the prongs of a fork similar to the one described above. With the variable of area thus controlled, he learned that it must always be given due consideration in the determination of the lower limit of tonal hearing. Judgments from ten highly practiced observers showed clearly that the threshold varies inversely as the area, within limits.

Investigators previous to Imai had used three different methods in the production of tone; namely, (1) vibrations of tuning forks, pipes, and reeds; (2) difference tones; and (3) interruption tones. Helmholtz, Stumpf, and Preyer favor the use of tuning forks. Where forks have been used the thresholds are, as a rule, noticeably lower than where other means have been employed. Schaefer views with suspicion all thresholds reported under 16 v.d., inclining to the belief that perceptions below that point are conditioned by overtones rather than by fundamentals. He doubts von Bezold's assertion that the fork by means of which he registered a threshold of 11 v.d. was free from overtones. Schaefer is doubtless in the right in his contention that von Bezold has not proved this point conclusively. Von Bezold's statement that the very low tone 11 v.d. was perceived by some observers with normal hearing, cannot be accepted unqualifiedly. In fact, the statement would have been more convincing had he admitted the probability of overtones. Unless an observer realizes the possibility of an overtone and is cautioned to discriminate between it and the fundamental, he will base his judg-

ment upon the first tone perceived which below 18 v.d. will always be an overtone, if the tone is not pure. Preyer reported the very conservative threshold of 18.6 v.d. Wundt's threshold of 14 v.d. with an Appunn reed and 16 v.d. with a wire fork undoubtedly held true with good observers. They compare very favorably with the results of Imai who found the threshold to be about 16 v.d. with the plain forks.

Imai, then, by an increase in area obtained a lower threshold than those recorded by the most reliable previous investigators. His observers who had a threshold of 16 v.d. with the plain fork were enabled to hear 12 v.d. with the most favorable size of disc-forks. This lowering of the conventional threshold of tonality by about 4 v.d. was of such importance as to justify a verification and extension of his experiment under most accurate conditions of control.

In the investigation here reported tuning forks were used for the production of the tone because they are among the best means for producing relatively pure tones, and there seemed to be no other way to change the area of the vibrating surface without at the same time altering the rate of vibration.

Two electro-magnetic forks were arranged tandem. The driver fork was constructed of soft steel and measured 65.5 cm. in length from the base to the tip of the prongs which were 1 cm. by 2 cm. in cross section. A series of eleven holes, 1 cm. in diameter and 2.8 cm. apart, bored in each prong made the fork lighter and more active. The fork hung vertically from a support by means of a large iron hook securely bolted to its base. The driven fork was made of soft steel rod and measured 57 cm. in length. The prongs were 1 cm. in diameter. By means of differential weights, vibration rates ranging from 18 to 12 v.d. could easily be secured. Three pairs of discs of fibre 6 mm. in thickness and 2.5 cm., 5 cm., and 10 cm. in diameter respectively, served a double purpose; they not only afforded the desired variation in area, but also, served as weights for the tuning of the fork. The two forks were tuned to the same pitch. The driving fork was kept at a distance, thus causing the driven fork in the observing room to vibrate without the disturbance of an interruption spark and sound. The amplitude of vibration was varied by means of variation in the strength of the energizing current through an adjustable resistance coil in the experimenting room.

All the experiments were conducted in the sound-proof room. Beginning with 18 v.d., the series followed a descending order until the threshold was reached. Each rate, however, was observed with the three different areas of vibrating surface as controlled by the discs of 10, 5, and 2.5 cm. in diameter. The observations were completed at each step in the order named before the forks were adjusted to the lower rate. The observer sat in a comfortable position with one hand on the switch which controlled the current through the driven fork, and waited attentively for the tone. The experimenter, after a warning signal, presented the fork with the central part of the disc exactly opposite the opening of the more sensitive of the two ears and as close to it as the amplitude of vibration would permit. After a very brief period of observation, with the current either on or off as desired, the observer immediately reported "tone" or "no tone". The observer's judgment thus determined the amplitude of the fork in the next presentation. When very near the threshold of intensity the experimenter recorded the amplitude of vibration upon smoked paper. This method was followed until the least intensity of tone which the observer could possibly hear, for a given rate and area, was reached. Each period of observation lasted one hour; during that time data for each disc at two different vibration rates could usually be obtained. Three complete series from 18 v.d. to the threshold with the three different discs were secured for each observer.

The nature of the experiment demanded that only experienced observers be employed. Ten members of a class in experimental psychology were available. Each of these submitted to an eight hour course of training which was divided into periods of two hours each. From this group of ten, the experimenter chose three whose work in the preliminaries was most satisfactory to act as observers throughout the entire series. The demonstrator of this class and the experimenter brought the total number of observers up to five, all of whom had had considerable training in psychological experimentation.

Before turning to the consideration of the results, a brief statement of the difficulties encountered in a study of this kind should be made. That there are difficulties may be inferred from the fact that different investigators have placed the threshold at points ranging from 8 to 30 v.d. Low tones are intrinsically weak and

require much more energy for their production than do the high tones which are relatively strong and clear. While little effort is required in sensing a tone in the normal range, it becomes a matter of strict attention to perceive the tones near the limen where the predominant overtone may be confusing. It is not easy to procure a perfectly pure tone in the central rigister where the instrument used is small, but it is next to impossible to produce an absolutely pure tone with the large forks of the lower limit. Furthermore, we are not accustomed to pay attention to liminal tones, therefore, when a person is asked to observe them under laboratory conditions, he is at first at a loss to know what to listen for. It takes hours of training before the individual feels the necessary degree of certainty in giving his judgments. But patience and practice tend to make him reasonably certain of his judgments. The development in accuracy of observations may be traced in the characteristic expressions, "I hear something!", later, "I think I hear the fundamental!" and finally, "I am sure I hear the fundamental!"

TABLE II. Average results of M.C.C.

	10 cm.		5 cm.		2.5 cm.	
v.d.	Ampl. in mm.	m.v.	Ampl. in mm.	m.v.	Ampl. in mm.	m.v.
18	.13	.04	.40	.13	1.33	.51
17	.23	.04	1.07	.31	2.93	1.18
16	2.00	2.00	3.43	1.05	5.50	2.20
15	2.23	1.78	3.43	1.91	5.33	1.98
14	3.07	1.76	3.50	3.33	9.80	4.20
13	5.23	1.96				

TABLE III. Average results of C. B.

	10 cm.		5 cm.		2.5 cm.	
v.d.	Ampl. in mm.	m.v.	Ampl. in mm.	m.v.	Ampl. in mm.	m.v.
18	.23	.18	.90	.47	2.20	.93
17	.43	.18	1.27	.49	3.27	.18
16	.87	.29	1.97	.38	4.00	.67
15	1.40	.13	2.63	.04	2.57	.42
14	2.33	.58	3.07	.29	5.60	.53
13	2.87	.91	4.53	.49		
12	4.50	.67				

TABLE IV. Average results of F.O.S.

	10 cm.		5 cm.		2.5 cm.	
v.d.	Ampl. in mm.	m.v.	Ampl. in mm.	m.v.	Ampl. in mm.	m.v.
18	.47	.24	2.47	.78	3.13	.62
17	1.23	.73	3.07	.18	5.07	1.29
16	1.60	.73	4.03	1.24	6.50	.87
15	3.43	3.35	4.60	1.20	7.37	.78
14	3.73	.79	6.00	1.67	9.00	2.33
13	3.70	.53	4.07	.89		
12	3.77	1.34				

TABLE V. *Average results of L. E. W.*

	10 cm.		5 cm.		2.5 cm.	
v.d.	Ampl. in mm.	m.v.	Ampl. in mm.	m.v.	Ampl. in mm.	m.v.
18	1.13	.57	1.77	.38	2.43	.78
17	1.50	.20	2.30	.60	3.47	1.51
16	2.73	.84	3.23	.62	5.33	.58
15	2.70	.20	4.63	.91	7.00	1.00
14	4.17	1.42	4.13	.24	6.40	1.40
13	5.40	2.13	7.07	.76		
12	6.10	.60				

TABLE VI. *Average results of T. F. V.*

	10 cm.		5 cm.		2.5 cm.	
v.d.	Ampl. in mm.	m.v.	Ampl. in mm.	m.v.	Ampl. in mm.	m.v.
18	.67	.04	1.13	.24	1.90	.27
17	1.23	.18	1.90	.13	3.13	.58
16	1.27	.18	2.40	.27	3.87	.44
15	1.77	.18	3.60	.27	5.50	.00
14	3.17	1.22	5.53	1.29		
13	5.67	.44				

TABLE VII. *Average results of five observers*

	10 cm.		5 cm.		2.5 cm.	
v.d.	Ampl. in mm.	m.v.	Ampl. in mm.	m.v.	Ampl. in mm.	m.v.
18	.54	.08	1.33	.24	2.20	1.05
17	.93	.09	1.92	.21	3.57	.72
16	1.69	.73	3.01	.58	5.04	.60
15	2.31	.33	3.78	.61	5.55	.52
14	3.29	1.03	4.45	1.16	7.70	.90
13	4.57	.96	6.37	2.68		
12	4.78	.37				

The quantitative results of this series of experiments are expressed in Tables II-VII and Fig. 1. The records show that, within the limits here studied, the larger area offers more favorable conditions for the perception of the lowest audible tone. To be more specific, the threshold varies inversely as the area of the vibrating surface; *i.e.*, the greater the area, the less the frequency of vibration, or the lower the threshold may be. In all of the curves, whether of individual series, or of averages of series, this law is demonstrated. In every case the frequency of vibration is less with the area of 10 cm. than with the area of 2.5 cm. The frequency of vibration with the area of 5 cm. usually falls between that of the area of 10 cm. and the area of 2.5 cm., but occasionally it is equal to one of these, but even then there is a decided difference in amplitude. The results show that the average threshold for the first five observers lies between 15 and 14 v.d. with the smallest area,

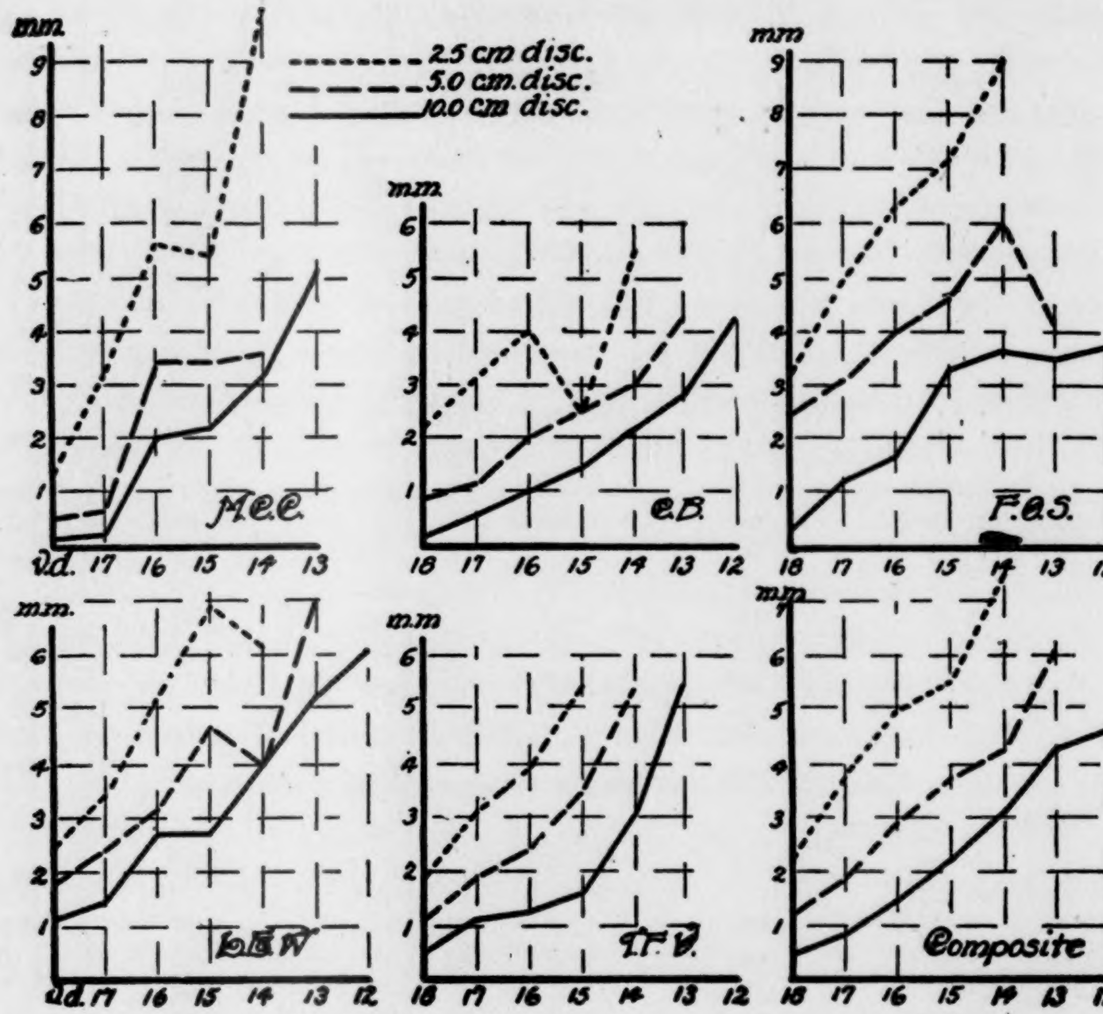


Fig. 1. (Tables. II-VII.)

between 14 and 13 v.d. with the middle area, and between 13 and 12 v.d. with the largest. This is as accurate a determination as can be made from the data secured. There are, obviously, certain judgments which cannot be accepted at their face value. The average results of F. O. S., as shown in Figure I, may be taken as an illustration. In this case there should be much hesitancy in placing the threshold at 13 v.d. for the middle area inasmuch as the amplitude required is considerably less than that required for 14 v.d. The drop in the curve for the smallest area for L. E. W. may be interpreted in a similar way. In either of the instances cited there is, however, the possibility that the observers actually did perceive a tone, the small amplitude being the result of better attention.

Within limits, then, the threshold is lowered by increase in area. This limit, under the conditions of the experiment, is approximately 10 cm. Results secured by Mr. Imai with a disc of 12 cm. in diameter are no better than those secured with a disc of 10 cm. Furthermore, at the beginning of my own investigation, the area was

increased to 20 cm. but the conditions were in no way improved by this still larger area.

Again, the threshold, as measured in terms of frequency of vibration, varies inversely as the amplitude of vibration; in other words, increase in amplitude lowers the threshold, within limits. It requires a greater amplitude to produce an audible tone of 17 v.d. than it does to produce one of 18 v.d. And so the amplitude gradually increases until the threshold is reached. Thus the curves have an almost uniformly ascending trend. Variations in attention offer the most probable explanation for the exceptions that occur in the lower part of the curves, but the confusion of the overtone with the fundamental is the most frequent cause of the irregularities in the upper part.

Mr. Imai conducted two sets of experiments on the variable of distance. From the first of these he deduced the law that, within limits, the maximal distance at which the fundamental tone is just perceptible varies directly as the pitch; *i.e.*, the higher the pitch the greater the distance it can be heard.

In the second series he used two different discs to vary the area, one 12 cm. in diameter and the other 6 cm. The difference in distance which resulted from this difference in area he found very slight,—not more than 1 mm. But this difference, together with incidental observations recorded throughout the course of the whole experimentation, led him to the conclusion that, within given limits, the maximal distance at which a threshold tone could be perceived varies directly as the area of the vibrating surface. Both of the above conclusions have been verified by our own experimental procedure.

The variables of a subjective nature encountered in the attempt to determine accurately the lower limit of tonality are not different in kind from those which are met in experimenting upon any other part of the tonal range. But the fleeting, momentary character of the low tones makes it extremely difficult to hold them in clear consciousness. Certain of the subjective variables, therefore, become more disturbing in the lower register than they do in the middle register. Every precaution must be taken to control them. Individual differences and variations within the individual have a vital bearing upon the problem.

Practice is one of the most significant variables but fortunately

it can be easily controlled. The difficulty of the task set the observer necessitated in every case a rather high degree of practice. When the series of experiments opened the observers had first to learn to distinguish between fundamental and overtone, and later, in the case of very low tones, to distinguish between fundamental and puffs of air with comparative ease and certainty. The records of three of the observers show clearly the effect of practice also in the lowering of the record. Musical education undoubtedly lessened the amount of special training required. Mr. Imai observed this also in his experiment. A few members of the Minneapolis Symphony Orchestra were given a series of these tests and nearly all of them heard the tone of 12 v.d. This superiority of musicians may be due to a selection—to a musical nature rather than to training, although both count.

Fatigue can easily become one of the most vitiating factors. Its effect is both physical and mental. It has been demonstrated in this laboratory that tones in the middle register may be listened to attentively for two hours without a disturbing effect. But low tones fatigue the ear very quickly and when thus incapacitated the power of analysis deteriorates and the determination of the threshold therefore becomes uncertain. Furthermore, it demands the closest attention on the part of the observer to sense the fundamental tone. There is, of course, a strain of attention and when mental fatigue occurs sensations and images of tones may be confused. It is highly essential, therefore, that the experimental series be of short duration and that it should be interrupted by frequent periods of rest.

The observers had a strong tendency to give the tone a definite location in space. One or two placed it in the back part of the head, very near the neck. One said that when he heard both the fundamental and the first overtone, the fundamental had a lower place in space than did the overtone. Another maintained that the first tone heard was usually an overtone and seemed very close to the ears, but the fundamental seemed to come out of the darkness from some place lower than the ears.

The larger area produced by the attachment of the discs caused puffs or whiffs of air to occur in connection with the tone. The larger the area, the more noticeable they are. Very early in the experimentation the puffs confused the observers, but after practice

they were scarcely noticed until the threshold had been reached when nothing but the puffs remained. They bear the same relation to the lowest audible tone that the thud, produced by striking the mallet against the König cylinder, bears to the highest audible tone. The observer ignores the thud until the threshold has been reached.

With three observers the fundamental bore a temporal relation to the overtone. The overtone was nearly always heard first, then the fundamental came out gradually from under the overtone, remained for an instant and then disappeared leaving nothing but the puffs. One observer said, "The fundamental arises as a faint impression in the ear and dies out quickly. It seems to emerge from beneath the overtone while the latter ceases. It does not come to consciousness suddenly, but gradually."

Low tones are intrinsically weak; therefore any factor that would increase the energy of the wave motion would tend to make a low tone more audible. Distance, *i.e.,* nearness to the ear, does this; area does also. In so far as mere audibility of the tone is concerned, amplitude, nearness, and area are the chief factors. But the element under observation is fusion of the individual vibrations into a tone. What has been brought out most clearly in this investigation is that the limit of fusion is set not only by vibration frequency but by the factors just named. Their total effect should be interpreted primarily in terms of the *form* of the wave. Nearness, large amplitude, and large area all unite to form a continuity of the wave. When the waves which impinge upon the tympanum assume the form of the sine curve we have probably the most favorable conditions for that fusion which is the essence of tonality.

In summary, the factors which must be taken into account in the accurate determination of the lower limit of tonality are of two kinds, the objective and the subjective. The objective factors are four: namely, area, amplitude, distance, and timbre. It has been shown (1) that the threshold varies inversely as the area of the vibrating surface; (2) that the threshold varies inversely as the amplitude of vibration; (3) that the amplitude of vibration varies inversely as the area of the vibrating surface; (4) that the maximal distance at which the fundamental tone is just perceptible varies directly as the pitch; and, (5) the maximal distance at which a threshold tone can be perceived varies directly as the vibrating surface. The subjective variables are in general the same as those

encountered in other experiments in audition. They are, differences in innate capacity, in degree of practice, in ability to concentrate attention and in mental content. The wide variation in threshold-values, as determined by the different investigators, may be accounted for by the lack of control of some of these variables.

VARIATION IN PITCH DISCRIMINATION WITHIN THE TONAL RANGE

BY

THOMAS FRANKLIN VANCE

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HISTORICAL STATEMENT

The object in this brief historical survey is to place before the reader only those results which are most closely related to the present problem with respect to the methods used and the aspects investigated. Therefore, investigations made with similar methods but having only a narrow field, as well as those exploring a large range by different means, will not be discussed.

Preyer may be considered the pioneer worker on the problem of pitch discrimination within the tonal range. Earlies investigators concerned themselves with the least perceptible difference at only a single point in the register. Delazenne, (1) in 1827, used a metal string 1147 mm. in length with a vibration rate of 60 v.d., which he divided into equal parts by a bridge. He found that if the bridge was removed only 1 mm. from the central position a difference of pitch could be detected by well trained ears when the two parts of the string were sounded in succession. Weber (20) declared that he was able to determine the pitch of tones so accurately through the ear that he never made an error of more than one vibration on a tone of 200 v.d. He thought it possible that this keen discrimination

might be due to beats. Sauveur (9) with two monochords tuned to the same pitch found that when one string was shortened by 1/2000 of its length a difference in the pitch of the two tones was recognizable, but he leaves no record as to the pitch of the tones. Schleiber (12) recorded a differential threshold of less than .5 v.d. on the tone of b. Seebeck (14) and two superior violin players differentiated without fail between two tuning forks vibrating at 1209 and 1210 v.d.

Preyer.—Preyer (7) used the Appunn tonmesser,—an instrument consisting of reeds which gave the following tones: from 500 to 501 in steps differing by 0.1 v.d., 504, 508, 512, 1000 to 1001 in steps of 0.2 v.d., 1008, 1016, 1024, 2048, and 4096 v.d., respectively. More than a thousand judgments as to whether the two tones compared were equal or different, were secured from twelve practiced observers. They were not required to tell the direction of the difference. To his own results he added those of Delezenne and Seebeck. Transcribing Delezenne's measurement in millimeters into terms of frequency of vibration, he found that the shorter string produced a tone of 120.2 v.d., while the longer one gave a tone of 119.8 v.d., thus making a difference of 0.4 v.d. which skilled observers could detect. Likewise, a simple computation revealed the fact that Seebeck had the very low threshold of 0.36 at 449 v.d. After some verification of Seebeck's results, Preyer inclined to the belief that the threshold might be brought as low as 0.25 v.d. The combined results give differential thresholds for four different places in the tonal range as follows: Delezenne at 120 v.d. found the threshold to be 0.4 v.d., Seebeck at 440 v.d. found it to be 0.4 v.d., Preyer at 500 v.d., 0.3 v.d., and at 1000 v.d., 0.2 v.d. From this study, inadequate as it may seem, Preyer draws the following conclusions:

(1) One-third of a vibration on 500 v.d. and five-tenths of a vibration on 1000 v.d. will always be recognized as different by the best observers, although the most sensitive, the most trained, and the most reliable ears tested could not recognize a difference of 1000 and 1000.25 v.d. nor of 500 and 500.2 v.d. (2) Very high tones and very low tones cannot be discriminated so accurately as tones of the middle region. Capacity for discrimination is keen between c and c^3, keenest for the region from a^1 to c^2, but beyond c^3 it decreases slowly until it becomes very unreliable at c^5. Fis^4 marks a second point of keenness of capacity. (3) The relative difference for pitch is dependent, in a high degree, on the number of vibrations

of the compared tones; and the absolute difference-sensitiveness does not decrease with the pitch. (4) The judgment concerning the place of tones in the tonal line is more uncertain than the judgment as to whether the two tones lie at different points. (5) Practice is an influential factor in pitch-discrimination. Extreme fineness of capacity is peculiar only to those who have much familiarity with tones.

Luft.—In the Leipsic laboratory, during the years 1884 to 1886, Luft (4) experimented upon a range of tones extending from 64 to 2048 v.d. Tuning forks were used for the production of the tones, the variable being mistuned from the standard by means of sliding weights. The forks 64, 128, and 256 v.d. were energized by the stroke of a hammer of India-rubber, forks of 512 v.d., by means of a violin-bow, and forks of 1024 and 2048 v.d., by a wooden hammer, which was padded with felt. Forks of 64 v.d. were held upon large resonator-boxes; 128, 256, and 512 v.d. were brought to the openings of resonator-tubes of paper; while 1024 and 2048 v.d. were permanently attached to resonator-boxes. He used the method of minimal change and employed from four to eight different steps in passing from a large difference to one that was just perceptible, with the following results:

Standard v.d.	64	128	256	512	1024	2048
Difference	.15	.16	.23	.25	.22	.36

"In the field of tonal quality within the region investigated the psychophysical law, according to which the absolute differences of sensation correspond to relative differences of stimulus which must be constant, finds no application. On the contrary the differential threshold within the interval mentioned approaches the constant average of 0.2 vibrations."

The threshold value slowly rises from 64 to 2048 v.d. with the single exception of 1024 v.d. Luft admits that the error here is probably due to an objective circumstance. His results can scarcely be compared with those of Preyer as the methods employed were quite different. It is a point worthy of observation, however, that with Preyer the point of finest discrimination lies at 500 v.d. while with Luft it lies at 64 v.d. Later, Luft found the threshold of 32 v.d. (by the same method employed with 64 v.d.) to be about .44 v.d. His results, therefore, do not contradict Preyer's statement that very high tones and very low tones cannot be distinguished as readily as tones of the middle register.

Luft noted that practice lowers the threshold and that the effect

of practice is not equally distributed in all parts of the range. The influence of practice is of special importance at 64 and 128 v.d. Luft lowered his own record from 0.85 to 0.3 v.d. at 128 v.d., and from 0.42 to 0.15 v.d. at 64 v.d. He believes that the only reason that the initial thresholds for the lower tones is higher than those of the central region, is, that the degree of practice for the former is not so great. The variable of practice was far less noticeable in the higher regions which was due, he believed, to the greater intensity and persistence of these tones. He even ventured to say that the individual variations of the differential threshold are, for the most part, due to practice.

Meyer.—In 1898, Meyer (5) published Professor Stumpf's thresholds for the discrimination of pitch. Tuning forks were used for the tones 100, 200, 400, 600, and 1200 v.d. The variable forks were mistuned by the insertion of a screw in the end of the prong,—a more accurate device than that of sliding weights. After discarding the method of minimal change as practically worthless for the problem in hand, and thereby questioning the validity of Luft's results, Meyer adopted the method of right and wrong cases. Forks of 100 and 200 v.d. were held in the hand and brought to the openings of the resonators, while forks of 400, 600, and 1200 v.d. were mounted on resonator-boxes and were energized by the blow of a hammer. Each individual experiment was performed three times and even more if the observer wished it, before a judgment was required. In this manner Meyer thought to equalize variations of intensity and time-interval, as well as fluctuations of attention. Stumpf's thresholds as determined by means of the Cattell-Fullerton table, from the data given, are as follows:

V. D.	100	200	400	600	1200
Differential threshold	.54	.25	.28	.24	.69

The author concluded his report thus:

"One sees, therefrom, that approximately the same difference of pitch is recognized with equal certainty at 200, 400, and 600 v.d. and with less, but likewise moderately equal certainty, at 100 and 1200 v.d. The differences in these cases are so small that they may be considered accidental. That the certainty of judgments declines in still higher and still lower tones is self-evident."

Stücker.—Stücker's work (16) is the most extensive study published on this particular subject. His observations covered the range between the limits 72 and 35000 v.d., or nine entire octaves. He

employed the following standard tones: d^{-1} (73.4), c^0 (130.5), c^1 (261), all the tones of the major scale up to c^2 (522), a^2 (870), a^3, g^3 (3100), c^5, g^5, c^6, g^6, c^7 and c^8. All of the tones up to and including c^2 were produced with tuning forks, a^2 and a^3 with a monochord, and the remaining ones with a Galton whistle. In each individual series he started with a large difference in the number of vibrations of the two instruments and then made the difference gradually smaller until the threshold was reached. Such a procedure was repeated a few times for the purpose of verification. Whether the observer indicated the direction of the difference or merely the difference is not stated. Given below are the average values of the relative and absolute sensitiveness of discrimination of his fifty observers for eight different levels, with his statement in summary:

Pitch	d^1	c^0	c^1	a^1	a^2	a^3	g^3	g^5
Rel. Disc.	.94	.74	.49	.32	.30	.44	.86	4.91
Abs. "	.7	1.	1.3	1.4	2.5	7.7	26.7	304

(1) Neither the absolute nor the relative sensitiveness to difference of the two tones remains constant in the different tonal regions. (2) The relative difference-sensitiveness is in general the greatest in the first and second accented octaves; in many cases, however, the second maximum lies in the third and fourth accented octaves. (3) With one-third of the entire number of observers the relative sensitiveness to difference in the second half of the first accented octave is nearly equal; namely, 0.2 and 0.3; when one compares the individual curves of sensitiveness with these, the places of greatest sensitiveness lie in the upper half of this region, while with unmusical individuals they occur in general in the lower half. (4) The degree of sensitiveness is subjected to fluctuations within an octave, which is repeated in each octave in the same proportion; it is the greatest for c, slightly less great for g and still less for f and h. (5) A number of persons possess a secondary maximum of sensitivity. (6) An unusually great sensitiveness in high tonal regions is a characteristic of musical persons.

Stücker points out that the discrimination was far more accurate in the lower regions when the second tone was lower, while in the higher region the opposite was true. The inference here is, that judgments are facilitated when the second tone is farther removed from the first and second accented octaves, which are most frequently employed in musical composition; i.e., when the second tone is the farther from this middle register, the judgment seems to be more accurate. He further adds, that the daily variation of non-musical observers is less than for musical ones.

A year later this same author (17) published a report supplementing the one just reviewed. In this he states the results obtained from three different types of observers, professional players of various instruments, singers, and individuals decidedly unmusical. The average values of the absolute differences for the three different sets of observers have been computed for seven levels in the tonal range, as follows:

	d^1	c^0	c^1	a^1	a^2	a^3	g^4
Players	.35	.37	.40	.56	1.20	2.64	13.0
Singers	.46	.48	.44	.71	1.62	3.07	14.0
Unmusical	12.62	2.20	2.80	4.80	9.96	24.00	130.0

Of special interest in this second article is the statement that with tenors and sopranos the finest discrimination is found beneath their voice register, but with bass and alto singers above their voice register; the difference is not between the voices of men and women, but only appears between the relative height and depth of the voice-register of both sexes.

The age difference, he maintains, is more significant than that between musical and non-musical observers. After the age of thirty, sensitiveness to difference declines and the range becomes restricted.

Schaefer.—In 1910, Schaefer (10) submitted a thesis to the Department of Psychology of the State University of Iowa on the subject, "The Curve for the Variation of Pitch Discrimination within the Tonal Range", which has not been published. The apparatus and method were practically the same as those used in the present investigation. For observers, he had fifteen normal individuals varying in musical ability and training. Five hundred tests were given on each of the tones 24, 32, 64, 128, 256, 512, and 2048 v.d. The average threshold for each of these in the order given is as follows: 3.3, 3.4, 2.9, 1.3, 1.5, 1.8, and 6.7 v.d. He summarizes his results thus:

(1) The form of the composite curve indicates that discrimination for the average normal individual is most difficult in the higher and the lower registers and becomes easier in the middle register. (2) The majority of the individual curves are of the same form as the composite. Curves of individuals having high thresholds are of about the same form as the curves of individuals having low thresholds. (3) There are notable individual differences. (4) Musical training does not influence to any large extent, the ability to perceive difference of pitch. (5) It is easier to detect difference in pitch than to name the direction of the difference.

STATEMENT OF THE PROBLEM

The primary purpose of this investigation has been to determine the prevalence of islands or gaps in pitch-discrimination within the tonal range. The pursuit of this aim has taken the form of an attempt to make a comparatively large number of complete individual measurements on pitch-discrimination within the tonal range with as many as possible of the hitherto unknown or disregarded sources of error under control. On the basis of frequently observed defects in the hearing of pitch, found in clinical cases, it is generally believed that such disturbances occur in varying degrees in normal persons. In the curves of two or three of Schaefer's observers, there are places where discrimination of pitch is less keen than the balance of the curves would seem to indicate that it ought to be; in the case of one observer the evidence of a gap was striking. Professor Titchener (19) deems such cases of sufficient importance to bring to the support of the Helmholtz theory of hearing. He says:

"Cases occur in which the range of hearing is normal, but the tonal scale is not continuous; there are tonal gaps, large or small, parts of the scale where the patient is completely deaf to tonal stimuli, though he can perfectly well hear the cases above and below."

The sources of error in a problem of pitch-discrimination are so great and insistent that successive investigators of the same problem are fully justified in a patient struggle to overcome them with progressive insight. In reading the various reports on the subject, one cannot help being impressed with the fact that very few, if any, of the investigators fully realized the significance of the many important variables which could easily—and doubtless did—vitiate the results. The disturbing factors, due to faulty apparatus and inadequate procedure, mentioned by Professor Seashore in his preliminary report (13), suggest the seriousness of the problem. From my own experience I am convinced that his statement in regard to these factors is in no way exaggerated. Rather, it has not been made sufficiently emphatic. The danger of false criteria entering into the judgments of the most conscientious observer, either consciously or unconsciously, can scarcely be realized by one who has not encountered them first-hand. The danger of identification, alone, is sufficient to make the investigator very cautious.

Apparatus and Method.—In this investigation the measurements

were made at six different levels in the register; namely, 64, 128, 256, 1024, and 2048 v.d. The tones were produced by the best grade of Kohl tuning forks. For 128, 256, 512, and 1024 v.d. Helmholtz resonators were used; the forks of 2048 v.d. were mounted on resonator-boxes; while resonance for 64 v.d. was produced by extending the Helmholtz resonator for 128 v.d. For 64 v.d. a second set of forks was found to be more satisfactory at a later stage of the experiment. These were made of round tool steel, 12 mm. in diameter. The prongs were 30 cm. in length and carried hard rubber discs 10 cm. in diameter.

The sounder was a simple device consisting merely of a lead pipe about one inch in diameter with one end bent into the form of a circle for the base, and the other in the shape of a U at right angles to the base. The U-end, when covered with several thicknesses of rubber, made a sounder of the required elasticity and softness. The placing of the sounder on leather sand-bags resting on a heavy metal stand eliminated, in large part, the accessory noise of the blow. The forks of the four central octaves were energized by striking the middle of the prong upon the sounder; the forks of 2048 v.d. were struck as lightly as possible with a felt-hammer; while those of 64 v.d. were set into vibration by striking them on the sand-bags.

To mistune the variable fork, in every case except those of the lower limit, a screw was inserted in the end of each prong and to these were attached nuts, varying in weight, to give the desired pitch. Such a device is a decided improvement over the method of sliding weights, inasmuch as the latter may allow a slight change in position, with a corresponding change in pitch, during the course of the experiment. This is especially true of the smaller forks. At 64 v.d. variation in pitch was secured by shifting the discs, which were firmly attached to the forks by large set screws. At each of the steps the successive differences of one, two, three, five, and eight vibrations were chosen—a range which was found to be sufficient for all but one or two observers. All of the forks were tuned to an accuracy of five-hundredths of a vibration per second.

The mode of procedure followed the plan suggested by Professor Seashore in his preliminary report (13) in almost every respect, in the four central octaves where it was possible to do so. A most careful attempt was made to keep the tones at a constant intensity

without resorting to the uniformity of mechanical devices. The experimenter simply relied on the accuracy of his own hand and ear in presenting the forks in such a way that the tones would be of equal strength. If at any time, through a lapse on the part of the experimenter, the difference of intensity seemed pronounced, the trial was repeated. Mechanical devices are particularly unsatisfactory in that the difference of intensity which is practically certain to occur, be it ever so slight, is constant and might thus become a criterion for identification. In the method of presentation by hand, this source of error is eliminated. The ideal presentation is that in which the tones are just loud enough to be heard without a strain of the attention, and extreme care was taken throughout to guage the tones by this standard. The duration of each tone, as well as the time-interval, was approximately one second. Whether the constant or the variable tone should be presented first, was decided by a key which had been arranged first by chance and then revised to the extent that the same order could be followed no more than three times in succession, and that in one hundred tests the two possible sequences should have the same frequency. The observers in every case were required to render their judgments in terms of "second tone lower", or "second tone higher", in accordance with the method of right and wrong cases. No doubtful judgments were allowed; when the observer felt uncertain after repeated tests he was simply requested to guess. As a rule each individual experiment was given but once, but whenever disturbances of any sort, either objective or subjective, were noted, the experiment was repeated. Observers were instructed to trust the first impression. Except with the lowest tones, where the judgments were given orally and were then recorded by the experimenter, the observers themselves kept the record by simply writing *H* or *L* as an abbreviation of the judgments "higher" or "lower." With one observer, however, the response was oral throughout because attention to the writing caused too much of a distraction. At least one hundred judgments were recorded at each level, but many observers required a considerably larger number before their thresholds could be satisfactorily determined. No series of observations extended long enough to cause any disturbing fatigue. The monotony of the experiment was broken at intervals by the checking of the record and by the adjusting of the forks. Fatigue caused previous to the experiment could

not be very well controlled as the observers had to be taken at times which best suited their convenience. The tests were, however, fairly well distributed throughout the hours of the day and those observers who did come at a late hour were always dismissed if they felt fatigue to a degree which they thought might interfere with their best work. The experiment was conducted in the sound-proof room and in every instance the observers were tested individually so that distractions of an objective character were reduced to conditions connected only with the actual experiment.

The experimental control was naturally most difficult at 64 v.d. The large size of the forks not only made them more difficult to handle but also increased the possibility of overtones. Still another problem was presented in obtaining sufficient resonance for these tones of low intensity. Overtones were especially distracting with the first pair of forks that were used, but it was possible to overcome them to some extent by setting the forks in heavy handles of iron and by putting heavy rubber bands upon the prongs. Yet the increased weight added to the difficulty of handling. Two different methods were tried with these forks; namely, bringing the forks to the openings of the resonators, described above, and presenting them to the ear of the observer without the aid of a resonator. Both of these methods are unsatisfactory. The resonator scarcely makes the tones loud enough to make the judgment one of certainty, and it is difficult for the experimenter to present the tones so that they are of equal intensity. Holding the forks to the ear has the advantage of making the tones louder, but here again the variable of intensity is left uncontrolled, and the possibility is open to the observer for obtaining clues from the position of the fork, from timbre, and from noises caused by movements in presentation. The fact that the tones could be distinctly heard by the second method gave it the preference. But when the results were compared with those obtained for 128 v.d. the thresholds seemed abnormally large. This pair of forks was therefore discarded for the forks with the discs, which were found to answer the purpose much better, for at least three reasons; namely, they were freer from overtones, the tones were louder and clearer because of the increased vibratory surface offered by the discs, and they were neither so heavy nor so long, which facilitated handling very materially. With these forks the method of presentation to the ear was adopted, but on

account of the louder tone it was possible to hold the forks farther from the ear. Being also lighter in weight, they could be energized in a more uniform manner, and it was easier to bring them more nearly to the same point opposite the ear; thus the variable of intensity and direction of source could be more adequately controlled. An opportunity was not offered for the retesting of all individuals whose thresholds had been determined by the first pair of forks, but in most cases where a second was possible, somewhat lower thresholds were obtained.

No particular comment in regard to the forks of 128 and 256 v.d. is necessary. They were energized and presented to the resonators with the conditions of duration, time-interval, and intensity carefully controlled. In each case the tones were perfectly clear and distinct. The forks producing these tones held up long enough to allow five individual experiments without restriking. But the control was not quite so satisfactory at 1024 v.d. The forks at this level would vibrate with sufficient energy for only two tests. A more forceful blow was also required, and it was necessary to bring them very close to the small resonators, indeed so close that they nearly touched it. All this, of course, made it more difficult for the experimenter to maintain a constant intensity. Again, the piercing character of the tone was annoying to some observers. The tones produced by forks of 128, 256, and 512 v.d. were not heard by the observers except when reinforced by the resonators. But the 1024 v.d. forks gave a high piercing tone before being presented to the resonator. The observer, as much as possible, ignored this tone and concentrated his attention on the tones as they were intensified by the resonators.

In the upper limit, the method was necessarily quite different. The small resonator-boxes on which the forks were mounted were held in the hand; the one fork was struck and dampened, and then the second in close succession. So delicate a stroke was necessary to produce a tone that the noise of the blow was but a slight distraction, if any. It was extremely difficult, however, to keep the intensity constant. To eliminate discrimination of the direction of source, the position of the left hand was shifted to bring the forks to exactly the same place before they were energized.

Of the fifty observers who made this study possible by giving it their time and thought, thirty-three were members of the elemen-

TABLE I. Absolute differential thresholds

Obs.	64 v.d.		128 v.d.		256 v.d.		512 v.d.		1024 v.d.		2048 v.d.	
	T	m.v.	T	m.v.	T	m.v.	T	m.v.	T	m.v.	T	m.v.
1	2.5	0.9	1.9	0.5	0.8	0.6	0.7	1.1	3.3	.0	5.3	0.4
2	3.5	0.1	0.8	0.6	0.6	0.8	1.0	0.8	2.2	1.1	2.5	3.2
3	3.3	0.1	0.6	0.8	0.7	0.7	0.8	1.0	2.5	.8	5.6	0.1
4	3.0	0.4	1.5	0.1	2.2	0.8	2.3	0.5	1.1	2.2	3.5	2.2
5	4.0	0.6	1.3	0.1	1.1	0.3	0.9	0.9	4.1	.8	7.3	1.6
6			0.7	0.7	0.8	0.6	1.2	0.6	2.4	.9		
7	4.0	0.6	0.7	0.7	1.2	0.2	1.7	0.1	3.5	.2	6.7	1.0
8	1.0	2.4	1.5	0.1	1.4	0.0	1.8	0.0	3.7	.4	6.5	0.8
9	6.4	3.0	2.7	1.3	0.7	0.7	1.7	0.1	5.3	2.0	6.5	0.8
10	1.5	1.9	1.4	0.0	1.1	0.3	1.8	0.0	2.4	.9	4.9	0.8
11	2.0	1.4	0.8	0.6	1.5	0.1	1.0	0.8	3.9	.6	5.6	0.1
12	3.7	0.3	0.7	0.7	1.5	0.1	2.4	0.6	4.3	1.0	3.5	2.2
13	1.5	1.9	1.0	0.4	0.7	0.7	2.0	0.2	3.8	.5	6.7	1.0
14	4.0	0.6	2.7	1.3	1.8	0.4	2.2	0.4	5.0	1.7	10.0	4.3
15	3.0	0.4	1.4	0.0	2.1	0.7	4.4	2.6	6.4	3.1	8.8	3.1
16	3.4	0.0	1.3	0.1	0.6	0.8	0.8	1.0	2.2	1.1	3.0	2.7
17	2.0	1.4	1.1	0.3	2.9	1.5	5.0	3.2	5.1	1.8	7.6	1.9
18	0.7	2.7	1.0	0.4	2.0	0.6	0.6	1.2	0.8	2.5	5.8	0.1
19	5.2	1.8	0.6	0.8	1.1	0.3	0.8	1.0	4.1	0.8	9.7	4.0
20	1.0	2.4	1.0	0.4	0.7	0.7	1.2	0.6	3.2	.1	6.1	0.4
21	3.8	0.4	2.3	0.9	1.6	0.2	1.7	0.1	3.6	.3	5.5	0.2
22	2.5	0.9	0.8	0.6	1.0	0.4	1.3	0.5	2.0	1.3	3.0	2.7
23			2.0	0.6	1.3	0.1	1.4	0.4	6.8	3.5	5.5	0.2
24	6.4	3.0	1.5	0.1	1.1	0.3	1.5	0.3	2.0	1.3		
25	4.0	0.6	2.0	0.6	2.5	1.1	2.5	0.7	2.4	.9	4.4	1.3
26	3.0	0.4	0.7	0.7	1.0	0.4	1.9	0.1	1.8	1.5	5.8	0.1
27	4.7	1.3	0.7	0.7	1.5	0.1	1.7	0.1	2.1	1.2	3.6	2.1
28	3.4	0.0	0.6	0.8	0.8	0.6	1.1	0.7	2.3	1.0	3.2	2.5
29	2.4	1.0	2.1	0.7	1.5	0.1	1.6	0.2	8.4	5.1	10.2	4.5
30	2.5	0.9	2.4	1.0	1.3	0.1	1.4	0.4	4.1	.8	5.7	0.0
31	6.4	3.0	2.4	1.0	2.0	0.6	1.5	0.3	3.9	.6	3.0	2.7
32	2.4	1.0	1.0	0.4	1.0	0.4	1.1	0.7	2.2	1.1	4.6	1.1
33			1.7	0.3	1.4	0.0	2.7	0.9	3.2	.1	8.8	3.1
34	3.3	0.1	1.5	0.1	1.5	0.1	1.4	0.4	2.5	0.8	4.9	0.8
35	6.4	3.0	1.1	0.3	1.7	0.3	1.0	0.8	6.4	3.1	9.9	4.2
36	8.8	5.4	1.1	0.3	0.8	0.6	2.2	0.4	3.5	.2	4.8	0.9
37	3.0	0.4	1.5	0.1	1.4	0.0	2.6	0.8	4.4	1.1	10.2	4.5
38	0.7	2.7	0.6	0.8	0.7	0.7	2.1	0.3	1.6	1.7	3.7	2.0
39	7.2	3.8	4.1	2.7	3.2	1.8	1.6	0.2	7.6	4.3		
40			0.9	0.5	1.2	0.2	0.9	0.9	0.7	2.6	4.3	1.4
41	2.4	1.0	0.8	0.6	0.7	0.7	2.1	0.3	1.8	1.5	3.5	2.2
42	4.0	0.6	2.0	0.6	3.2	1.8	4.9	3.1		7.0	7.0	1.3
43	1.3	2.1	0.8	0.6	0.7	0.7	1.1	0.7	1.8	1.5	5.3	0.4
44	3.0	0.4	1.1	0.3	0.4	1.0	1.2	0.6	2.6	.7	3.0	2.7
45	3.1	0.3	1.4	0.0	1.5	0.1	1.4	0.4	3.2	.1	8.0	2.3
46			1.4	0.0	1.3	0.1	2.6	0.8	2.7	.6		
47	3.0	0.4	1.2	0.2	1.6	0.2	1.7	0.1	2.4	0.9		
48	3.0	0.4	1.4	0.0	1.1	0.3	0.8	1.0	2.7	.6		
49	5.0	1.6	4.0	2.6	2.5	1.1	6.1	4.3				
50	1.3	2.1	1.0	0.4	0.7	0.7	1.0	0.8	0.9	2.4	1.2	4.5
Mean	3.4	1.5	1.4	.57	1.4	.5	1.8	.76	3.3	1.31	5.7	1.56
Median	3.00		1.2		1.3		1.5		3.0		5.5	

TABLE II. Relative differential thresholds

Obs.	64 v.d.	128 v.d.	256 v.d.	512 v.d.	1024 v.d.	2048 v.d.
1	.31	.12	.03	.01	.03	.02
2	.44	.05	.02	.02	.02	.01
3	.41	.04	.02	.02	.02	.02
4	.38	.09	.07	.04	.01	.01
5	.50	.08	.03	.01	.03	.03
6		.04	.03	.02	.02	
7	.50	.04	.04	.03	.03	.03
8	.13	.09	.04	.03	.03	.03
9	.80	.17	.02	.03	.04	.03
10	.19	.09	.03	.03	.02	.02
11	.25	.05	.05	.02	.03	.02
12	.46	.04	.05	.04	.03	.01
13	.19	.06	.02	.03	.03	.03
14	.50	.17	.06	.03	.04	.04
15	.38	.09	.07	.07	.05	.03
16	.43	.08	.02	.01	.02	.01
17	.25	.07	.09	.08	.04	.03
18	.09	.06	.06	.01	.01	.02
19	.65	.04	.03	.01	.03	.04
20	.13	.02	.02	.02	.03	.02
21	.48	.14	.05	.03	.03	.02
22	.31	.05	.03	.02	.02	.01
23		.13	.04	.02	.05	.02
24	.80	.09	.03	.02	.02	.02
25	.50	.13	.08	.04	.02	.02
26	.38	.04	.03	.03	.01	.02
27	.59	.04	.05	.03	.02	.01
28	.43	.04	.03	.02	.02	.01
29	.30	.13	.05	.03	.06	.04
30	.31	.15	.04	.02	.03	.02
31	.80	.15	.06	.02	.03	.01
32	.30	.06	.03	.02	.01	.02
33		.11	.04	.04	.03	.03
34	.41	.09	.05	.02	.02	.02
35	.80	.07	.05	.02	.05	.04
36	.11	.07	.03	.03	.03	.02
37	.38	.09	.04	.04	.03	.04
38	.09	.04	.02	.03	.02	.01
39	.90	.26	.10	.03	.06	
40		.06	.05	.01	.01	.02
41	.30	.05	.02	.03	.01	.01
42	.50	.13	.10	.08		.03
43	.16	.05	.02	.02	.01	.02
44	.38	.07	.01	.02	.02	.01
45	.39	.09	.05	.02	.03	.03
46		.09	.04	.02	.02	
47	.38	.08	.05	.03	.02	
48	.36	.09	.03	.01	.02	
49	.63	.25	.08	.10		
50	.16	.06	.02	.02	.01	.01
Average	.4	.09	.04	.03	.03	.02

tary class in psychology in the University, sixteen others were advanced students in psychology, and one other a member of the staff in psychology. It is important to note that the fifty represent a selected group. The thirty-three from the elementary class were chosen from a class of one hundred or more because their differential thresholds at 435 v.d. were less than 8 v.d., as determined from a test given to the class for purposes of demonstration. The advanced students had likewise shown in previous tests that their thresholds for discrimination of pitch were easily less than 8 v.d. Their closer association with the work in the department of psychology also tended to make them slightly better as a group than the elementary students. This basis of selection must be borne in mind in the consideration of the results, for our composite curve is not an average curve; it is superior to the average. It was gratifying to find that all of the observers took keen interest in the problem and made a sincere effort to give the work their best attention. Their knowledge of the fact that they were chosen because of their former good record helped them to maintain an interest.

RESULTS

The Composite Curves.—Table I includes the individual thresholds in terms of the absolute difference of vibrations for the six points in the range. The odd numbers of the observers refer to women, and the even, to the men. The thresholds are given in column T, and the mean variation in column m.v. At the foot of the table are the mean, the median, and the mean variation of the group. In Table II the same records are reduced to the relative threshold expressed in terms of the fractional part of a whole tone, at the respective levels. The figures in italics at the head show the number of vibrations in a whole tone at each of these respective levels. The record of Table I is shown graphically in Fig. 1 and that of Table II in Fig. 2. By an error the decimal point was left out before each of the numbers 1, 2, 3, and 4, in Fig. 2.

There is evidently no essential difference between the mean and the median curves; they run practically parallel throughout their course, coming a little closer together at 256 v.d. than at any other point. But inasmuch as the mean allows the extremes an influence out of proportion to their importance, the median must be considered the truer representative figure.

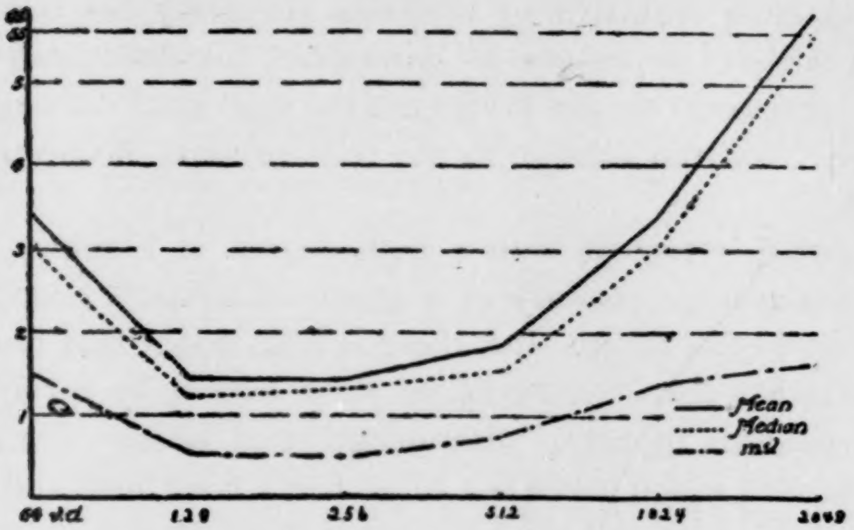

Fig. 1. Mean, median, and mean variation—absolute (Table I).

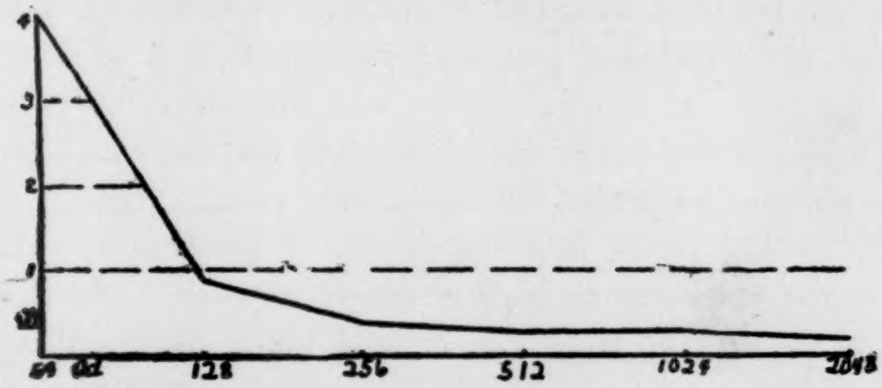

Fig. 2. Mean—relative (Table II).

The average capacity of discrimination, as measured in terms of absolute difference, is practically the same for 128 and 256 v.d. From this central register the curves rise slowly to 512 v.d., from which begins a rapid rise that becomes even more rapid from 1024 on up to 2048 v.d. The curve of mean variation (Fig. 1) follows the general trend of the composite, which means that the thresholds for this group of fifty form a more compact grouping in the central register, while at both the upper and the lower limits they are more widely separated.

The relative curve declines rapidly at first and then very gradually, reaching its lowest point at 2048 v.d. In other words discrimination of pitch, as measured in fractional parts of a whole tone, decreases somewhat abruptly from 64 to 128 v.d., but very slowly from that point to the upper limit of the range studied. In the relative curve the minimal value is at 2048 v.d. which is the region of the maximal value in the curve of absolute difference.

The absolute difference of vibration frequency has been adopted as the vehicle of expression in this report, for the reason that it is slightly more concrete and brings out the individual differences more strikingly. Its true relation to the relative must, however, be kept in mind.

The curves represent with a high degree of accuracy, it is believed, the average capacity of a group of observers such as have had a part in this study. But there is little doubt that the form of the curves has been influenced, to some extent, by certain factors other than those of actual discrimination of pitch. There are objective factors which could not be perfectly controlled and which in some cases have led to confusion, but in other cases have resulted in identification. The former necessarily raised the threshold, while the latter lowered it. The subjective variables of attention and practice are important inasmuch as attention is seldom at is best and then only for short duration, and the degree of practice might always be greater. The thresholds are therefore not quite as low as they would be under the most ideal conditions.

With tuning forks, it is impossible to produce as satisfactory a tone at the extremes as in the central register. Discrimination of pitch at 64 and at 2048 v.d. is thus made most difficult and the observer has a tendency to pick up other criteria than pitch upon which to base his judgments. Differences of intensity, change in the direction of the source of sound, and noises accompanying the control of the experiment are the chief factors which cause disturbance. They lead to confusion, rather than to identification, because the method used necessitated their approximately equal distribution between the higher and the lower tones; that is to say, that they occurred in a chance order, were therefore unpredictable, and consequently could not be used as safe criteria for accurate judgments; for example, if an observer was inclined to judge the more intense tone the higher, there would be an increased probability of error whenever the lower tone happened to be more intense. Had the forks been energized by a mechanical device, rather than by the free hand, these variables would have been constant and would have become a means of identification, rather than a source of confusion. At the higher limit it was difficult to keep the tones of equal loudness. The tones produced by the small forks are very fine and persistent, and a slight variation in the forces of the blow produced

a perceptible change in the intensity of the tones, which was often confusing. Whether or not the greater intensity favored a judgment of higher or lower varied with the individual. For some, the pitch being nearly equal, the louder tone was considered the higher, while for others the reverse experience was true.

It is in the lower limit, however, that the most abrupt rise in the threshold is to be found. As has been previously mentioned, various methods of presentation were given a trial, but none of them, excepting with a very few observers, gave results which were comparable with those obtained at 128 v.d. Only two observers had a lower threshold for 64 and for 128 v.d., (Nos. 8 and 18). For observer No. 20 the thresholds for the two tones were the same, while No's. 10, 13, 29, 30, 38, and 50 were the only remaining ones whose thresholds for 64 v.d. did not exceed that of 128 v.d. by more than 0.5 v.d. In other words, forty observers have a threshold for 64 which is more than one-half of a vibration higher than for 128 v.d. That this difference would have been less had it been possible to rule out all the factors of confusion is probable.

But not all of the variables cause confusion. Those which are constant soon come to be associated with one of the two possible judgments and this, in time, brings about a lowering of the threshold. Just what is seized upon as a means of identification one cannot always say. The auditory capacity of analysis is very keen and often the slightest variable which occurs in a particular setting is selected as a clue for the proper response. Slight variations in timbre are among the most frequent sources of identification. It is impossible to make two forks exactly alike and the unavoidable structural difference may be perceived in the nature and composition of the overtones. The forks of the lower limit are particularly susceptible to variation in timbre. If these differences are perceptible, the error of identification is sure to appear. Even with presentation by hand there is the possibility of the experimenter's falling into some characteristic habit of presenting the forks, which may be identified eventually. He may form the habit unconsciously of striking one fork at a different angle from that of the other, or the time-order may have some constant peculiarity which gives a clue.

The errors due to identification are without a doubt the most serious with which the experimenter has to contend. But in an experiment such as this the error of identification is usually discover-

able by comparing the thresholds of one level with those of the other levels. Whenever an observer has a threshold at any particular level considerably lower than the tentative norm would warrant, the chances are, that the error of identification has had a part to play. The record of No. 20 is wanting in the table for 2048 v.d. because he had discovered some criterion other than pitch upon which to base his judgments. In fact he made nearly a perfect record with a difference of one vibration,—a lower threshold than his records at the other levels would warrant. Two other observers had a similar experience in the upper limit, but when the method was slightly changed, they lost their clue and were forced to rely on pitch. In the lowest level Nos. 28 and 40 were influenced in some way by criteria other than pitch, the latter to such an extent that his results were worthless. As has been said, just what criteria were selected by these observers is not known. Their introspections fail to reveal them, the observers contending throughout—and with undoubted conviction—that they were judging on pitch alone. Such illustrations show that the experimenter connot be too careful in his attempt to keep the judgments confined to pitch.

The subjective variables of attention and practice also play more important rôles at the extremes than in the central register. To secure a low threshold at these levels closer attention is necessary and, as these tones are rarely heard, the degree of practice is much less than for tones of the central register. Practice for these tones is only to be had in the laboratory as they are seldom used in musical compositions. Observers Nos. 2, 28, 47, and 50 were the only ones who had the advantage of practice for these extreme tones and their thresholds at these levels are all below the average.

Summing up, we have found that the curve of pitch-discrimination shows the threshold of absolute difference to be keenest from 128 to 256 v.d.; from 256 to 512 v.d. it takes a gradual rise; and from 512 to 2048 v.d., a rapid rise. On the lower side, from 128 to 64 v.d., the rise is very sudden. As expressed by the curve of relative difference, there is a continual decline from the lower to the higher limit; this decline, however, is very rapid from 64 to 128 v.d., much less pronounced from 128 to 256 v.d., and from 256 to 2048 v.d. the curve becomes very nearly a straight line. It will be of interest now to compare the above results with those of other investigators.

Comparative Curves.—Figure 3 represents the composite results

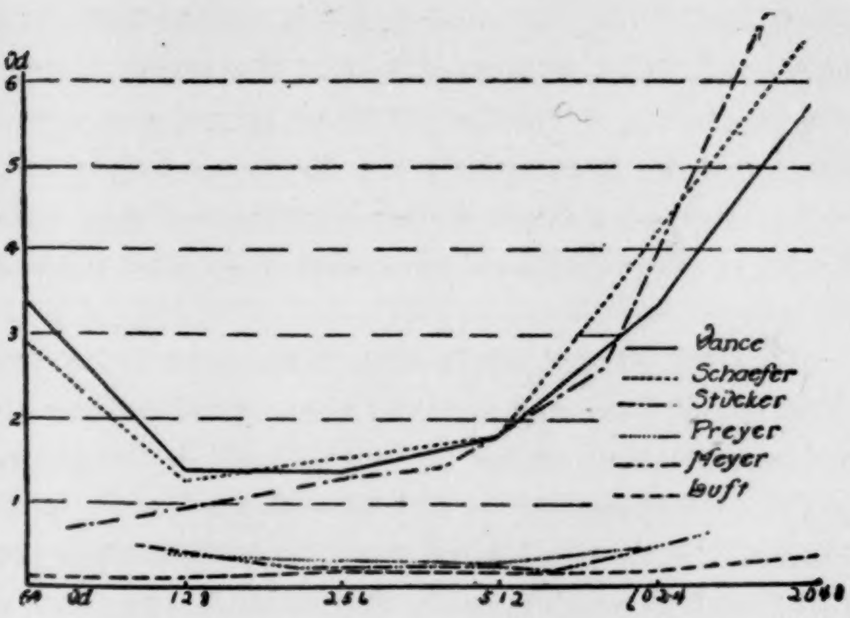

Fig. 3. Composite curves for six different investigators—absolute.

of the six different investigators who have approached the problem by the same general method. The curves of Stücker (16), Schaefer (10), and the writer show considerably higher thresholds than those of Luft, Preyer and Meyer. This difference may be explained. In a pioneer work, such as Preyer's (7), numerous sources of error as yet undiscovered must have had much influence upon the results. His low thresholds can be attributed, in some degree at least, to the error of identification. Since this error may creep in when the best grade of tuning forks is used, there is little doubt but that it must have played an important rôle with the tonmesser. Furthermore this instrument, the reed, is not reliable for fine pitch differences. Luft's (4) values, especially at the extremes, must likewise be questioned. Whenever such low results are obtained at 64 and at 2048 v.d., there must be conclusive evidence that they are not due primarily to discrimination of pitch, but to some other factor which permits of identification. Luft has given no such proof. Furthermore, in a problem of this kind, the method of minimal change, which he used is unreliable, as Meyer has well pointed out, in that it introduces factors other than those of pitch and the threshold value is not quite comparable to the threshold value in our method of constant stimuli. Professor Stumpf's curve, drawn by Meyer (5), shows exceptional ability and is probably accurate. The low thresholds can be adequately explained by the less extended range, by extraordinary natural capacity, and by a high degree of training in

experimental work. On the other hand it should be kept in mind that the curves of Stücker, Schaefer, and the writer, represent the results of a much larger number of observers, many of whom do not have exceptionally fine capacity for discrimination of pitch. The curves are therefore on a much higher level than if they were drawn exclusively from the results of observers who had unusually fine ability.

In the curves of Preyer, Schaefer, Meyer, and the writer the minimal threshold lies somewhere near the central region; but in the other two discrimination seems to be the best in the lowest level. With Preyer the finest capacity is at 500, with Luft at 64, with Meyer at 600 (although the thresholds for 200, 400, and 600 are practically equal), with Stücker at 73.4, with Schaefer at 128, and with the writer at 256 v.d. The maximal threshold is to be found in the highest part of the range in every case. The second maximum lies with Preyer, Meyer, Schaefer and the writer at 64 v.d.

An examination of these curves raises the question as to the cause of the variations. Individual differences are, of course, the principal cause but the nature of the objective control is undoubtedly a very important factor, especially at the extremes. If the apparatus and the method of the three investigators, who had a large number of observers, had been equally refined at the different steps, these grosser differences would probably not have occurred. As it is, they are most pronounced at the extremes where the control was the most difficult. The experimental control at 128, 256, and 512 v.d. can be made so perfect that no observer will be able to pass consistent judgments on any criterion other than pitch. For this reason the results of Stücker, Schaefer, and the writer agree, approximately, within this region.

Inasmuch as the curves take the same general direction, the variations in the upper limit are about what would be expected when one considers the difficulties to be encountered, together with the fact that one of the experimenters used an entirely different apparatus. But from 130.5 to 73.4 v.d., Stücker's curve continues in the same general direction which it has had throughout the entire course, while the other two curves have changed their direction. In other words, Stücker found the absolute difference for 73.4 v.d. to be less than for any other point in the line, while both Schaefer and I found at 64 v.d. the second maximum which is noticeably

greater than for any other point except at 2048 v.d. Luft's results seem to confirm those of Stücker, but Meyer's curve, as well as Preyer's, shows a rise at the lower limit. Indeed the ratio between the thresholds of Meyer for 100 and 200 v.d. is very similar to the ratio between the thresholds for 64 and 128 v.d. obtained by Schaefer and myself. I have no hesitancy in concluding, therefore, that sensitiveness in the great octave is, in general, not so keen as in the small octave. But for reasons already given, it does not follow that the difference is actually as great as the numerical results of this study would seem to indicate. In the light of the experience of the present study, however, Stücker's findings in the lower limit must be held in question. It seems more probable that his observers had learned to make judgments on some criterion other than pitch. Just what that may have been cannot be stated definitely as that author has failed to give any detailed account either in regard to method or to apparatus. It is only known that the tone in question was produced by a tuning fork. The possibilities of error with the large tuning forks are, however, sufficiently great to warrant the statement that Stücker's low record is due, not altogether to discrimination of pitch, but that secondary criteria have been operative in giving the low thresholds.

Individual Differences.—An examination of Table I discloses the fact that the observers may be classified in two general divisions. In the first there are thirty-seven whose curves follow the course of the composite curve in that the smallest values are to be found in the central register on either side of which a slow or a rapid rise is evident. In the second division, are thirteen whose curves do not conform to any general type. In the irregularity of the curves of this second division lies the only possible evidence of gaps which this study has developed.

The curves of the first division may, in a general way, be given a three-fold classification; namely, (1) those which show a relatively low threshold at some point in the central region and relatively high thresholds at the extremes, (2) those in which the thresholds are fairly uniform throughout the entire range, and (3) those curves in which the threshold for 64 is lower than for 2048 v.d.

Division I.—In Figure 4 are the five curves of the first group which show, the most strikingly, the relatively low thresholds in the central register and the higher thresholds at the extremes. These

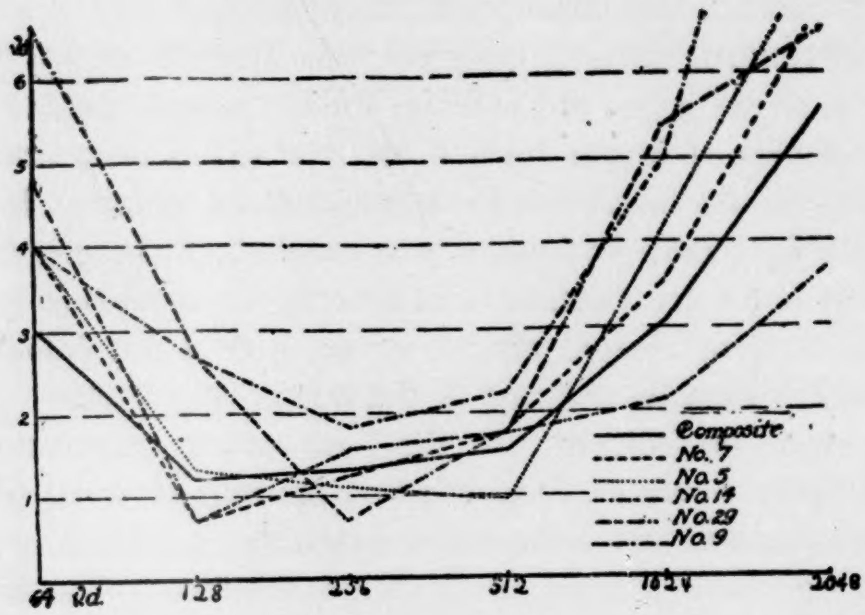

Fig. 4. Individual curves of observers Nos. 5, 7, 9, 14, and 27, which show relatively low values in the central region and high values at the extremes. The solid line is the composite curve of the fifty observers.

curves all resemble the composite more or less closely. At the extremes, however, all excepting that of No. 27 rise above the composite, but in the central region, at 128, 256, and 512 v.d., one-third of the fifteen thresholds pass beneath it. The normal variation of the point of keenest discrimination is well illustrated in this figure. Nos. 7 and 27 made the best record at 128, Nos. 9 and 14 at 256, and No. 5 at 512 v.d. In fact, all but one of the entire number of observers made their lowest record at one of these central levels.

These curves represent the results of observers who were the most unreliable. Very few of these values indicate the physiological threshold. One could not say that the high values at the extremes should be interpreted to mean that all of the observers in question were unable to perceive smaller differences on account of physiological incapacity. It is much more probable that the difficulty is psychological. Individuals of this type do not adapt themselves so readily to new situations under experimental control. When new adjustments must be made their work is relatively poor and continues on a low plane until time has been given for the proper adjustment after which their work may be on a par with that of individuals who adapt themselves more quickly to new situations.

Figure 5 represents the results of the six individuals who are most typical of the second group. All of these observers are men, but they are not of equal rank in previous work in discrimination of

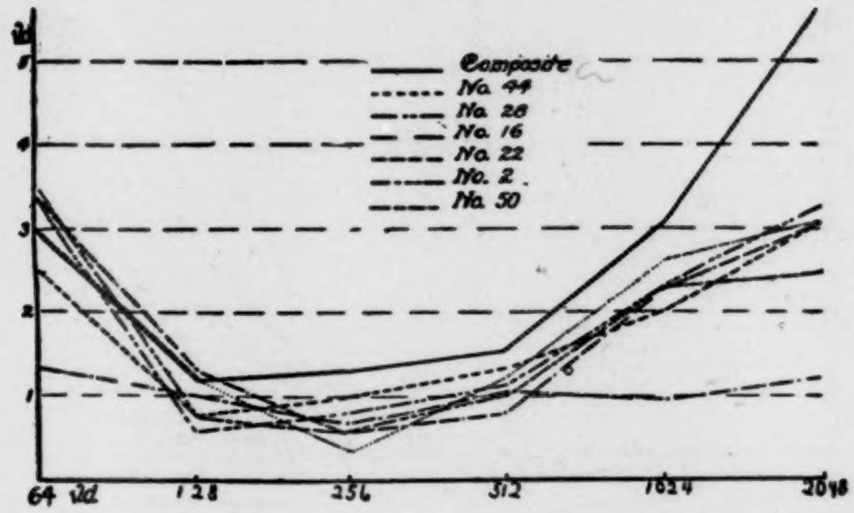

Fig. 5. The composite curve of the fifty observers and the individual curves of Nos. 2, 16, 22, 28, 44, and 50, which are characterized by a high degree of uniformity in the threshold values throughout the range.

pitch. Nos. 2, 32, and 48 were graduate students in psychology and were trained in other tests of discrimination; with the remaining three, however, previous training was very limited. The striking difference between these curves and those of the former group is, as would be expected, with reference to the extremes. The values for 64 and especially for 1024 and 2048 v.d. are lower than in curves of the first type; they approach, therefore, a more uniform level,—a goal which is most nearly approximated by No. 48. In contrast to the former group, these curves fall below the composite at practically every point; only four values are actually higher than the composite, while two more are equal, and these are at the lower limit. From the standpoint of consistency, the curves of Class II can easily be judged the better. Observers who give such results are reliable. With a state of secondary passive attention, they are able to meet the new situation in an easy and natural manner and are little disturbed by unusual difficulties which may be presented. In addition, exceptional ability in analyzing a problem enables them to select the proper element or elements upon which to base their judgments, even though there be disturbing factors. They are so consistent that the experimenter can feel a high degree of assurance that their records represent a close approach to the physiological threshold.

The curves of the five individuals who are most representative of the third group are shown in Figure 6. The peculiar character of these curves, in contrast to those already considered, lies in the lower limit. Here the thresholds are very much lower than for Class I

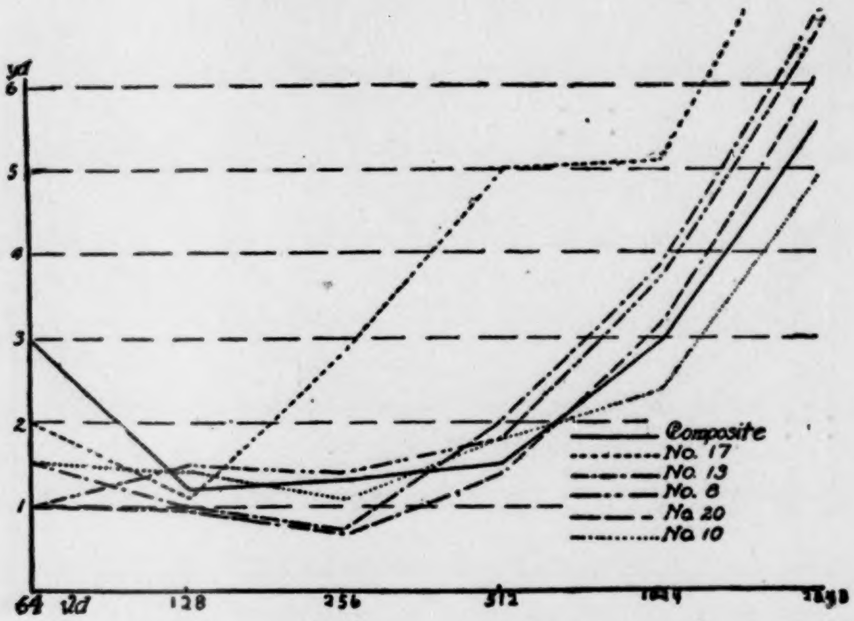

Fig. 6. The composite curve of the fifty observers and the individual curves of 8, 10, 13, 17, and 20, which show relatively low values at the lower limit and high values at the upper limit.

and considerably lower than they are for Class II. But in the upper extreme the curves are similar to those of the first group, with one exception,—the curve of No. 17. Indeed the average results of these five observers form a curve which closely approximates Stücker's curve, the essential difference being, that the latter is tilted at a slightly different angle, due to the fact that Stücker's thresholds at 73.4 v.d. are lower than ours and higher in the vicinity of 2048 v.d.

The similarity of our results to those of Stücker in the lower extreme might invite the same criticism which we advanced against him. It might be said that our low threshold at 64 v.d. was due to the discovery of some variable other than pitch upon which the judgment was based. There is, of course, the possibility that this occurred, but reference to Figure 7, in which the composite curves of the three groups may be compared, leads to the belief that such a criticism does not have much weight with respect to these particular observers. It is to be observed that the minimal thresholds of the first two groups lie at 256 v.d. with a gradual rise on either side of this point. The point of keenest discrimination for Class III, however, lies at 128 v.d. with here again a rise on either side proportional to that which we find in the other two groups. In other words, the form of the latter curve from 64 to 256 v.d. is similar to the form of the other two from 128 to 512 v.d. We should expect to find a higher threshold for 64 when the minimum is

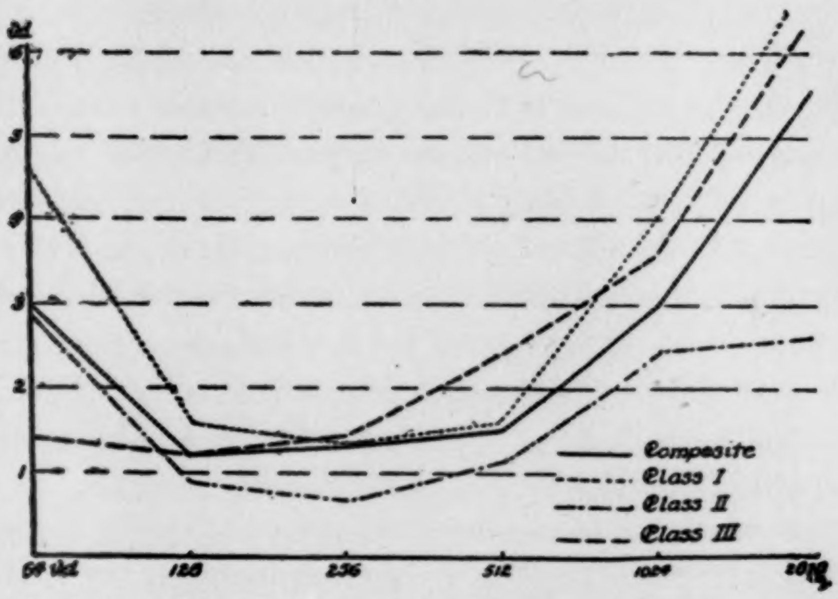

Fig. 7. The composite curves of the fifty observers and the three different classes.

at 256 than when it is at 128 v.d., and this is what occurs. With conditions such as they are, we are inclined to regard the results of this class of observers, at 64 v.d., as fairly accurate. The observers in question naturally do better work on the low tones. If such an interpretation is true, it would not be just to say that the curve of Class III is less consistent than the curve of Class II; each represents a different type and is consistent with itself throughout.

There are still to be considered the results of the thirteen observers which are not exactly comparable to any of the classes described above, because of certain irregularities occurring in their curves. It must be determined whether or not these ridges or elevations may be explained on the basis of daily fluctuations, in which case a sufficiently large number of observations would result in smooth curves, or whether the variations are due to natural weaknesses for the regions where they are found. The extent to which this latter explanation must be invoked, indicates an answer to the question of the frequency of gaps in the registers of individuals who have apparently normal hearing.

In Table I, the observers who have such curves are Nos. 4, 8, 11, 12, 18, 19, 25, 38, 40, 41, and 45. It will be noted, however, that at no place are the deviations from the normal very pronounced. All of them, excepting perhaps one, are doubtless due to certain factors which would have been eliminated by a large number of judgments. Daily variations, the relative amount of practice, and the accuracy with which the increments used corresponded to the true

thresholds, are the most important factors which have contributed toward the irregularities. With No. 5, for example, the experiment was begun at 256 v.d., so that the greater amount of practice would give the neighboring tones the advantage. With No. 11, the tone of 256 v.d. was first tried with a difference of 2 v.d., which was too large, resulting in an almost perfect series. Had the order in which these differences were presented been reversed, the threshold would probably have been very close to 1 v.d. Indeed, a number of these observers were given additional tests to determine whether or not these variations from the normal would hold. In each case as Table III will show, the curves became fairly smooth.

The curve of No. 4 is abnormal at 1024 v.d. in its relatively low threshold of 1.1 v.d. During an experimentation of one hour he made a record of eighty-eight per cent. on two hundred judgments with a difference of 2 v.d. But on the following day, a difference of 1 v.d. gave only fifty-two per cent. of the right cases. A larger number of observations would doubtless have resulted in a threshold more equal to that of the tone an octave lower.

But there is one observer, No. 18, who gives some evidence of a

TABLE III. *Irregular results which additional observations have corrected*

Observer	*64*	*128*	*256*	*512*	*1024*	*2048*
19	5.2	0.6	1.1	0.8	4.1	9.7
		0.6	0.6	1.1		
41	2.4	0.8	0.7	2.1	1.8	3.5
				1.8	3.5	
35	6.4	1.1	1.5	1.1	6.4	9.9
		1.8	2.2	2.6		
38	0.7	0.6	0.7	2.1	1.6	3.7
			1.1	.9	1.3	
12	3.7	0.7	1.5	2.4	4.3	3.5
				1.8	3.3	

slight weakness in the region of 256 v.d. His threshold at this point was derived from four hundred judgments. The first half of the number with a difference of 3 v.d. gave a threshold of 2.9 v.d., while the second half with a difference of 2 v.d. gave a threshold of 2.3 v.d. At no time was he able to approach a threshold of 1 v.d. On the other hand, with the tones above and below, he made low and consistent thresholds. It is difficult to account for this high threshold at 256 v.d.; the observer himself could offer nothing as a basis for explanation. The affective element, association and imagery, and inherent characteristics of volume and intensity may have played varying rôles in causing the discrepancy. At any rate the differ-

ences are not sufficiently great to be regarded as representing gaps.

We have found, then, from this study of the curves of discrimination of pitch of fifty normal observers no clear evidence of tonal gaps. The grosser irregularities which might arouse the suspicion of a gap are due to certain factors which have not been perfectly controlled. It is highly probable that with more extended observations the irregularities would have been eliminated. It must be kept in mind, however, that this conclusion has reference only to observers with apparently normal auditory capacity; with respect to individuals whose audition is unquestionably recognized as pathological, this study has nothing to offer.

Relation of Musical Training and Expression to Discrimination of Pitch.—The question naturally occurs in a study of this kind as to the nature and extent of the correlation between musical education and pitch-discrimination. It seemed obvious that if a correlation existed it would be between discrimination and musical expression rather than between discrimination and mere technical training. The Pearson method of rank difference was used to determine the correlation. The mean of the six levels in the range for each of thirty-eight observers gave a value for the ranking of the individuals according to their capacity for the discrimination of pitch. The records of the remaining twelve were not included as most of them were advanced students whose greater experience in work in the laboratory might possibly put them in a slightly better class, while with one or two others, information regarding their musical training was not at the time available. The ranking according to expression was not quite so simple. For this purpose an evaluation was made of the answers to the questionnaire, which was an exact duplicate of the one published by Professor Seashore in his Preliminary Report (13). To recall, there are three questions under the topic "Musical Expression": namely, (1) Favorite selections you can sing (by ear? by note?), (2) Favorite selections you can play (by ear? by note?), (3) Singing or playing in public (parts, occasions, etc.). The individuals were instructed to give as specific information as possible. But the comparison of the two functions showed no correlation whatever.

It was still believed, however, that there must be some difference between the discriminating capacity of those who seemed to be the

most musical and those who appeared to be the least, as far as previous experience was concerned. Again the questionnaire, to which reference has been made was resorted to, but this time the questions were designed to reveal the amount of training. They were as follows: (1) Musical training in public schools, (2) Private vocal lessons (when, where, how long, etc.), (3) Private instrumental lessons (when, where, how long, etc.). The observers were then equally grouped in two divisions, the first group consisting of the better ones in training and expression and the second of the poorer ones. The mean threshold for each group for the different levels is recorded below:

Table IV

V.d.	64	126	256	512	1024	2048
Group I	3.8	1.2	1.2	1.8	3.8	6.8
Group II	3.1	1.5	1.5	1.6	3.1	4.9

We find, then, that the group whose members have had greater musical education and more practice excel in capacity for discriminat-

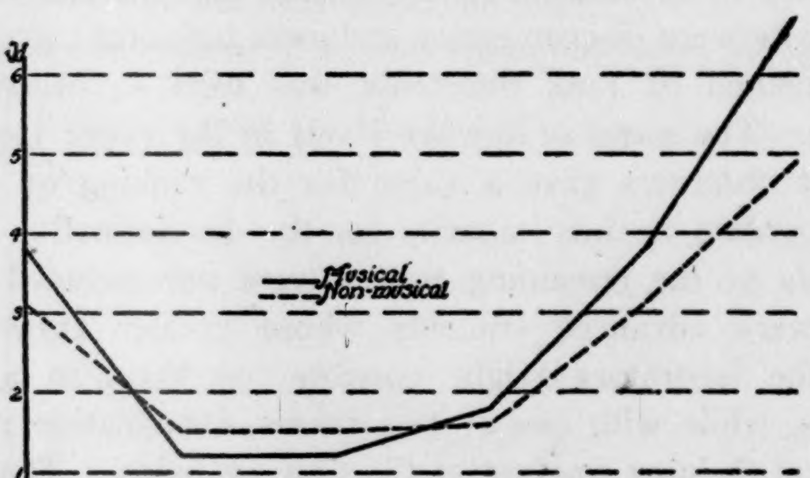

Fig. 8. Comparison of the musical and the non-musical.

ing pitch only at 128 and 256 v.d. But it is in this region that the above factors would have the most influence. Their effect upon the differential threshold of either extreme would be small because these tones are seldom used either in singing or playing. There is, then, some correlation between musical ability and discrimination of pitch in the central register. This is in general agreement with the conclusions of both Mount (6) and Smith (15) who found a fair degree of correlation between musical expression of pitch and dis-

crimination of pitch at 435 v.d. But we cannot agree with Stücker (17) in his assertion that musical observers, in general, show keener discrimination in the upper limit than the non-musical ones. Much depends upon the standard of classification for musical observers. Stücker may have reference only to observers of unusually fine ability in music. For such, his statement may be true, but for observers whose ability is not so exceptional it scarcely holds.

The frequency and distribution of the false judgments.—From the 39700 judgments it has been possible to determine definitely not only the frequency of the false judgments but also the way in which they have been distributed in the various levels. So large an amount of data should show whether or not there is a preference for one or the other order, and if so what relation this preference bears to sex, voice-register, and pitch.

In computing the number of errors, the results of sixty-two individuals, thirty-two women and thirty men, for at least three different tonal regions were available. The nature of the error in the wrong judgments at 64 v.d. was not recorded; the observer sat with closed eyes and gave oral judgments and the experimenter merely recorded the number of the errors. At the other levels, with one or two exceptions, the observer recorded *H* or *L* for each pair of tones. For 128, 256, 512, 1024, and 2048 v.d., then, the distribution of errors could be accurately studied. Just one-half of the sixty-two observers had a record for each of these five steps, for the other half discriminations were made at from three to four levels. Two different computations were therefore made, the first including the results of the entire number of observers and the second, only those which are complete for the five different levels. The total of the complete results could thus be used as a check upon the total of the incomplete results. Reference shall be made to the first, however, only in so far as it differs from the second.

TABLE V. Distribution of errors

Section 1

Computed from the results of sixty-two observers

A	B	C	D	E
128	8300	23.40	10.78	12.62
256	8600	25.13	13.31	11.82
512	10100	26.03	13.79	12.24
1024	8300	25.42	13.61	11.81
2048	4400	25.41	14.00	11.41
Total	39700	25.09	13.04	12.05

Section 2

Computed from the results of thirty men

128	4700	23.21	10.34	12.87
256	4800	25.00	12.83	12.17
512	6100	27.46	13.84	13.62
1024	4500	24.67	12.50	12.17
2048	2100	23.52	11.61	11.91
Total	22200	25.11	12.39	12.72

Section 3

Computed from the results of thirty-two women

128	3600	23.67	11.36	12.11
256	3800	25.29	13.92	11.37
512	4000	23.78	13.48	10.30
1024	3800	26.32	14.95	11.37
2048	2300	27.13	16.13	11.00
Total	17500	25.07	13.86	11.21

Table V is a record of the errors computed from the results of the total number of the observers. Column A represents the vibration-rate of the fork; B, the total number of judgments; C, the total percentage of error; D, the percentage of error when the second tone was lower; and E, the percentage of error when the second tone was higher. The greater number of judgments in the central register is due to the fact that irregularities occurring here necessitated further experimentation to determine whether they were due to subjective factors which were permanent or merely transient, or possibly to objective factors.

The final average of the percentage of right cases approaches to within .09 per cent. of the ideal of 75 per cent. When the different levels are considered collectively, the false judgments amount to 13.04 per cent. when the second tone is lower, and to 12.05 per cent. when the order of succession is reversed. There seems then to be a slight though not significant preference for the order in which the second tone is higher.

A difference is observable in the distribution of error at the various levels. At 128 v.d. more errors by 1.84 per cent. occur when the second tone is higher, but at the other levels there is a greater percentage of error with the opposite order. As shown in Table V the differences between Column D and E increase gradually from 256 to 2048 v.d. In the first computation, however, made from the complete results of a smaller number of observers, the order of second tone higher gives the smaller per cent. of error at each level. At 128 v.d., the difference in favor of this order is only .48 per cent., but

at 256 it amounts to 2.03 per cent., at 512. to 3.65 per cent., at 1024 to 2.68 per cent., and at 2048 v.d. to 2.46 per cent. On the average, then, judgments of difference in pitch are more accurate when the second tone is higher, *i.e.* given two successive tones of the same pitch, there is a slight tendency to hear the second as the higher, excepting at 128 v.d., where fewer errors are made when the reverse order is followed.

Difference of sex.—When the results are studied with respect to sex it is found that the above conclusion would not be valid for a group of individuals in which there was a much larger percentage of men than women.

In the study of differences of sex it is found that the women on the average, show a decided preference for the order in which the second tone is higher at every step except at 128 v.d., where the difference seems to be slightly in favor of the second tone lower. But it is at this latter level that the men show a very strong preference for the second tone lower, while in the other levels the difference in favor of either order is insignificant. This variation of sex affords additional evidence that normal illusions are greater with women than with men.

An arrangement of results according to voice registers of the observers brought out nothing new. The difference seems to be essentially between the voices of men and women. Had our observers been highly specialized singers, there might have been some difference showing itself in the different voice registers.

TABLE VI. Variation with sex

V.d.	*64*	*128*	*256*	*512*	*1024*	*2048*
20 women	3.2	1.2	1.4	1.7	3.8	6.6
16 men	2.7	1.1	1.0	1.6	2.4	4.8

The foregoing table and the accompanying figure show the results for the twenty women and the sixteen men who had a similar amount of training in experimental procedure. At first sight there seems to be a decided difference between the sexes, inasmuch as the thresholds for men are lower throughout the whole range than those for women. While there are differences in favor of the men, care must be taken not to attach too much significance to them. The differences at 1024 v.d., of 1.4 and at 2048 v.d. of 1.8, seem to be considerable, yet they are not much greater than should be expected when the total results are considered. At 256 v.d. the varia-

tion of .4 appears high when compared with the difference of .1 in the octave just above and just below. With such noticeable variation in the central region it is not so surprising to find much larger differences at the extremes where objective factors are not so well controlled. Smith (15) reports practically the same difference of sex as is shown in these results. He finds that at the ages of 17 to 20 and at maturity, the men surpass the women by an average of 0.3 v.d. at 435 v.d. It is evident that the men's curve presents a more satisfactory form than does that of the women, in that there is not so high a variation between the points of keenest discrimination and the

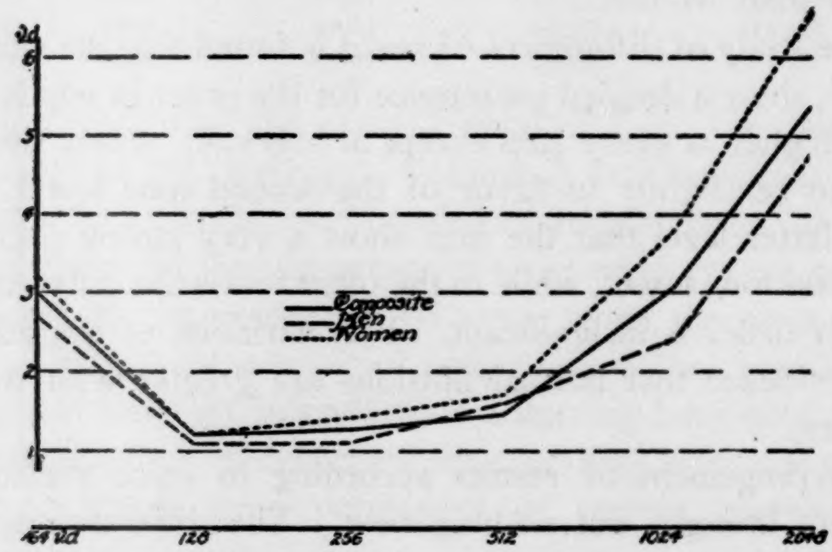

Fig. 9. The comparative curves of twenty women and sixteen men together with the composite of the fifty observers. (Table VI).

extremes. It may be that the cause of this arises from a possible inherent difference between the sexes in the method of meeting new situations. Or it may be that the men adapt themselves more quickly to experimental conditions and for this reason it has been easier to reach their physiological threshold.

Stücker (17) contends that the greatest sensitiveness to small differences of pitch lies with tenors and sopranos in the lower half of their voice registers, but with singers of bass and of alto parts, as a rule, in the upper half. In other words, the differences is not between the voices of men and the voices of women but between the relative height and depth of the voice register of both sexes. As none of my observers could be classed as professional singers, the results have little to offer either positively or negatively, in regard to Stücker's statement. Table VII indicates that only the results of

the soprano singers can be harmonized with the conclusion of Stücker. The finest sensitivity of the tenors is in the central part of their register and not in the lower, as he finds it to be; the basses made the best record in the lower part of their register, rather than in the upper; the baritones have done better in their register; and finally, the altos do better in the lower register and not in the upper. But these facts are not necessarily contradictory to Stücker's, inasmuch as the observers in this experiment represent only average ability as singers. One should have plotted the curves of a relatively large number of highly practiced singers before he would be able to add a conclusive word in answer to the problem which Stücker has suggested.

TABLE VII. Average thresholds classified according to voice register

Soprano	(16)	3.8	1.5	1.6	2.1	4.1	7.1
Tenor	(4)	4.1	1.5	1.2	1.2	2.3	4.3
Baritone	(15)	3.2	1.4	1.3	2.1	2.4	4.7
Alto	(8)	3.5	1.5	1.2	1.3	4.3	6.0
Bass	(7)	2.5	1.1	1.3	1.6	2.8	4.3

SUMMARY

(1) For individuals selected because of a slight superiority at 435 v.d., the composite absolute curve of pitch-discrimination within the limits of 64 and 2048 v.d. shows the keenest discrimination at 128 and 256 v.d. On either side of this central register, there is a rise in the curve which is relatively abrupt toward the lower limit but much more gradual toward the higher extreme.

(2) The relative curve takes the form of a continual decline from the lower to the higher limit. From 64 to 128 v.d. the decline is comparatively steep, but from 128 to 2048 v.d., it is very gradual, approaching approximately a horizontal line in the upper half of the register.

(3) Individual differences, factors which lead to confusion and to identification, and variation in practice and in attention are the principal conditions upon which the form of the curve depends. The variations in the curves of the different investigators are explainable on the basis of the varying degrees of influence of these conditions.

(4) Most of the individual curves conform more or less closely to one of the following types of curves; namely, (a) a curve in

which there is a relatively low value at some point in the central register and relatively high values at the extremes, (b) a curve in which the thresholds are fairly uniform throughout the entire range, and (c) one in which the threshold for 64 is considerably less than for 2048 v.d.

(5) There is very little evidence of tonal gaps. The grosser irregularities in a few curves, which at first seemed to indicate the presence of a gap, disappeared with more extended observations.

(6) A correlation between musical ability and discrimination of pitch occurs only in the central register.

(7) The women make more accurate judgments when the second tone is higher; their preference for this order increases in direct proportion to the pitch, within limits, excepting at 128 v.d. where the reverse order is slightly preferred. The men make fewer mistakes at 128 v.d. when the second tone is lower, but at the other levels no particular preference for either order of succession is observable.

(8) The men surpass the women in discrimination of pitch at every level in the register; this variation between the sexes is the greatest at the extremes.

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THE DURATION OF TONES, THE TIME INTERVAL, THE DIRECTION OF SOUND, DARKNESS AND QUIET, AND THE ORDER OF STIMULI IN PITCH DISCRIMINATION

BY

DAVID ALLEN ANDERSON

I. Most favorable duration of the tones

In this investigation to ascertain the relative favorableness of different durations of tone in pitch discrimination, the tones were produced by tuning forks from the "standard pitch discrimination set" as described by Professor Seashore (1) reenforced by Koenig adjustable resonators suspended behind a revolving slit-disc which was driven by a synchronous motor (2,3).

The tuning forks were tuned to an accuracy of ± .015 v.d. They were held firmly by the fingers near the end of the stem and energized by striking the middle of the prong lightly against a sounder made of ¾ in. lead pipe covered with a soft rubber tubing and resting on a leather cushion filled with sand. When they had been set in motion the forks were held directly in front of the mouths of the resonators during the passage of the open slits in the intervening revolving disc. Revolving discs made from cardboard, in which were slits cut in appropriate sectors, regulated the duration of tones and the interval between them. The disc proper prevented the passage of the vibrations from the forks to the resonators while the slits admitted of their free passage. The length of the slit determined the duration of the tone and the size of the sector between governed the length of the interval. When a slit passed a fork the resonator would take up the vibrations. The result was a clear and pure tone, clean cut at beginning and end. The intensity was kept as regular as possible without maintaining an identifiable uniformity. An effort was made to change the forks from hand to hand and to govern the duration of time between the energizing of the forks and the hearing of the tones in such a way that the observers could get no clue regarding the order in which the tones were to be given. Whether the higher or lower tone was to be given last was regulated by a key prepared beforehand according to chance, except that not more than three consecutive cases of one kind were allowed.

The two resonators were fastened side by side immediately behind the revolving disc.

The speed of the synchronous motor used in driving the revolving discs is controlled by a tuning fork and gives an accuracy in the time element far beyond the requirements of this experiment.

A time interval of ⅛ second was chosen arbitrarily and was kept constant throughout. Ten judgments constituted a group and ten groups or columns (100 trials) confined to one duration made up a set. When a group of ten judgments had been taken and recorded in a column, another group followed, and so on until the set was completed. When one set had been given it was followed by a set of another duration and so on throughout the series. Generally about four sets were given at a sitting.

The observers were Professor C. E. Seashore, and three graduate students, namely, G. H. Mount, L. E. Widen and W. R. Miles; all of whom were at the time pursuing experimental problems in the laboratory and had quite extended experience in observing tones. Each of them had a threshold of 1 v.d. or less on the basis of 75 per cent. correct judgments; hence 1 v.d. was used as the pitch interval throughout the tests. The observers were permitted to choose a location in the room which seemed favorable, and comfortable with the understanding that it was to be kept unchanged.[1] They listened to the two tones, judged the latter as higher or lower than the former, and recorded the decision (H for higher and L for lower).

TABLE I. Effect of differences in duration of tones
(Time interval ⅛ second; pitch interval 1 v.d.)

	S.			*Mo.*			*W.*			*Mi.*			*Ave.*
Duration	%	m.v.	n.	%	m.v.	n.	%	m.v.	n.	%	m.v.	n.	
⅛ sec.							72	2	500				71.6
¼ sec.	58	5	500	73	9	500	76	4	500	67	2	500	68.4
⅜ sec.	68	6	500	81	5	500	78	3	500	76	5	500	76.0
½ sec.	75	3	500	84	4	500	76	3	700	74	4	500	77.5
1 sec.	76	8	500	82	4	500	79	3	500	84	4	500	80.4
2 sec.	87	5	300							89	2	300	88.3
1st tone 2, 2nd, ½ sec.	77	1	200							86	2	200	82.0

%, per cent. of right cases; *n*, number of trials; *m.v.* mean variation for successive hundreds of trials.

[1] Two observers each made one change in position but their records in both positions were so distributed throughout the series as not to interfere with the results. The records of both were materially improved by the shift. The influence of position of the observer with reference to the origin of the tone is discussed later.

Table I shows a general tendency in favor of the shorter durations. There is practical uniformity in this general tendency among the several observers, there being but three steps that are exceptions: Mo.'s average at ½ second is higher than his average at 1 second, and W and Mi. make higher averages with a ⅜ second duration than when it is ½ second in length. However, the increase in ability with increase in duration of tone is comparatively small.

Introspections indicate that the most favorable feeling attending the hearing of tones of a certain duration may or may not parallel the percentage of correct judgments. It is also noted that when tones are of any long duration, judgment is usually made as soon as the essential character of the second tone is perceived without waiting for its cessation. Some tests were thereupon made on S and Mi. using durations of 2 seconds and ½ second in each couplet. This experiment consisted of two hundred judgments by each of the observers and resulted in a general average of 82 per cent. correct judgments. (See last line of Table I.). There seems to be therefore no advantage in making the second tone more than ½ second in duration.

Taking all things into consideration, it appears that the initial tone should have a duration of about 1 second, while the second tone need not exceed ½ second in duration. The demands for economy justify these limits even though longer intervals result in a slight increase in efficiency.

II. Most favorable time-interval between tones

The problem was to determine whether the time-interval between tones should be of a definite length and, if so, what it should be. In this problem the laboratory conditions, dates of experimenting, observers, apparatus, and methods were the same as in the preceding section. The discs were made so as to vary the interval and keep the duration uniform. The time intervals tested were 0, 1/16, ⅛, ¼, ½, 1, 2, 3, and 4 seconds. The duration of tones used was kept constant at ½ second.

Upon examination of Table II we notice that the gross averages for the several intervals are fairly uniform while the records of each individual vary widely at each step in the series. For this group the intervals tested are practically equally favorable, with possibly a slight tendency in favor of the shorter. Introspections

TABLE II. Effect of difference in time interval between tones

(Duration ½ sec.; pitch interval 1 v.d.)

Interval in	*S.*			*Mo.*			*W.*			*Mi.*			
seconds	%	m.v.	n.	%	m.v.	n.	%	m.v.	n.	%	m.v.	n.	Ave.
1/16 sec.	78	9	300				95	1	300	87	4	300	86.6
1/16 sec.	73		100	80	5	500	87	8	500	89	7	500	84.6
⅛ sec.	68	10	200	74	7	500	87	7	500	92	1	500	82.5
¼ sec.	63		100	81	5	500	87	6	500	90	2	500	84.7
½ sec.	61		100	78	5	500	87	6	500	91	4	500	83.4
1 sec.	74		100	75	5	500	83	7	500	84	7	500	80.4
2 sec.	67	4	200							86	1	200	76.7
3 sec.	71		100							80		100	80.5
4 sec.	78		100							78		100	78

%, per cent. of right cases; *n*, number of trials; *m.v.* mean variation for successive hundreds of trials.

indicate that conditions of stress, strain, annoyance and fatigue are experienced when long intervals are used. It is advisable to economize time and energy by adopting a short interval or by excluding it altogether.

The results here reported are in accord with, and supplement in so far as they cover the same ground, the studies of Wolf (4), Angell and Harwood (5), and Whipple (6), though these investigators were concerned chiefly with the influence of the time interval upon tone memory and imagery. The shortest interval tested by any of these experimenters was one second and comparatively few records were taken where they used an interval of less than two seconds. Their results are fairly uniform in showing the advantage in a short interval.

III. Most favorable direction of the sound

The problem was to ascertain whether any one direction of the source of the sound was most favorable in discrimination of pitch. The experiments were performed in July 1911 in the open air away from every influence of limiting walls or surfaces aside from the ground and the grass. The sounds were produced by tuning forks with resonators.

The observers were Professor Seashore, Jessica E. Strawbridge, T. F. Vance and E. T. Walker. The three mentioned were chosen from among the graduate students in the University of Iowa. All but one of the observers were practiced in pitch discrimination. A pitch interval of 1 v.d. was used with *S* and *St.* and 2 v.d. with *V* and *Wa.*

In the experimenting the observers were seated about ten feet from the source of the tone. Beginning in a position directly facing the source of the stimulus the observers turned to the left ninety degrees at a time so as to hear the tones exactly from the front, the right, the back and the left in successive series of trials. Making twenty judgments in each position, they continued to turn until one hundred judgments were made from each of the four directions. Table III gives the results in percentage of correct judgments of each individual in each of the four positions and the averages for the group, showing that in the open air there seems to be no significant effect of direction of the sound upon ability in pitch discrimination.

TABLE III. Effect of direction of sound

	S.	*St.*	*V.*	*Wa.*	*Ave.*
Front	71	64	56	72	65.7
Right	66	55	58	72	62.7
Back	64	53	63	69	62.2
Left	76	62	62	69	62.2

A similar set of experiments was then tried in a room 15 x 18 feet square with the observer in a selected series of positions with reference to direction of sound, relation to walls, and distance. On the whole the same conclusion was reached as for out-of-doors. However, strong individual preferences were expressed. There was no clear evidence of correlation between feeling of favorableness and actual ability.

IV. Darkness and quiet

The experiments herein reported were designed to ascertain the influence of occupying the "dark room" upon accuracy in judgments in pitch discrimination. The work was done in July 1911 in room No. 210. L. A. which is a "measurement room", (15 x 18 x 13 feet, well lighted and occupied by apparatus and laboratory furniture, quite resonant), and in the light, sound, and jar proof room described in Vol. III of these Studies (7).

The tuning forks with resonators were used as before, without the discs. The observers were Professor Seashore and graduate students F. O. Smith and E. T. Walker. The position of the observers with reference to the origin of the tones was kept uniform in the two rooms throughout the experimenting. With *S* a pitch-interval of 1 v.d. was used, with *Sm.* and *Wa.* 2 v.d. *Wa.* was an unpracticed observer.

Each observer made two hundred judgments in each of the two rooms. One hundred in the "dark room" were followed immediately by the same number in "the measurement room".

TABLE IV. Effect of darkness and quiet

	S.	*Sm.*	*Wa.*	*Ave.*
"Dark Room"	80	54	64	
	73	55	81	67.8
"Measurement Room"	75	51	56	
	75	62	75	65.7

The results indicate that accuracy in judgment is about equally favored in the two rooms. Introspection shows that while the silence and freedom from distraction of the dark room are soothing, they also make the observer more critical about the stimulus. As one observer said, "Any irregularity in the tones was extremely annoying in the dark room, but in room 210 the accessory sounds made the stimulus seem smooth and soft."

In view of these results, including introspections and observations, it appears that accuracy in judgment of pitch of clearly audible tones will be as high in an ordinary laboratory-room as in a quiet room. The freedom from distractions in the dark and quiet room has a soothing effect upon the observer but, owing to the absence of distracting influences, the observer in the quiet room detects minor qualities and characteristics of tones and methods of procedure which would not come to consciousness in the ordinary laboratory. The normal noises and lights of an ordinary room seem to soften and smooth the stimulus.

V. The order of stimuli

During the progress of the experiments reported in the preceding sections one observer noted that his first judgment in a column (group of ten made in rapid succession) nearly always designated the second tone as higher and that such judgment was frequently in error. In order to ascertain whether or not this was a general tendency, the experimenter examined his records of more than 15000 judgments made by eight observers and written down in 1517 columns of ten judgments each.[2] The recording of judgments in successive columns corresponded to the method of presenting tones in successive groups of ten trials each. The trials in the individual

[2] A few columns include twenty judgments each.

groups followed each other in rapid succession, but there was a slight pause between groups. Therefore the first judgment in each of the 1517 columns or groups is the only one now under consideration. Table V shows the record made by each of the eight observers.

TABLE V. Effect of order of high and low

Observers	Number of judgments	Percentage of errors when judging High	Low
S.	362	77	23
Sm.	20	86	12
St.	20	100	0
Mo.	450	50	50
W.	310	31	69
Mi.	295	64	36
Wa.	40	57	43
V.	20	33	67

Now by actual count of all errors made in the list of over 15000 judgments, there is found no important constant tendency, 50.8 per cent. of the errors falling on the side of low. Without any constant tendency the errors should be about equally distributed between high and low in Table V. But such is not the case with observers *S., Mo., W.* and *Mi.,* for whom the record is extensive; *S.* and *Mi.* tend to judge "high", *W.* to judge "low" and *Mo.* happens to be exactly neutral. For the other observers the records are too few to be of much significance. We must therefore conclude that there are probably fixed individual tendencies to judge high or low but there is no constant group tendency. The order of tones therefore remains a factor which must be kept under control in experimenting.

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THE EFFECT OF INTENSITY AND ORDER ON THE APPARENT PITCH OF TONES IN THE MIDDLE RANGE

BY

ROLLAND M. STEWART

In the study of various phases of pitch discrimination in this laboratory the problem of controlling the intensity of sound has proved persistent. The writer therefore undertook to ascertain some of the principal features of this tendency. The work was begun with two differential forks, standard 128 v.d., mounted on tripods and energized electrically. Before these, a Helmholtz resonator was suspended which could be swung freely into position in front of either fork. The intensity was judged subjectively by the experimenter and was varied by the distance of the resonator from a point of greatest efficiency in front of the fork. After some preliminary work this apparatus was abandoned on account of the disturbance coming from the interruption of the current. The forks were remounted on handles easy of manipulation and were struck on a sounder and presented alternately in front of the same resonator but these forks proved to be unwieldy and it was found practically impossible to avoid the error of identification. They were therefore abandoned again and it was decided to attempt the test with a set of standard forks at 435 v.d.[1]

The real difficulty with 128 v.d. forks lay in the fact that one fork was tuned by sliding weights. The final test was therefore made with a series of differential forks, standard 435, which were tuned permanently to given fixed increments in the making and had no adjustable parts. The forks were presented to the resonator at a uniform rate according to a pre-determined scheme of distribution of weak and strong sounds according to prepared keys. The keys used were known only to the experimenter and were prepared by determining the order of the weak and strong sounds by chance

[1] This incident has proved in the light of more recent work to be of far greater importance than was known at the time, since, as is shown in the accompanying article by Miss Hancock, the law of intensity effect is quite different at 128 v.d. from what it is at 435 v.d.

except that a sufficient number of substitutions were made to get the same number of trials of each kind.

Two keys were used. Key I called for 150 observations: 50 cases of the same pitch and varied intensity; 50 cases with a pitch increment and constant intensity; and 50 cases with a pitch increment and varied intensity—thus high and strong, high and weak, low and strong, low and weak.

Key II called for 192 observations, 32 of which were introduced to exclude or to determine a measure of the so-called time error, *i.e.*, a tendency to call a tone either high or low simply because it is the second tone of a pair. For these trials the same fork was sounded with the constant intensity in both presentations. The remaining 160 observations were divided equally among the four variations mentioned in Key II; namely, high strong, high weak, low strong, low weak.

All observations were made in the light, sound, and jar proof room. Trained observers were used for the most part. Three of these, *D*, *F* and *H*, had engaged in extensive practice in the study of the problem of pitch discrimination. Four, *A*, *C*, *E* and *G*, were working in related problems at this time. Only one observer, *B*, was inexperienced in this problem, but she proved to have a fine ear for pitch discrimination where the intensity was constant.

TABLE I. The effect of the intensity and the order of the tones

	Key I		Key II			
Obs.	Intensity Constant	Intensity Varied	Intensity Constant	Intensity Varied	Weak	Strong
A	77	76	90	86	83 L	67 H
B	95	76	96	81	74 H	70 H
C	80	80	100	77	76 H	57 L
D	75	74	73	76	61 L	63 L
E	75	60	100	66	89 H	96 L
F	63	68	90	80	60 L	58 H
G	84	71	90	57	67 L	67 L
H	77	62	100	87	67 H	52 H
Ave.	78	71	92	76		

In all about 7500 observations were taken as a basis for these records. Table I shows the effect of the variation of intensity on the judgment of pitch. The records are given separately for Key I and Key II for each observer; and, in the last two columns of the table the same records for the two keys are combined and distributed with reference to weak and strong. In the former the

numbers indicate the per cent. of right cases under the four conditions named and, in the latter, what per cent. of the weak and strong respectively were called high (H) or low (L) when actually equal in pitch.

Examination of these figures shows that the varying of the intensity causes confusion which results in a poorer record than when the intensity is constant. This is true in the records both under Key I and Key II. The ratio of the right cases for "intensity constant" as compared with "intensity varied" is on the average 78: 71 in Key I, and this tendency is true of all individuals except one (F) for Key I. For Key II the corresponding ratio of the per cent. of the right cases is 92: 76.

The analysis of the results with reference to weak and strong proves that, for these observers, there is no constant tendency to identify weak or strong and high or low. However, examination of the individual records shows that this is not due to the absence of constant tendency in individuals but rather to the balancing of opposite tendencies in different individuals of the group. The percentages in the table show these individual tendencies to be quite marked. On the whole four (A, B, D and F) have a constant tendency to call the weak low and the strong high; while the other four (C, E, G and H) show the reverse tendency.

Table II contains the distribution of the same records, showing other details in regard to the intensity-pitch illusion. The notation of the table is self-explanatory; the abbreviations HS, HW,

TABLE II. Redistribution of same data as in Table I

Key I	Observers	A	B	C	D	E	F	G	H	Av.
Intensity	High called low	25	6	11	37	25	38	11	32	23
Constant	Low called high	26	4	31	13	25	35	23	11	21
Key I	HS called low	11	2	27	34	73	23	16	50	29
Intensity	HW called low	21	44	12	20	12	33	17	17	22
Varied	LS called high	43	31	21	24	7	43	11	36	27
	LW called high	18	12	59	22	73	25	59	50	40
Key II	HS called low	5	4	27	15	55	15	62	7	24
Intensity	HW called low	22	47	5	27	22	32	15	12	23
Varied	LS called high	20	23	5	40	22	22	25	15	22
	LW called high	7	1	45	15	35	10	67	20	24
	Tendency of second tone	53H	74L	50	66H	69H	81L	62H	53H	

HS, HW, LS and LW stand for High Strong, High Weak, Low Strong, and Low Weak respectively.

LS, LW, standing respectively for high strong, high weak, low strong, low weak. A comparison of the strength of the illusion for each individual in the second and third horizontal sections of this table with the distribution of errors in the first horizontal section (for "intensity constant") shows that these personal equations are sufficiently large to be recognized as fairly prominent individual tendencies that must be taken into account in any comparison of the pitch of two tones.

The above named conclusions are fully substantiated by preliminary observations on 22 observers whose records are not included in the above table because they were taken under somewhat varying conditions. Of these 22 observers 9 called the weak high and 10 low; 10 called the strong low and 12 the strong high. Although the introspections were studied quite carefully, no satisfactory explanation could be found to show why these errors occur. It was first thought that the primary tendency was to identify strong with high and to assume that when the opposite tendency appeared this was due to a reaction, conscious or unconscious, as a correction to this tendency which the observer might expect in himself. But this interpretation is probably not true since it is shown in the article referred to above that for low tones the tendency is just the reverse. We are therefore left without any satisfactory interpretation of the phenomenon and with the impossibility of knowing what direction the illusion will take in a given individual.

The order of the sound was so distributed as to eliminate that source of error for the main purpose of this experiment. The bottom section of Table II shows that there is no constant tendency in the time-error for these observers as a group: there is about as strong tendency to call the second tone low as to call it high. It is, however, clear that quite marked individual tendencies exist as in the 74 per cent. low for *B*, the 81 per cent. low for *F*, the 66 per cent. high for *D* and the 69 per cent. high for *E*. The conclusions on this point in the foregoing article by Anderson (Section V) are thus sustained, both as to the divergence in the direction of the tendency and the characteristic magnitude of the error.

THE EFFECT OF THE INTENSITY OF SOUND UPON THE PITCH OF LOW TONES

BY

CLARA HANCOCK

In pitch discrimination tests, a difference of intensity in the sounds compared has proved so important a source of error as to require investigation. A series of experiments were conducted by Stewart, as reported in the foregoing article, (1), with forks of 435 v.d. for the purpose of discovering the effect of intensity variation. It was on his tests that Professor Seashore based the following statements in his preliminary report on, "The Measurement of Pitch Discrimination." (2):

"Extensive experiments show (1) that both trained and untrained observers may be influenced by intensity in their pitch judgment; (2) that, although there is a tendency among the untrained, especially the ignorant, to judge the loud tone the higher, it may work either way; (3) that the same individual may show one tendency at one time and the reverse at another; (4) that for trained observers the two tendencies are almost equal; and (5) that the tendency is more serious for large than for small intensity differences."

Later, during a series of experiments on accuracy in singing the tones of forks of from 109 to 308 v.d., Miles (3) found results that differed materially from those of Stewart. He found that an increase in intensity of the standard tone regularly caused a lowering in the pitch of the reproduction when that was of medium intensity, and that when the standard tone was presented with medium intensity, if it was reproduced loudly, it was sharped. This indicated that the effect of intensity on pitch discrimination might not be the same for low tones as for high ones, and that the conclusions of Stewart's experiments might not be applicable to tones of low pitch.

The following series of experiments were undertaken to determine whether or not, in the sounds of tuning forks of 128 v.d., a difference of intensity produces any more constant illusion as to pitch than in the higher tones.

A set of forks was tuned from 128 v.d. upward with intervals of 1, 2, 3, 5, 8, 12, 17, 23, 30, 38 and 47 v.d. Later it was found necessary to tune a few lower than the standard for corresponding increments. The range is from 123 to 175 v.d. In the first series of tests no resonators were used. The difference in intensity was controlled by the force with which the forks were struck on the sounder and by the distance they were held from the ear. The aim was to have the faint sound just loud enough to be distinctly heard as a tone, and the loud one as loud as possible without interfering with its quality as a tone. The method of limits was used. The standard, 128 v.d., was always made faint, and the variables loud. The sounds were presented in two orders in alternating series: (1) a series with the standard first, followed by the variable; and, (2) a series with the variable first, followed by the standard. With the last observers several series were given also with no difference in intensity, for the purpose of comparison. In these the order was variable.

The point in each series at which the observer's judgment changed from "high" to "low", or "low" to "high" was taken as the tone which, when loud, was perceived as equal to the standard 128 v.d. when faint. Many of the observers' judgments changed several times through a range of several vibrations, and in these cases the mean between the highest and the lowest change of judgments was used. As the range of uncertain judgments varied considerably among the different observers, that, as well as the amount of the error, is stated in the tables.

The results of this series of tests are shown in Table Ia. The average error made in comparing the loud sounds with the faint ones, and the average range through which the judgments were not constant or certain are indicated for each observer. When the faint sound (128 v.d.) was given first, a loud sound actually several vibrations higher was selected as of the same pitch. When the loud sound was given first, in five cases the average error was zero, and in one case a sound 1 v.d. lower than the standard was selected; in the case of the thirteen other observers the error was in the same direction as before. In both series, the average of the errors is 6 v.d. Most of the observers found it more difficult to judge when the faint sound was second than when it was first, and there is greater variation among the judgments.

TABLE I.

(a)

	Intensity Equal		Intensity Different			
			Standard first		Standard second	
Obs.	Error	(Range)	Error	(Range)	Error	(Range)
Mc.G	0	(3)	3	(9)	2	(7)
Go.	1	(3)	2	(6)	0	(2)
Ch.			6	(0)	7	(0)
Cu.	2	(5)	6	(5)	9	(8)
Bo.			9	(1)	7	(5)
Ar.			14	(4)	9	(2)
Ge.			13	(4)	11	(5)
On.			7	(12)	29	(10)
Va.			5	(3)	8	(3)
Th.			3	(1)	0	(2)
Sa.			8	(1)	6	(1)
Le.			1	(4)	1	(2)
Ba.			13	(8)	18	(8)
Pi.	0	(2)	4	(2)	2	(3)
Ma.	0	(2)	2	(0)	0	(1)
Gr.	0	(1)	4	(2)	0	(5)
So.	1	(3)	13	(7)	9	(5)
Dm.	0	(5)	8	(3)	1	(5)
Li.	0	(1)	5	(1)	0	(1)
S.			8	(0)	3	(0)

(b)

Li.	0	(3)	1	(7)	0	(1.5)
Cu.	0	(4)	3	(6)	4	(8)
Ma.	0	(0)	4	(0)	2	(0)
Mc.G.	0	(0)	4	(2)	2	(3)
Ge.	2	(2)	7	(0)	7	(0)
So.			9	(1)	9	(1)

(c)

Cu.	5
Ge.	1.5
Ma.	2
So.	3

There seemed to be a possibility that the disturbance caused by having the loud sound close to the ear affected the judgment of its pitch, and for that reason further tests under other conditions were given to several observers. The same forks were used, but a glass funnel was held to the ear to prevent any effect on the sound that might result from the shape of the ear lobe. The results are shown in Section b. While the error in judgment was considerably reduced for several observers, it was still in the same direction as before.

Further tests were given, using a resonator with the forks, so placed that the sound was in the median plane instead of close to

one ear. The method was changed in this series. Pairs of forks were selected of different intervals from .5 v.d. to 8 v.d. according to the judgments of the observer. Small intervals were used first, and these were increased until the judgments were given correctly and with certainty. In these tests, the observer knew that the same pair of forks was being used for several times in succession, the weak one being sometimes first and sometimes second; in this way he had an opportunity to verify or contradict a judgment in the following one. He did not know what forks were used, nor the results of the former experiments. The interval at which the judgments were "equal" or were about equally divided between "high" and "low", was taken as the amount of the observer's error. This varied from 1.5 to 5 v.d., and was in the same direction as the previous results. Further, to test the fact of the existence of the illusion, the same fork was sounded strong and weak; and also the pairs of forks used before were reversed, the lower being sounded louder. With the combinations reversed, the judgments were always correct, even with as small an interval as .5 v.d., indicating an apparent increase of the interval; when the same fork was used for the two sounds, the louder was judged to be low, except that one observer, Ma., in the case of one fork, called the sounds equal in pitch. Ma. also reported an apparent rise in the pitch of the loud fork as it was taken from the resonator.

These tests show that the illusion of the earlier tests was not due to a disturbance at the ear. A repetition of the test with one observer, Cu., in which the last method was used, but without the resonator, shows an illusion of 6 v.d., about the same as that in the first series of tests.

Both experienced and inexperienced observers were used, some with special ability and some without. There seems to be no correlation between the amount of the error and either practice or musical ability. Their only effect is to make the judgments more constant.

The illusion of pitch which is due to strength of tone is then clearly established for relatively low tones and found to be different from that of tones of the middle range. But what is the situation for high tones? Preliminary tests were made of eleven observers with 512 v.d. forks. They show, on the part of six observers, a decided illusion in the same direction as with 128 v.d. forks; one, Ma. (as in the 128 v.d. test) reported a change in pitch in the

same direction when the fork was brought to the resonator and removed from it. With three others there appears to be no illusion, and with one it is reversed, the strong tone being called higher. Similar tests on thirteen observers at 1024 v.d. show the same direction of the illusion as low tones in one case, a reversal of it in three cases, and no illusion in the other nine cases.

The conclusions of this investigation may therefore be summed up as follows:

(1). With forks of 128 v.d., a difference of intensity causes an illusion in pitch which is constant in direction, though variable in amount, a louder sound being judged lower than a faint one of the same pitch. The magnitude of the error seems to increase with increasing strength of tone, the average illusion for the differences in strength here used being about 6 v.d. Many observers show considerable confusion on account of the difficulty of comparing tones of different intensity. There is no constant relation between this confusion and the amount of the illusion. Musical ability, and experience seem to lessen the confusion, but not the amount of the error.

(2). At 512 v.d. the tendencies seem to be approximately as Stewart found them at 435 v.d., with a somewhat stronger tendency to judge the loud tone the lower. At 1024 v.d. the effect of intensity difference seems to be less disturbing than in lower tones.

(3). In general, difference in the intensity of tone is always a disturbing factor in pitch: the illusion is strong and constant in direction for relatively low tones; with rising pitch, it decreases and may vary in direction.

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THE MEASUREMENT OF TIME-SENSE AS AN ELEMENT IN THE SENSE OF RHYTHM

BY

FELIX BRUENE ROSS

The object of this series of experiments was to make an accurate measurement of the time sense as an element in the sense of rhythm, primarily for the purpose of standardizing a test for the measurement of individual differences.

It was necessary to procure some means of producing regularly recurrent sound stimuli and of varying the interval under control. Hitherto kymographs have been mostly used for the running of some sort of "time-sense apparatus", such *e.g.*, as that of Meumann. But it can be easily shown that a clock-work is not sufficiently accurate for the finer measurements of this kind.

We are fortunate in having in the laboratory a synchronous motor which very well fills the needs of this experiment. This motor has been described by Lorenz (1) and Seashore (2).

Fig. 1. The horizontal type of synchronous motor with rhythm attachment.

Fig. 1. shows the motor with the accessaries as here used. The top wheel is a balance wheel set on the axle of the motor proper. The pointer attached to this wheel makes contact with the knife-edge on the long movable and insulated arm. This arm may be set by hand for any point on the scale. To secure accuracy the two clamps on the scale plate are used as stops.

A telephone receiver was placed in the circuit of the above make-and-break contrivance in circuit with a battery. The momentary make-and-break of the current produced a distinctly audible and clear click which was used as the stimulus.

The balance wheel carrying the contact arm revolved clockwise at the rate of one revolution per second. If the long arm were held stationary the time interval would be constant,—one second. To change the interval it was only necessary to swing the arm through the required distance as indicated on the scale.

The motor with all the above mentioned accessaries was tested for accuracy by the spark method of recording and it was found that the limit of accuracy for the apparatus as thus operated is .0008 seconds.

The motor was located in a distant room and the telephone receiver connected with it suspended eight feet from the floor in the center of the room in which the measurement was made. A few preliminary trials were given in order that the observer, or observers, as the case might be, might have a clear understanding of the nature of the experiment. This done, a signal was given and the test began. A single test consisted in sounding ten clicks in succession with the understanding that the first five marked equal intervals but that in the last five there would be one short interval: and it was the task of the observer to detect this one. Seven different steps of change were used in successive groups of trials: namely, .02, .03, .05, .08, .12, .17 and .23 seconds. Twenty tests were given on each increment beginning with the largest and taking them in order. The right and wrong cases were counted and the records checked accordingly. The amount of deviation which would yield 75 per cent. correct judgments was computed, using only records between 65 and 90 per cent. right judgments. The average of the thresholds thus computed was taken for as many of the above steps as yielded records within the limits of 65 and 90 per cent.

The results here reported are based on four group measurements and a series of individual measurements. The group measurements were made in a large room into which noises from the halls and streets penetrated freely and proved a disturbing element. In the first three group measurements such disturbances as are characteristic of a large group were present. But in taking the fourth group measurement these objectionable features were eliminated by limiting the observers to a small number in this group. All individual measurements were taken in the dark-and-quiet room of the laboratory.

The results of the first group measurement taken on 200 observers, mostly sophomores, in two divisions of 100 each, are shown in Fig. 2. Those of the second and third, which were taken on 256

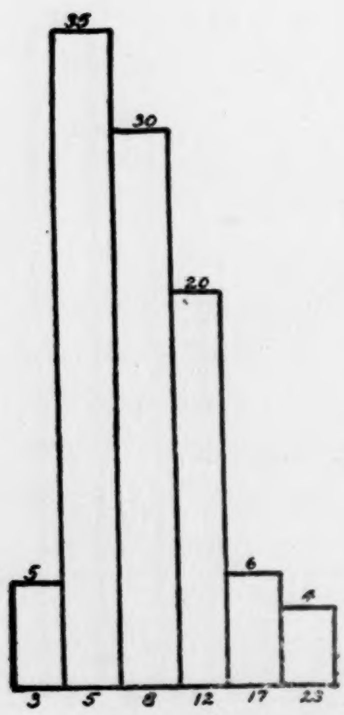

Fig. 2.

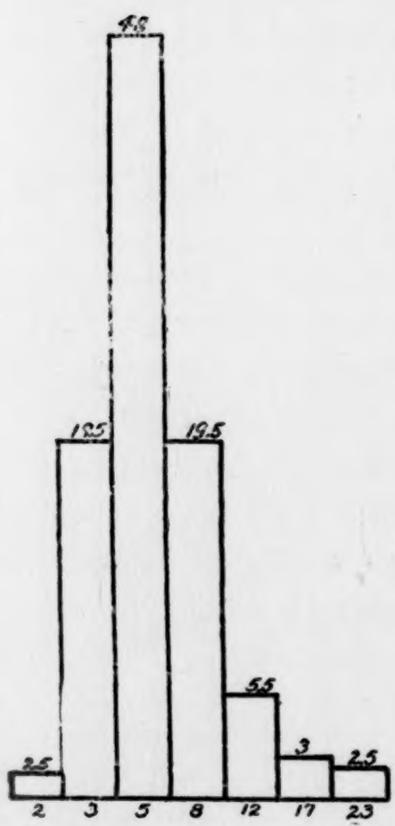

Fig. 3.

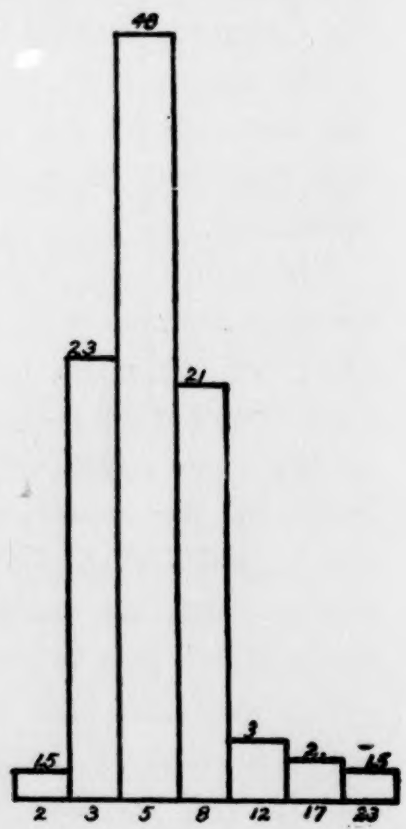

Fig. 4.

observers each, mostly sophomores, in two divisions each, with approximately an equal number in each division, are shown in Figs. 3 and 4 respectively. The distributions for these large groups of one hundred or more are all similar. This would indicate that this distribution is characteristic for different groups and for the same group at different times.

The third group measurement is a repetition of the second taken on the same persons as accurately as possible, and by the same method and under similar conditions, except those of practice, the object being to determine to what extent the distribution for a group is stable—the coefficient of correlation was computed and was found to be .69, P. E. r .027. This is a fairly satisfactory correlation considering that there are factors in the test not yet under control.

In order to determine the effect of the disturbances due to large groups, the same test was made on a group of thirteen in a small and relatively quiet class room. The result is shown in Fig. 5a.

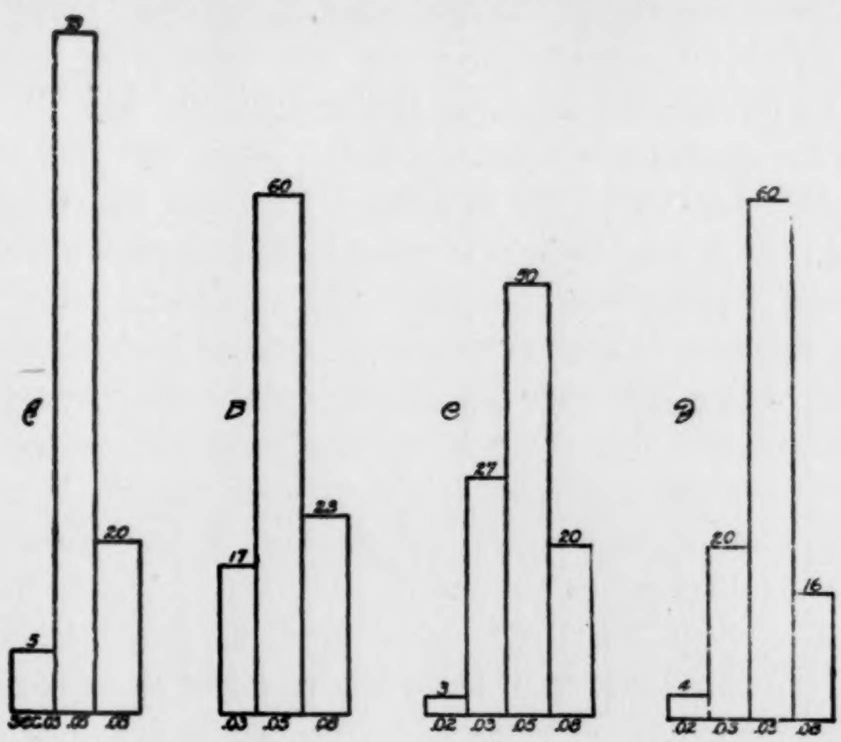

Fig. 5a. Fig. 5b. Fig. 5c. Fig. 5d.

Here the mode is the same (.05) as before but the extremes are eliminated. This group was an advanced class made up mostly of graduate students. The absence of poor records must therefore be attributed to two factors, namely, adaptation and skill in observing as a result of laboratory training and the more favorable conditions due to the smallness of the group. This points to the conclusion that in a large class of inexperienced observers most of the records poorer than .08 second may be due to lack of adaptation to the test or to disturbances in the room. The fine records on .02 second in a few cases of a large group may be due to the fact that in the larger group there are more chances of finding cases of exceptional ability.

To determine whether or not the distribution would be altered under even more rigorous and favorable conditions the same 13 observers were taken individually into the dark-and-quiet room where the test was made with most rigorous experimental control of the environment. The result is shown in Fig. 5b. which takes the same general form as 5a. The average is practically the same, .055 for the group and .054 for the individual tests. The Pearson coefficient of correlation between their individual records and those made in the group is .85, P. E. r .058, which would seem to indicate that both measurements are reliable.

To determine further the validity of the large group measurements 50 from the 200 in the first large group were taken for very careful individual measurements in the light and sound-proof room. The distribution for these is shown in Fig. 5c. The average threshold for these same observers in the group test was .07 second whereas their average in the individual tests was .05 second. This proves that the individual test is more reliable than the large group test because it gives finer records. The coefficient correlation between the two sets of measurements is however low, being only .23, P. E. r .07; that means that not all cases show improvement in the individual test and that there is relative instability in the records. That this instability is greater for the poorer records is proved by the fact that the average record for the best 20 in the above group of 50 was .054 second in the group and .047 second in the individual tests—a very small difference—and the correlation of the group and the individual records for these 20 best cases is .50 P. E. r .10 or more than twice as great as for the whole group of 50 cases. The significant thing in this is that it proves that ordinarily we may trust the finer records, whether in group or individual tests; the uncertainty is largely in the apparently poor records. This view is further supported by the distribution in that the better half of the surfaces of frequency is relatively stable for all the cases here considered.

It is still further supported by the results obtained from taking individual measurements of the 40 whose records were the very poorest in the last large group measurement. These results are shown in Fig. 5 d. The average threshold being .055, whereas in the group test, for the same 40 observers, it is .106. The correlation between the two sets of measurements is only .24, P. E. r, .085.

It is therefore probable that the poor records in the large group measurement are due to some cause other than a lack of appreciation of time on the part of the individual, and that this cause lies in the fact that with the large group there are within the group and peculiar to it sources of error that are not present in a small group or with the individual alone.

To determine the constancy of the record of an individual and the effect of practice, eight observers were given eight successive daily individual tests. After the first day in this series, only the two steps which were closest to the observers threshold of the day previous were used and in each of these 50 trials were made. The results are shown in Table I.

TABLE I. The effect of practice

Obs.	1	2	3	4	5	6	7	8	Ave.	% Gain
1	.051	.039	.039	.027	.030	.023	.024	.028	.032	45
2	.060	.063	.055	.032	.032	.032	.030	.030	.040	50
3	.052	.046	.042	.040	.040	.035	.037	.036	.041	29
4	.045	.047	.039	.032	.032	.028	.028	.025	.031	45
5	.039	.038	.039	.030	.040	.038	.039	.035	.037	11
6	.073	.064	.057	.050	.044	.043	.045	.044	.052	40
7	.040	.037	.040	.040	.034	.034	.039	.030	.037	25
8	.040	.038	.038	.037	.034	.034	.030	.029	.034	27
Ave.	.050	.047	.044	.036	.036	.033	.034	.031	.038	34

These eight observers were of approximately average ability, as may be seen by comparing the records for the first day with the form of distribution in the norms established for groups as shown above. In every case there was improvement as a result of the practice. The amount of gain for each individual, the average gain for the group for each successive day, and the daily fluctuations in the individual records may be seen in the Table.

This unquestionable gain with practice proves that the test was not elemental. The introspections also confirm this view, showing that the real difficulty is in the point of view, the method of imagery, the strain of attention, the method of counting, or some other such feature not essential to the simple experience of time sense. It is factors of this sort that form the basis for improvement by practice. To the extent that they are present this measurement, as a psychophysic test, is vitiated. Our aim is to make the test elemental, so that it shall be adapted for use in the measurements of individual difference. To this end the records and the observations in this series of measurements show that we must eliminate group distur-

bances and simplify the required mental attitude and strain to such extent that a test approximating the physiological limit shall be attainable without practice. As has been demonstrated in the above experiments, the first requirement may be complied with by making the test by individuals or in small groups in favorable surroundings. The second requirement must be met chiefly by simplifying the attitude of observation and the method of replying or recording. To do this our next step will be to take an individual test in which the interval-making sounds are not broken up into sets of ten, as here, or any other small group, but continued for a period of about five minutes each. This will secure better adaptation and will do away with the need of counting. Then the simplest sort of signal such as a motion of the hand may be used to designate those intervals which are recognized as short. Such simplifying of procedure should make this test approximately elemental. Norms are now being worked out on that basis.

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SOME STANDARDIZING TESTS ON STERN'S TONE VARIATOR[1]

BY

REUL H. SYLVESTER

The tone variator is a tone producing instrument of the blown bottle type. It was designed especially for use in gradual changes in pitch, and is made in different sizes. The one here used is of medium size and range, the bottle being 8 cm. in diameter and 17 cm. in length, with a mouth 1 cm. in diameter, giving a pitch of 200 v.d. to 400 v.d. The bottom of the bottle is a piston which moves upward and downward under the control of a gear and parobolic curve mechanism and can be set for different pitches according to a graduated scale. The tone is produced by a stream of air directed across the mouth of the bottle through a tube set at such an angle as to produce a whistling sound. The instrument is fully described by Stern,[2] who calls attention to some possible limitations to its accuracy, but offers no measure of reliability.

The experiments here reported were tests of the reliability of the tone variator, the intention being to use it, if it proved reliable, in studies of pitch discrimination. The constant air pressure was suppled by Whipple tanks.[3] Their pressure was regulated by weights and by a screw clamp applied to the soft rubber tube leading from the tanks to the variator. A water manometer, attached to this tube, gave the pressure readings. Where used in this report, these readings have been translated into grams per square centimeter. For reasons which will be given later, settings of the variator were always made in such a way that the movement of the piston was downward to the desired point, and by means of a special attachment the setting shaft control was firmly clamped each time. The pitch of the tone emitted was read by means of the

[1] These measurements were made in 1909. A synopsis of a report of them is given in the Proceedings of the Iowa Academy of Science for 1910, p. 195.

[2] Der Tonvariator. *Zeitschr. Physiol. u. Psychol. d. Sinn.*, 1902, XXX; 422-432.

[3] Whipple, G. M. A compressed Air Device of Acoustic and General Laboratory Work. Amer. Jour. of Psych. 1902, XIV p. 107 ff.

tonoscope.[4] In order to eliminate the interference at the mouth of the bottle by the funnel of the manometric capsule on the tonoscope, the electrical form of transmission, the phonette,[5] was used. Clamped with its lower edge in contact with the shoulder of the variator body, the phonette receiver faced the mouth of the variator in a position at right angles to the direction of the stream of air.

This apparatus, though complicated, was easily managed by one operator. With the tonoscope running, batteries turned on and the variator set at the desired pitch, the Whipple tanks were started and the taking of records began. After twenty readings had been recorded, the variator was set at another pitch and the record taking continued. With the exception of the resetting of the variator and the shifting of the tank weights, the apparatus ran continuously during two hours of experimentation. The pitch was read in tenths of a vibration, with accuracy. Every effort was made to secure the most favorable conditions for accuracy. It is not probable that the variator would be operated with such care in ordinary use.

Preliminary tests indicated that besides the changes in pitch which can be made in the intended way by moving the piston, variations of more than 15 v.d. may be made by varying the pressure of the air stream, and 10 v.d. variations may be made by changing the position of the mouth-piece. The results of experiments planned for a study of these two features are given in Table I, which shows the pitch variations for four scale readings, three pressures, and three mouth-piece positions. The limits of pressure and mouth-piece gap were determined by preliminary tests in which it was found that the extremes used here are the maximum ones at which the variator gives a fairly steady tone. Each pitch record given in this table is the average of between 40 and 50 readings. To this rather small amount of data, statistical checks of variability cannot be closely applied, but the mean variations given in the table are of considerable value. Their irregularity is largely due to the varying behavior of air currents in different settings, pressure, and mouth-piece gap combinations, some combinations causing more fluctuations than others. No records are given for the 400 v.d. setting because with the mouth-piece in the "flush" position there is a hissing sound which partly obscures the real tone and makes

[4] Seashore, The Tonoscope. (In this Volume.)
[5] General Acousticon Company, New York.

its pitch fluctuate, and because when used at low air stream pressure and wide gap mouth-piece position no sound is produced.

TABLE I. Tonoscope readings for the various combinations of pressures, settings, and mouth-piece gaps.

Pressure	Setting	3.8 mm. gap Pitch	m.v.	1.9 mm. gap Pitch	m.v.	"flush" Pitch	m.v.	Average Pitch	m.v.
2 gm.	200	187.9	.1	192.2	.2	193.8	.5		
	250	239.6	.1	242.4	.3	244.3	.3		
	300	288.0	.3	290.1	.3	292.4	.7		
	350	334.4	.4	340.9	.4	341.1	.7		
Average		262	.2	266	.3	268	.5	265	.3
3 gm.	200	191.7	.1	197.2	.3	202.1	1.2		
	250	243.5	.1	248.1	.3	251.1	.5		
	300	293.4	.3	296.9	.2	299.2	.6		
	350	345.4	.3	349.1	.6	345.3	.5		
Average		268	.2	273	.4	274	.7	272	.4
4 gm.	200	192.3	.1	201.3	.5	209.0	1.3		
	250	244.2	.1	250.9	.3	255.9	.4		
	300	294.5	.1	298.3	.4	302.4	.6		
	350	345.3	.2	352.0	.1	350.5	.8		
Average		269	.1	276	.3	279	.8	275	.5
Grand average		266	.2	272	.4	274	.7		

Inspection of this table indicates that the nine combinations of different pressures and mouth-piece positions, the 4 gm. pressure and a 1.9 mm. mouth-piece gap gives tones with pitches that are nearest the setting scale readings. The pitches, 201.3, 250.9, 298.3, and 352.0 vary from their settings of 200, 250, 300, and 350 an average of less than 1.5 v.d., while the averages of the other eight combinations vary from 2.2 v.d. to 10.1 v.d. from their settings. Therefore the intention of the designer of this instrument must have been that it be used with somewhere near 4 gm. pressure and a 1.9 mm. mouth-piece gap. If however, one disregards closeness to the setting scale pitch, he finds a lower pressure and a wider mouth-piece gap to be more desirable. With this 4 gm. pressure and 1.9 mm. gap the tone sounds forced and there is a prominent hissing in it, especially at the higher pitch settings. The tonoscope shows that under these conditions the pitch continually fluctuates. Obviously this is responsible for some of the larger m.v.'s in the table, it being impossible to catch the readings on the same phase of the fluctuations. When the mouth-piece is in the widest gap position, the pitch fluctuates least, the m.v.'s are least and to the ear the tones are the most clear, smooth and pure. Therefore this wide gap is more reliable than the 1.9 mm. gap. Evidence in favor of the lower pres-

sures is not found in the m.v.'s but to the ear the tones are clearer and purer.

Careful study of the table reveals no important tendencies that are obscured by condensing the data into averages. Hence the use of averages in the following consideration. They show first that an increase of pressure causes a rise in pitch. This is more marked in the lighter pressures, an average rise of 7 v.d. (265 v.d. to 272 v.d.) resulting from changing the pressure from 2 gm. to 3 gm. while, from changing the pressure from 3 gm. to 4 gm. the rise in pitch is but 3 v.d. (272 v.d. to 275 v.d). Averages in the table also show that when the mouth-piece is at the widest gap position at which a sound can be produced, the tone is comparatively low and changing the mouth-piece toward the "flush" position raises the pitch. This is more marked in the wider gap positions, an average rise of 8 v.d. (266 v.d. to 272 v.d.) resulting from changing it from a 3.8 mm. gap to a 1.9 mm. gap, while for changing it from a 1.9 mm. gap to "flush", the rise in pitch is but 2 v.d. (272 v.d. to 274 v.d.).

Mention has been made of the varying behavior of air currents at different settings. For this size bottle there seems to be the least disturbance at the 250 v.d. setting. Again using averages (and these averages as well do not smooth out or bury any important tendencies) one finds that for the 250 v.d. setting, the average of all records is 247 v.d. with a m.v. of .3 v.d. and that for the settings 200, 300, and 350, the averages of all records are 196 (m.v. .5), 295 (m.v. .4) and 345 (m.v. .5) respectively. In other words the pitch varies less from the scale reading than do the pitches produced at the other settings, and the mean variation is less. Hence the conclusion that a variator of this size is most reliable with the length of air column which gives a pitch near 250 v.d.

So much for conclusions from quantitative results. Certain general observations should be added. The manner in which the instrument's mouth-piece is attached is very unsatisfactory. It should be absolutely firm, accurately adjustable, and provided with a setting scale showing the width of the gap, or in some other way indicating the position. The piston sometimes settles downward. For the above experiments it was necessary to fit it with a clamping device. The securing of a steady stream of air is a serious problem. As shown in the tables, a slight pressure change causes a consider-

able change in pitch. The Whipple tanks are perhaps the best contrivance available, but they demand close care and are at best an occasional hindrance to the experimenting.

From the results of these tests, it is obvious that the tone variator can be relied upon only as an instrument of approximate pitch and relative intervals, and that it is not suitable for use in research which requires accuracy in pitch. The fact that its pitch varies with pressure and mouth-piece position, and probably with temperature, humidity, and other conditions makes an absolute reading impossible. A variation in one of these conditions would throw any scale out of proportion. But even if it cannot be relied upon for careful quantitative work, it is a desirable piece of apparatus for the psychological laboratory. Its loud clear tone and its ready manner of changing pitch make it especially valuable for general class experiments and demonstrations of consonance, beats and combination tones.